# GIRRAWEEN HIGH SCHOOL

# MATHEMATICS

Year 11 Extension 1 Task 2 — Thursday 28st June 2007

Instructions
(a) Write all answers on your own paper
(b) Show all necessary working
(c) Marks may be deducted for careless or badly arranged work
(d) Attempt all questions and start each question on a new page

Time allowed 90 minutes

**Question 1** ***(15 marks)*** — **Marks**

(a) Find the acute angle between the lines $2x - y + 5 = 0$ and $x + 3y - 4 = 0$ to the nearest degree — 3

(b) The acute angle between $x - 2y - 3 = 0$ and $mx + y + 3 = 0$ is $45^0$. Find the possible value(s) of m. — 4

(c) Let A ( 3 , 2 ) and B ( - 5 , - 2 ) be two points in the number plane. Find the point C that divides the interval ;

- (i) internally in the ratio 3 : 1 — 3
- (ii) externally in the ratio 2 : 3 — 3

(d) The point N (-2 , 1) divides the interval M (-4 , 3 ), P ( 4, -5) internally in the ratio $k : 1$ . Find the value of $k$. — 2

**Question 2** ***(21 marks)***

(a) Find the exact value of ;

- (i) $\tan 75^0$ — 3
- (ii) $\sin 105^0$ — 3
- (iii) $\cos 15^0$ — 3

(b) Solve for $\theta$ to the nearest degree, $0^0 \leq \theta \leq 360^0$

- (i) $\sin 2\theta = \sqrt{3} \cos 2\theta$ — 3
- (ii) $\tan^2 \theta = 3$ — 3
- (iii) $\sec \theta = 2$ — 2
- (iv) $\tan \theta - \sec^2 \theta + 3 = 0$ — 4

**Question 3** ***(15 marks)***

(a) Given that $\sin\alpha = \frac{3}{5}$ and $\sin\beta = \frac{2}{3}$ and that $0 \le \alpha \le 90^0$ and $90^0 \le \beta \le 180^0$,

find

(i) $\cos\alpha$ and $\cos\beta$ 4

(ii) $\sin 2\alpha$ 2

(iii) $\tan(\beta - \alpha)$ 3

(b) Find the exact value of the following;

(i) $\frac{2\tan 15^0}{1-\tan^2 15^0}$ 2

(ii) $\sin 105^0 \cos 105^0$ 2

(iii) $\cos^2 75^0 - \sin^2 75^0$ 2

**Question 4** ***(15 marks)***

(a) Prove the following identities;

(i) $\frac{\sin(\alpha - \beta)}{\cos\alpha\cos\beta} = \tan\alpha - \tan\beta$ 3

(ii) $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\text{cosec}\,\theta$ 3

(iii) $\frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$ 3

(b) If $t = \tan\frac{\theta}{2}$ find an expression for the following in terms of $t$

(i) $\tan\theta$, $\sin\theta$ and $\cos\theta$ 3

(ii) $\frac{\cos\theta}{1-\sin\theta}$ 3

**Question5** *(21 marks)*

(a) Solve the following correct to the nearest degree for $\theta$, $0^0 \le \theta \le 360^0$

(i) $\tan 2\theta = 3\tan\theta$ 4

(ii) $5\sin\theta + 4\cos\theta = 5$ using the $t$ results ($t = \tan\frac{\theta}{2}$) 4

(b) (i) Express $2\sin x + 4\cos x$ in the form $R\sin(x + \alpha)$

Clearly stating the values for R and $\alpha$ (to the nearest degree) 3

(ii) Hence or otherwise solve for $x$, $0^0 \le x \le 360^0$ the equation

$2\sin x + 4\cos x = 3$ ( to the nearest degree ) 3

(c) (i) Let $t = \tan 22\frac{1}{2}^0$ . Express $\tan 45^0$ in terms of $t$ 2

(ii) Hence find the exact value $\tan 22\frac{1}{2}^0$ 3

(d) (i) By expressing $\cos 3\theta$ as $\cos(2\theta+\theta)$ or similar, find an expression for

$\cos 3\theta$ in terms of powers of $\cos\theta$ 3

(ii) If $\cos\theta = \frac{1}{3}$, find the value of $\cos 3\theta$ 2

YEAR 11 EXTENSION 1

TASK 2 - JUNE 2007 - SOLUTIONS

## Question 1.

a) $2x - y + 5 = 0$ ; $x + 3y - 4 = 0$

$y = 2x + 5$ ; $y = -\frac{1}{3}x + \frac{4}{3}$

$m_1 = 2$ ; $m_2 = -\frac{1}{3}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \frac{2\frac{1}{3}}{\frac{1}{3}} = 7$

$\theta = 81.87° = 82°$ (3m)

b) $x - 2y - 3 = 0$ ; $mx + y + 3 = 0$

$2y = x - 3$ $\quad$ $y = -mx - 3$

$y = \frac{1}{2}x - \frac{3}{2}$ $\quad$ $m_2 = -m$.

$m_1 = \frac{1}{2}$

$\tan\theta = \left|\frac{\frac{1}{2} + m}{\frac{2 - m}{2}}\right|$, $\tan 45 = 1$

$1 = \left|\frac{1 + 2m}{2 - m}\right|$

$\therefore\ 1 = \frac{1 + 2m}{2 - m}$ OR $1 = \frac{1 + 2m}{m - 2}$

$2 - m = 1 + 2m$ , $m - 2 = 1 + 2m$

$1 = 3m$ , ~~$3m = 3$~~ (4m)

$m = \frac{1}{3}$ , $m = -3$

c) (i) $A(3,2)$ $B(-5,-2)$ $3:1$

$C_x = \frac{1(3) + 3(-5)}{4}$ $\quad$ $C_y = \frac{1(2) + 3(-2)}{4}$

$C = (-3, -1)$ (3m)

(ii) $A(3,2)$ $B(-5,-2)$ $-2:3$

$C_x = \frac{3 \times 3 + -2 \times -5}{-2 + 3}$ $\quad$ $C_y = \frac{3 \times 2 + -2 \times -2}{-2 + 3}$

$C = (19, 10)$ (3m)

(d) $-2 = \frac{-4 \times \ldots + 4 \times k}{k + 1}$

$-2k - 2 = 4k - 4$

$6k = 2$

$k = \frac{1}{3}$ (2m)

## Question 2

a) i) $\tan 75 = \tan(45 + 30)$

$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ (3m)

ii) $\sin 105° = \sin(60° + 45°)$

$= \sin 60 \cos 45 + \sin 45 \cos 60$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$ (3m)

iii) $\cos 15 = \cos(45 - 30)$

$= \cos 45 \cos 30 + \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$ (3m)

(b) (i) $\sin 2\theta = \sqrt{3} \cos 2\theta$

$\frac{\sin 2\theta}{\cos 2\theta} = \sqrt{3}$

$\tan 2\theta = \sqrt{3}$ (3m)

$2\theta = 60°, 240°, 420°, 600°$

$\theta = 30°, 120°, 210°, 300°$

(ii) $\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

$\theta = 60°, 120°, 240°, 300°$ (3m)

(iii) $\sec\theta = 2$ $\quad$ $\cos\theta = \frac{1}{2}$

$\theta = 60°\ 300°$ (2m)

(iv) $\tan\theta - \sec^2\theta + 3 = 0$

$\tan\theta - (1 + \tan^2\theta) + 3 = 0$

$\tan\theta - \tan^2\theta + 2 = 0$

$\tan^2\theta - \tan\theta - 2 = 0$

$(\tan\theta - 2)(\tan\theta + 1) = 0$ (4m)

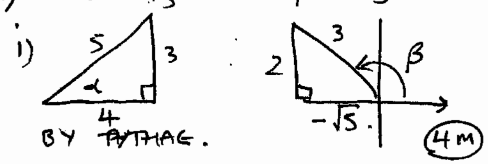
$\tan\theta = -1$ $\quad$ $\tan\theta = 2$

$\theta = 135°, 315°$ $\quad$ $\theta = 63°, 243°$

## Question 3.

a) $\sin\alpha = \frac{3}{5}$ $\quad$ $\sin\beta = \frac{2}{3}$

i)




(4m)

$\therefore\ \cos\alpha = \frac{4}{5}$ $\quad$ $\cos\beta = -\frac{\sqrt{5}}{3}$

ii) $\sin 2\alpha = 2 \sin\alpha \cos\alpha$

$= 2 \cdot \frac{4}{5} \cdot \frac{3}{5}$

$\sin 2\alpha = \frac{24}{25}$ (2m)

(iii) $\tan(\beta - \alpha) = \frac{\tan\beta - \tan\alpha}{1 + \tan\beta \tan\alpha}$

$= \frac{-\frac{2}{\sqrt{5}} - \frac{3}{4}}{1 + -\frac{2}{\sqrt{5}} \cdot \frac{3}{4}}$

$= \frac{-8 - 3\sqrt{5}}{4\sqrt{5} - 6}$ (3m)

(b) (i) $\frac{2\tan 15°}{1 - \tan^2 15} = \tan 2(15)$

$= \tan 30$

$= \frac{1}{\sqrt{3}}$ (2m)

(ii) $\sin 105 \cos 105 = \frac{1}{2} \sin\{2(105)\}$

$= \frac{1}{2} \sin 210$

$= -\frac{1}{4}$ (2m)

(iii) $\cos^2 75 - \sin^2 75 = \cos(75 + 75)$

$= \cos 150$

$= -\frac{\sqrt{3}}{2}$ (2m)

## Question 4

a) i) $\frac{\sin(\alpha - \beta)}{\cos\alpha \cos\beta} = \text{LHS}$

$= \frac{\sin\alpha \cos\beta - \sin\beta \cos\alpha}{\cos\alpha \cos\beta}$

$= \frac{\sin\alpha \cos\beta}{\cos\alpha \cos\beta} - \frac{\sin\beta \cos\alpha}{\cos\beta \cos\alpha}$

$= \tan\alpha - \tan\beta$

$= \text{RHS}$. (3m)

(ii) $\frac{\sin\theta}{1 - \cos\theta} + \frac{\sin\theta}{1 + \cos\theta} = \text{LHS}$

$= \frac{\sin\theta(1 + \cos\theta)}{1 + \cos^2\theta} + \frac{\sin\theta(1 - \cos\theta)}{1 + \cos^2\theta}$

$= \frac{2\sin\theta}{\sin^2\theta} = \frac{2}{\sin\theta}$

$= 2\csc\theta = \text{RHS}$ (3m)

(iii) $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \text{LHS}$

$= \frac{1 - (1 - 2\sin^2\theta)}{1 + (2\cos^2\theta - 1)}$ (3m)

$= \frac{2\sin^2\theta}{2\cos^2\theta} = \tan^2\theta = \text{RHS}$

(b) (i) $\tan\theta = \frac{2t}{1 - t^2}$ (1m)

$\cos\theta = \frac{1 - t^2}{1 + t^2}$ (1m)

$\sin\theta = \frac{2t}{1 + t^2}$ (1m)

(ii) $\frac{\cos\theta}{1 - \sin\theta} = \frac{\frac{1 - t^2}{1 + t^2}}{1 - \frac{2t}{1 + t^2}}$

$= \frac{\frac{1 - t^2}{1 + t^2}}{\frac{1 + t^2 - 2t}{1 + t^2}} = \frac{1 - t^2}{1 - 2t + t^2}$ (3m)

$= \frac{(1 - t)(1 + t)}{(1 - t)(1 - t)} = \frac{1 + t}{1 - t}$

Question 5.

(a) i) $\tan 2\theta = 3\tan\theta$

$$\frac{2\tan\theta}{1-\tan^2\theta} = 3\tan\theta$$

$2\tan\theta = 3\tan\theta - 3\tan^3\theta$

$3\tan^3\theta - \tan\theta = 0.$

$\tan\theta(3\tan^2\theta - 1) = 0$ (2M)

$\tan\theta = 0 \quad \theta = 0, 180, 360$

or $\tan\theta = \pm\frac{1}{\sqrt{3}}$

$\theta = 30°\ 150°\ 210°\ 330°$ (2m)

ii) $5\sin\theta + 4\cos\theta = 5.$

$$\frac{10t}{1+t^2} + \frac{4(1-t^2)}{1+t^2} = 5.$$

$10t + 4 - 4t^2 = 5 + 5t^2$

$9t^2 - 10t + 1 = 0$

$(9t-1)(t-1) = 0$

$t = 1$ or $\frac{1}{9}$

$\theta/2 = 45, 225$ or $\theta/2 = 6{\cdot}34°\ 186{\cdot}34°$

$\theta = 90, 13°$ (4M)

(b) i) $R\sin(x+\alpha)$

$= R\{\sin x\cos\alpha + \cos x\sin\alpha\}$

$\therefore R\sin x\cos\alpha = 2\sin x$

$R\cos\alpha = 2$

$\therefore R\sin\alpha = 4$ (3M)

$\tan\alpha = 2 \quad \alpha = 63°26'$

$R^2\cos^2\alpha + R^2\sin^2\alpha = 20$

$R^2 = 20$

$R = \sqrt{20} = 2\sqrt{5}.$

ii) $2\sin x + 4\cos x = 3$

$\therefore \sqrt{20}(\sin\{x + 63°26'\}) = 3$

$\sin(x + 63°26') = \frac{3}{\sqrt{20}}$

$x + 63°26' = 42°8', 137°52, 402$

$x = -21°18, 74°26, 338°42$

$\boxed{x = 74°, 339°}$ (3M)

(c) i) $\tan 45 = \dfrac{2\tan 22\tfrac{1}{2}°}{1-\tan^2 22\tfrac{1}{2}°}$

$= \dfrac{2t}{1-t^2}.$ (2m)

ii) $1 = \dfrac{2t}{1-t^2}$

$1 - t^2 = 2t$

$0 = t^2 + 2t - 1$

where $t = \dfrac{-2 \pm\sqrt{4+4}}{2}$

$t = -1 \pm \sqrt{2}$

But $22\tfrac{1}{2}°$ IS IN $1^{ST}$ QUAD

$\therefore \tan 22\tfrac{1}{2} = \sqrt{2} - 1$ (3m)

d) i) $\cos 3\theta = \cos(2\theta + \theta)$

$= \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$= (2\cos^2\theta - 1)(\cos\theta) - 2\sin\theta\cos\theta\sin\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta\sin^2\theta$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$

$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta.$ (3m)

(ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$= 4(1/3)^3 - 3(1/3)$

$= 4/27 - 1$

$\cos 3\theta = \dfrac{-23}{27}$ (2m)