Year 11 Mathematics Task 1   Thursday 22nd March 2007

Instructions: a) Write all your answers on your own paper.
                b) Show all necessary working.
                c) Marks may be deducted for careless or badly arranged work.
Time Allowed: 90 minutes

Question 1 *(21 marks)*   *Marks*

a) Calculate $\sqrt{\dfrac{36.41-19.57}{23.62-11.39}}$ correct to four significant figures   2

b) Classify each of these real numbers as rational or irrational;

    (i) π *1*

    (ii) $\sqrt{\dfrac{4}{11}}$ *1*

    (iii) $0.12\dot{5}$ *1*

    (iv) $\sqrt[3]{64}$ *1*

c) Write 0.0000000607 in scientific notation   2

d) Simplify;

    (i) $16^{-\frac{3}{4}}$ 2

    (ii) $12a^7 \times 6a^9 \div 9a^5$ 2

e) Find the exact value of $\dfrac{x^4 z}{y^4}$ where $x=\left(\dfrac{2}{3}\right)^2, y=\left(\dfrac{4}{3}\right)^4, z=\left(\dfrac{8}{3}\right)^3$ 3

f) Express the following as a fraction on its simplest form;

    (i) $0.\dot{2}\dot{1}$ 3

    (ii) $0.3\dot{4}\dot{5}$ 3

Question 2 *(21 marks)*

a) Simplify;

    (i) $6\sqrt{2} \times 5\sqrt{7}$

                      1

    (ii) $\sqrt{18} + \sqrt{128} - \sqrt{242}$

                      2

    (iii) $\dfrac{5\sqrt{7} \times \sqrt{3}}{\sqrt{28}}$

                      2

b) Expand and simplify;

    (i) $\sqrt{6}\left(\sqrt{3} - 3\right)$

                      2

    (ii) $\left(2\sqrt{6} + 3\right)\left(2\sqrt{6} - 3\right)$

                      3

    (iii) $\left(3\sqrt{2} - \sqrt{5}\right)^2$

                      3

c) Express with a rational denominator;

    (i) $\dfrac{3}{5\sqrt{2}}$

                      2

    (ii) $\dfrac{\sqrt{7}}{3 - \sqrt{7}}$

                      3

d) Show that $\dfrac{1}{9 - 4\sqrt{5}} - \dfrac{4}{2 + \sqrt{5}}$ is a rational number.

                      3

Question 3 *(21 marks)*

a) Expand and simplify;

    (i) $4(x + 3) - 5(2x - 3)$

                      2

    (ii) $(2 + 3a)^2$

                      2

    (iii) $\left(m^2 - 4m + 16\right)(m + 4)$

                      2

    (iv) $(x + 1)^2 (x - 1)^2$

                      3

b) Factorise;

    (i) $36 - 25k^2$

                      2

    (ii) $x^2 + 5x - 36$

                      2

    (iii) $a^3 + 8$

                      2

    (iv) $4m^2 + 4m - 15$

                      3

    (v) $x^3 + 9x^2 - 4x - 36$

                      3

Question 4 *(24 marks)*

a) Factorise and simplify;

   (i) $\dfrac{2p+2q}{p+q}$ *2*

   (ii) $\dfrac{ac+ad+bc+bd}{a^2+ab}$ *3*

b) Simplify;

   (i) $\dfrac{x+1}{2}+\dfrac{x+2}{3}$ *2*

   (ii) $\dfrac{x}{x-1}-\dfrac{x}{x+1}$ *3*

   (iii) $\dfrac{1}{x^3-1}+\dfrac{x+1}{x^2+x+1}$ *3*

c) Simplify;

   (i) $\dfrac{3a^2b}{4b^3c}\times\dfrac{2c^2}{8a^3}\div\dfrac{6ac}{16b^2}$ *2*

   (ii) $\dfrac{x^2+x-2}{x+2}\times\dfrac{x^2-3x}{x^2-4x+3}$ *3*

   (iii) $\dfrac{m^3+1}{6m^2}\times\dfrac{3m}{m^2+m}$ *3*

   (iv) $\dfrac{ab-2b^2}{6a^2b}\div\dfrac{a^2-4ab+4b^2}{3a}$ *3*

Question 5 *(20 marks)*

a) Solve the following equations, writing solutions in simplest exact form;

   (i) $x+\dfrac{x}{3}=7$ *2*

   (ii) $2m(2m+9)=0$ *2*

   (iii) $3a^2+2a-8=0$ *3*

   (iv) $2h^2+8h+3=0$ *3*

   (v) $\dfrac{2}{x+3}+\dfrac{x+3}{2}=\dfrac{10}{3}$ *3*

   (vi) $|x+2|=2x-5$ *4*

b) Solve $x^2-6x+3=0$ by completing the square.   *3*

Question 6 *(18 marks)*

a) Solve each inequation, and graph your solution on the number line;

    (i) $3 - 2x < 7$ *2*

    (ii) $-7 \leq 5x + 3 \leq 3$ *3*

    (iii) $|3x - 5| > 4$ *4*

b) Solve the following simultaneous equations;

    (i) $2x + y = 9$ *3*
                $x + y = 5$

    (ii) $2x + 3y = 28$ *3*
                $3x + 2y = 27$

    (ii) $x^2 + y^2 = 146$ *3*
                $x + 2y = 21$

$$\frac{36.41 - 19.57}{23.62 - 11.39} = 1.173 \quad ②$$

a)(i) irrational ①
(ii) irrational ①
(iii) rational ①
(iv) rational ①

b) $0.000\,000\,060\,7 = 6.07 \times 10^{-8}$ ②

(ii) $16^{-\frac{3}{4}} = \frac{1}{(\sqrt[4]{16})^3}$
$= \frac{1}{2^3}$
$= \frac{1}{8}$ ②

(iii) $12a^7 \times 6a^9 \div 9a^5$
$= 8a^{11}$ ②

e) $\frac{x^4 z}{y^4} = \left(\frac{2}{3}\right)^8 \times \left(\frac{8}{3}\right)^3 \times \left(\frac{3}{4}\right)^{16}$
$= \frac{2^8}{3^8} \times \frac{2^9}{3^3} \times \frac{3^{16}}{2^{32}}$
$= \frac{3^5}{2^{15}} = \frac{243}{32768}$ ③

d) $0.2\dot{1} = \frac{21}{99} = \frac{7}{33}$ ③

(ii) $0.3\dot{4}\dot{5} = \frac{345-3}{990} = \frac{342}{990} = \frac{19}{55}$ ③

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a)(i) $6\sqrt{2} \times 5\sqrt{7} = 30\sqrt{14}$ ①

(ii) $\sqrt{18} + \sqrt{128} - \sqrt{242}$
$= 3\sqrt{2} + 8\sqrt{2} - 11\sqrt{2}$
$= 0$ ②

(iii) $\frac{5\sqrt{7} \times \sqrt{3}}{\sqrt{21}} = \frac{5\sqrt{3}}{2}$ ②

b)(i) $\sqrt{6}(\sqrt{3}-3) = \sqrt{18} - 3\sqrt{6}$
$= 3\sqrt{2} - 3\sqrt{6}$ ②

(ii) $(2\sqrt{6}+3)(2\sqrt{6}-3)$
$= 24 - 9$
$= 15$ ③

(iii) $(3\sqrt{2}-\sqrt{5})^2$
$= 18 - 6\sqrt{10} + 5$
$= 23 - 6\sqrt{10}$ ③

c)(i) $\frac{3}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{10}$ ②

(ii) $\frac{\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{3\sqrt{7}+7}{9-7} = \frac{3\sqrt{7}+7}{2}$ ③

d) $\frac{1}{9-4\sqrt{5}} = \frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}} = \frac{9+4\sqrt{5}}{81-80} = 9+4\sqrt{5}$ ③

(ii) $\frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}} = \frac{9+4\sqrt{5}}{4-5} \times \frac{2+\sqrt{5}}{2-\sqrt{5}}$

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a)(i) $4(x+3) - 3(2x-5)$
$= 4x+12 - 10x+15$
$= -6x + 27$ ②

(ii) $(2+3a)^2$
$= 4 + 12a + 9a^2$ ②

(iii) $(m^2 - 4m + 16)(m+4)$
$= m^3 + 4^3$
$= m^3 + 64$ ②

b)(i) $(x+1)^2(x-1)^2$
$= (x^2-1)^2$
$= x^4 - 2x^2 + 1$ ③

(ii) $x^2 + 5x - 36$
$= (x+9)(x-4)$ ②

(iii) $36 - 25k^2$
$= (6+5k)(6-5k)$ ②

(iv) $4m^2 + 4m - 15$
$= 4m^2 + 10m - 6m - 15$
$= 2m(2m+5) - 3(2m+5)$
$= (2m+5)(2m-3)$ ③

(v) $x^3 + 9x^2 - 4x - 36$
$= x^2(x+9) - 4(x+9)$
$= (x+9)(x^2-4)$
$= (x+9)(x+2)(x-2)$ ③

c)(i) $\frac{3a^2b}{4b^3c} \times \frac{2c^2}{8a^3} \div \frac{6ac}{16b^2}$
$= \frac{3a^2b}{4b^3c} \times \frac{2c^2}{8a^3} \times \frac{16b^2}{6ac}$
$= \frac{1}{2b^2a^2}$

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a)(i) $\frac{x}{x-1} - \frac{x}{x+1} = \frac{x(x+1)-x(x-1)}{(x-1)(x+1)}$
$= \frac{x^2+x - x^2+x}{(x-1)(x+1)} = \frac{2x}{(x-1)(x+1)}$

(ii) $\frac{x}{x-1} - \frac{x}{x+1} = \frac{x(x+1)-x(x-1)}{x^2-1}$
$= \frac{2x}{(x-1)(x+1)}$ ②

(iii) $\frac{1}{x^2-1} + \frac{x+1}{x^2+x+1} = \frac{1+(x+1)(x-1)}{(x-1)(x^2+x+1)}$
$= \frac{x^2}{(x-1)(x^2+x+1)}$ ③

(ii) $\frac{x^2+x-2}{x^2+2x-3} \times \frac{x^2-3x}{x^2-4x+3}$
$= \frac{(x+2)(x-1)}{(x+3)(x-1)} \times \frac{x(x-3)}{(x-3)(x-1)}$
$= \frac{x}{2a^2}$

(iii) $\frac{x^2+x-2}{x+2} = \frac{(x+2)(x-1)}{(x+2)} = x$

(iv) $\frac{6m^2}{m^2-m+1} \times \frac{m^3+1}{m^2+m}$
$= \frac{6m^2}{m^2-m+1} \times \frac{(m+1)(m^2-m+1)}{m(m+1)}$
$= \frac{6m}{m^2+m}$

(ii) $\frac{3a^2b}{4b^3c} \div \frac{6ac}{3a}$

(iii) $ab - 2b^2 \div \frac{a^2-4ab+4b^2}{3a}$
$= \frac{b(a-2b)}{6a^2b} \times \frac{3a}{(a-2b)^2}$
$= \frac{1}{2a(a-2b)}$ ③

a)(i) $\frac{2p+2q}{p+q} = \frac{2(p+q)}{(p+q)} = 2$ ②

(ii) $\frac{ac+ad+bc+bd}{a^2+ab}$
$= \frac{(a+b)(c+d)}{a(a+b)}$
$= \frac{c+d}{a}$ ③

Question 5 — (20)

a)(i) $x + \dfrac{x}{3} = 7$

$3x + x = 21$

$4x = 21$

$x = \dfrac{21}{4}$ ②

(ii) $2m(2m+9) = 0$

$m = 0$ or $m = \dfrac{-9}{2}$ ②

(iii) $3a^2 + 2a - 8 = 0$

$3a^2 + 6a - 4a - 8 = 0$

$3a(a+2) - 4(a+2) = 0$

$(a+2)(3a-4) = 0$

$a = -2$ or $a = \dfrac{4}{3}$ ②

(iv) $2h^2 + 8h + 3 = 0$

$h = \dfrac{-8 \pm \sqrt{64-24}}{4}$

$= \dfrac{-8 \pm \sqrt{40}}{4}$

$= \dfrac{-8 \pm 2\sqrt{10}}{4}$

$= \dfrac{-4 \pm \sqrt{10}}{2}$ ③

(v) $\dfrac{x}{x+3} + \dfrac{x+3}{2} = \dfrac{10}{3}$

$12 + 3(x+3)^2 = 20(x+3)$

$12 + 3x^2 + 18x + 27 = 20x + 60$

$3x^2 - 2x - 21 = 0$

$3x^2 - 9x + 7x - 21 = 0$

$3x(x-3) + 7(x-3) = 0$

$(x-3)(3x+7) = 0$

$x = 3$ or $x = \dfrac{-7}{3}$ ③

(vi) $|x+2| = 2x-5$

$x+2 = 2x-5$ or $x+2 = -(2x-5)$

$x = 7$ $x+2 = -2x+5$

        $3x = 3$

        $x = 1$ NOT SOLUTION

$\therefore x = 7$ ④

b) $x^2 - 6x + 3 = 0$

$x^2 - 6x = -3$

$x^2 - 6x + (-3)^2 = 6$

$(x-3)^2 = 6$

$x - 3 = \pm\sqrt{6}$

$x = 3 \pm \sqrt{6}$ ③

Question 6 — (18)

a)(i) $3 - 2x < 7$

$-2x < 4$

$x > -2$

②

(ii) $-7 \le 5x+3 < 3$

$-10 \le 5x < 0$
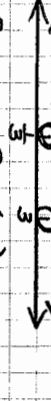
$-2 \le x < 0$
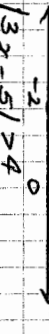
③

(iii) $|3x-5| > 4$

$3x-5 > 4$ or $-(3x-5) > 4$

$3x > 9$ $-3x+5 > 4$

$x > 3$ $-3x > -1$

        $x < \dfrac{1}{3}$

$\therefore x < \dfrac{1}{3}$ or $x > 3$

④

b)(i) $2x+y = 9$ (−)

$x + y = 5$

$\therefore x = 4, \; y = 1$ ③

(ii) $2x + 3y = 28$

$3x + 2y = 27$

$6x + 9y = 84$ (−)

$6x + 4y = 54$

$5y = 30$

$\therefore x = 5, \; y = 6$ ③

(iii) $x^2 + y^2 = 146$

$-(27-2y)^2 + y^2 = 146$

$-84y + 4y^2 + y^2 = 146$

$5y^2 - 84y + 295 = 0$

$(5y - 59)(y - 5) = 0$

$5y = 59$ or $y = 5$

$y = \dfrac{59}{5}$ or $y = 5$

$\therefore x = \dfrac{-13}{5}, \; y = \dfrac{59}{5}$ or $x = 11, \; y = 5$ ③