

Girraween High School
Mathematics Extension

Year 11
Task 1 (Half Yearly)

April, 2003
Time: 80 min

- Instructions:
1. Attempt all questions.
 2. Write your answers on your own paper.
 3. All necessary working must be shown.
 4. Marks will be deducted for careless or badly arranged work.

Question 1 (19 marks)

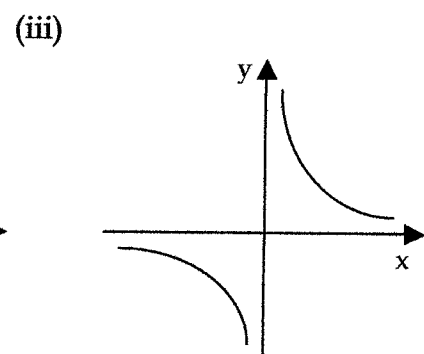
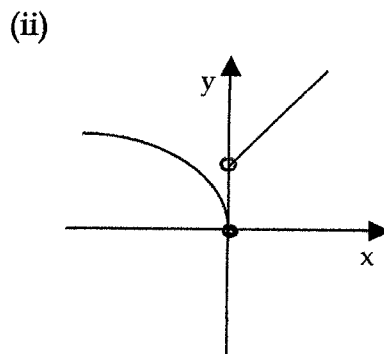
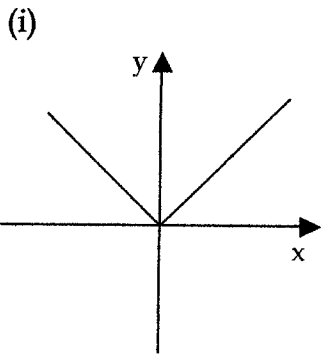
- a) Solve (i) $\frac{x-3}{2} - \frac{x+1}{4} = 5$ (ii) $\frac{3}{x^2} = 2$ 4
- b) Solve $\frac{1}{1-x} > 2$ 3
- c) Factorise $\frac{x^3-1}{x^2-1}$ 3
- d) Solve for $0^\circ \leq \theta \leq 360^\circ$:
- (i) $\sqrt{3} \sin \theta = \cos \theta$ (ii) $\sin(\theta - 30^\circ) = \frac{\sqrt{3}}{2}$ (iii) $2 \cos^2 \theta - \cos \theta = 1$ 9

Question 2 (23 marks)

- a) If $f(x) = 1 - 2x^2$, find:
- (i) $f(2)$ (ii) $f(-x)$ (iii) $f(x+1)$ 4
- b) What is the domain of each of these functions:
- (i) $y = x^2 + 2$ (ii) $y = \frac{1}{x}$ (iii) $y = \sqrt{x-1}$ 6
- (iv) $y = \sqrt{x^2 - 9}$ (v) $y = \sqrt{4 - x^2}$ (vi) $y = \frac{1}{\sqrt{1-2x}}$ 6
- c) If $f(x) = \begin{cases} x^2 & x > 0 \\ x+1 & x \leq 0 \end{cases}$
- (i) evaluate $f(-2) + f(2)$ 2
- (ii) find the domain and range of the function. 2

d) State whether these curves represent functions:

3



Question 3 (21 marks)

- a) Evaluate: (i) $10!$ (ii) $\frac{6!}{3!}$ (iii) $\frac{100!}{98!}$ 4
- b) How many ways can 4 boys and 3 girls be arranged in a line if:
 (i) there are no restrictions? 1
 (ii) the boys are together? 2
 (iii) the boys and girls alternate? 2
- c) How many ways can 5 different English books, 4 different Science books and 2 different History books be arranged on a shelf if the books in each subject are to be together? 3
- d) How many ways can the letters of the word SUCCESS be arranged if:
 (i) there are no restrictions? 2
 (ii) the C's must be together? 2
- e) There are two small rows of chairs, with 3 in the front row and 4 in the second row. How can 7 people be seated if:
 (i) 2 particular people must sit in the second row. 2
 (ii) Jan will not sit on the same row as Norman and Stan must sit in the first row. 3

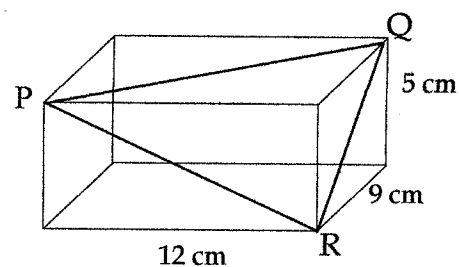
Question 4 (16 marks)

- a) A debating team of 4 students is to be selected from 10 students. How many team combinations are possible if:
 (i) there are no restrictions? 1
 (ii) Talitha must be included in the team? 2

- c) A standard pack of 52 cards has 4 suites: Hearts, Diamond, Clubs and Spades
Each suite has 13 cards: Ace, King, Queen, Jack, 10,9,8,7,6,5,4,3,2
Five cards are selected at random from a standard pack of 52 cards.
- (i) How many different selections of 5 cards are possible? 1
 - (ii) What is the probability that the 5 cards selected are hearts? 2
 - (iii) What is the probability that 3 kings are in the selected 5 cards? 3
- d) In a conference room there is a round table surrounded by 10 chairs. The group attending the conference consists of 7 women and 3 men.
- (i) How many seating arrangements are possible if there are no restrictions? 1
 - (ii) How many seating arrangements are possible if the 3 men sit together? 2
- Two of the people are elected to attend the national conference. What is the probability that they:
- (iii) sit next to each other? 2
 - (iv) sit opposite each other? 2

Question 5 (21 marks)

- a) For this rectangular prism, find the size of $\angle PQR$, correct to the nearest degree.



5

- b) A vertical tower AB stands on level ground with A being the top point. Points C and D are at ground level. The angle of elevation of A from C is 25° and the angle of elevation of A from D is 30° and $\angle CBD = 60^\circ$. Let the height of the tower AB be h .
- (i) Draw a sketch showing all this information. 2
 - (ii) Show $CB = h \cot 25^\circ$ 2
 - (iii) Show $CD^2 = h^2 \cot^2 25^\circ + h^2 \cot^2 30^\circ - 2h^2 \cot 25^\circ \cot 30^\circ \cos 60^\circ$ 3
 - (iv) Find the height h , correct to 1 decimal place, if the distance $CD=50$ m 3
- c) Prove:
- (i) $\frac{\cot \theta \cos \theta}{\cot \theta + \cos \theta} = \frac{\cos \theta}{1 + \sin \theta}$ 3
 - (ii) $\frac{\sin^2 \theta}{1 + \cos \theta} + \frac{\sin^2 \theta}{1 - \cos \theta}$ is independent of θ . 3

Solutions Maths Extension Yr 11 Task 1 2003

①

Question 1. (19)

a) i)

$$\frac{x-3}{2} - \frac{x+1}{4} = 5$$

$$2x-6 - (x+1) = 20$$

$$2x-6 - x-1 = 20$$

$$x-7 = 20$$

$$x = 27$$

②

ii) $\frac{3}{x^2} = 2$

$$x^2 = \frac{3}{2}$$

$$x = \pm \sqrt{\frac{3}{2}}$$

②

b) $\frac{1}{1-x} > 2$

Discontinuous at $x=1$

Solve $\frac{1}{1-x} = 2$

$$1 = 2-2x$$

$$2x = 1$$

$$x = \frac{1}{2}$$

Check points



$\therefore \frac{1}{2} < x < 1$

③

c) $\frac{x^3-1}{x^2-1} = \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$

$$= \frac{x^2+x+1}{x+1}$$

③

d) i) $\sqrt{3} \sin \theta = \cos \theta$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^\circ \text{ or } 210^\circ$$

③

ii) $\theta - 30^\circ = 60^\circ \text{ or } 120^\circ$

$$\therefore \theta = 90^\circ \text{ or } 150^\circ$$

\therefore Solution is 90° or 150°

③

iii) $2 \cos^2 \theta - \cos \theta = 1$

$$2 \cos^2 \theta - \cos \theta - 1 = 0$$

$$(2 \cos \theta + 1)(\cos \theta - 1) = 0$$

$$\therefore \cos \theta = -\frac{1}{2} \text{ or } 1$$

$$\therefore \theta = 120^\circ, 240^\circ, 0^\circ, 360^\circ$$

③

Question 2. (23)

a) $f(x) = 1-2x^2$

i) $f(2) = -7$

①

ii) $f(-x) = 1-2(-x)^2$

$$= 1-2x^2$$

①

iii) $f(x+1) = 1-2(x+1)^2$

$$= 1-2(x^2+2x+1)$$

$$= -2x^2-4x-1$$

②

b) i) $y = x^2 + 2$

Domain: all real x

Question 3 (21)
a) i) $10! = 3\,628\,800$

ii) $y = \frac{1}{x}$

Domain: all real $x, x \neq 0$

ii) $\frac{6!}{3!} = 6 \times 5 \times 4$

$= 120$

iii) $y = \sqrt{x-1}$

Domain: $x-1 \geq 0$

iii) $\frac{100!}{98!} = 100 \times 99$

$= 9900$

$x > 1$

②

b) i) $7! = 5040$

iv) $y = \sqrt{x^2-9}$

Domain: $x \geq 3, x \leq -3$

ii) boys together

$$4! \times 4! = 576$$

iii) boys & girls alternate

$$B \ G \ B \ G \ B \ G \ B$$

$$4! \times 3! = 144$$

v) $y = \frac{1}{\sqrt{1-2x}}$

Domain: $-2 \leq x \leq 2$

c) $5E \ 4S \ 2H$

$$(5! \times 4! \times 2!) \times 3!$$

$$= 34560$$

c) i) $f(-2) + f(2)$

$$= (-2+1) + 2^2$$

②

ii) Domain: all real x

Range: all real y

②

d) all curves represent functions

③

e) i) $(4 \times 3) \times 5!$

$$= 1440$$

ii) $\frac{6!}{3!} = 120$

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iii) $(4 \times 3) \times 5!$

$$= 1440$$

②

d) i) $9! = 362880$ (1)

ii) $7! \times 3! = 30240$ (2)

iii) $P(\text{next to each other}) = \frac{8! \times 2}{9!} = \frac{2}{9}$ (2)

or, if one person is seated the probability that the person is next to her is $\frac{2}{9}$

iv) $P(\text{opposite each other}) = \frac{1}{9}$ (2)

(some reasoning as above)

Question 5 (21)
By Pythagoras.

$P^2 = 12^2 + 5^2$ $P^2 = 12^2 + 9^2$
 $\therefore P^2 = 169$ $PQ^2 = 225$
 $QR^2 = 5^2 + 9^2$
 $QR^2 = 106$

In ΔPQR , cosine rule.

$\cos \angle PQR = \frac{PQ^2 + QR^2 - PR^2}{2 \times PQ \times QR}$
 $= \frac{225 + 106 - 169}{2 \times 13 \times \sqrt{106}}$
 $\cos \angle PQR = 0.52449 \dots$
 $\hat{=} 58^\circ$ (5)

ii) J S N
 $(4 \times 3 \times 2) \times 4!$
 $24 \times 4! = 576$
 but Jan and Norma can swap rows $\therefore 2 \times 576 = 1152$ (3)

Question 4 (16)

i) ${}^{10}C_4 = 210$ (1)

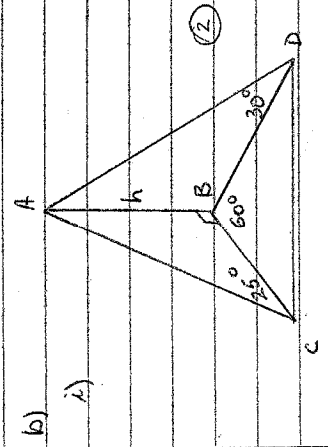
ii) T + 3 others = ${}^9C_3 = 84$ (2)

c) i) ${}^{52}C_5 = 2598960$ (1)

ii) $P(\text{Hearts}) = \frac{{}^{13}C_5}{{}^{52}C_5} = \frac{33}{66640}$
 or $\hat{=} 0.000495$ (2)

iii) $P(3K) = \frac{{}^4C_3 \times {}^48C_2}{{}^{52}C_5} = \frac{94}{54145} = \frac{2}{579}$ (3)

or $\hat{=} 0.001736 \dots$



LHS

i) $\frac{\cot \theta \cos \theta}{\cot \theta + \cos \theta} = \frac{\cos \theta \cdot \cos \theta}{\sin \theta \cdot \cos \theta + \cos \theta} = \frac{\cos^2 \theta}{\cos \theta (\sin \theta + 1)} = \frac{\cos \theta}{1 + \sin \theta}$ (3)

ii) $\frac{\sin^2 \theta}{1 + \cos \theta} + \frac{\sin^2 \theta}{1 - \cos \theta} = \frac{\sin^2 \theta (1 - \cos \theta) + \sin^2 \theta (1 + \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{\sin^2 \theta - \sin^2 \theta \cos \theta + \sin^2 \theta + \sin^2 \theta \cos \theta}{1 - \cos^2 \theta} = \frac{2 \sin^2 \theta}{\sin^2 \theta} = 2$ which is independent of θ (3)

In ΔABC
 $\tan 25^\circ = \frac{h}{CB}$

$CB = \frac{h}{\tan 25^\circ}$

$CB = h \cot 25^\circ$ (2)

Similarly $BD = h \cot 30^\circ$

Using the cosine rule in ΔCBD :

$CD^2 = h^2 \cot^2 25^\circ + h^2 \cot^2 30^\circ - 2 \cdot h \cot 25^\circ \cdot h \cot 30^\circ \cos 60^\circ$
 $CD^2 = h^2 [\cot^2 25^\circ + \cot^2 30^\circ - 2 \cot 25^\circ \cot 30^\circ \cos 60^\circ]$

$2500 = h^2 \times [1.13 \cdot 0.845]$
 $643.6 = h^2$
 $\therefore h = 25.3688$
 $\hat{=} 25.4$ (3)