



SYDNEY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT
TRIAL EXAMINATIONS 2007

FORM VI

MATHEMATICS EXTENSION 1

Examination date

Monday 6th August 2007

Time allowed

2 hours (plus 5 minutes reading time)

Instructions

- All seven questions may be attempted.
- All seven questions are of equal value.
- All necessary working must be shown.
- Marks may not be awarded for careless or badly arranged work.
- Approved calculators and templates may be used.
- A list of standard integrals is provided at the end of the examination paper.

Collection

- Write your candidate number clearly on each booklet.
- Hand in the seven questions in a single well-ordered pile.
- Hand in a booklet for each question, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Keep the printed examination paper and bring it to your next Mathematics lesson.

Checklist

- SGS booklets: 7 per boy. A total of 1000 booklets should be sufficient.
- Candidature: 117 boys.

Examiner

MLS

QUESTION ONE (12 marks) Use a separate writing booklet.

Marks

- (a) If $(x - 2)$ is a factor of the polynomial

1

$$P(x) = 2x^3 + x + a,$$

find the value of a .

- (b) Given that $\log_a b = 2.8$ and $\log_a c = 4.1$, find $\log_a \frac{b}{c}$.

1

- (c) Shade the region on the number plane satisfied by $y \geq |x + 2|$.

2

- (d) Solve the inequality $\frac{5}{x - 4} \geq 1$.

3

- (e) State the domain and range of $y = \cos^{-1} \frac{x}{4}$.

2

- (f) Evaluate $\int_0^3 \frac{dx}{9 + x^2}$.

3

QUESTION TWO (12 marks) Use a separate writing booklet.

Marks

- (a) Evaluate $\lim_{x \rightarrow 0} \frac{\sin \frac{x}{3}}{2x}$.

1

- (b) The point A has coordinates $(-2, 1)$ and the point B has coordinates $(b, -3)$.
The point $P(13, -9)$ divides the interval AB externally in the ratio $5 : 3$.
Find the value of b .

2

- (c) Using the substitution $u = e^x$, find

3

$$\int \frac{e^x}{\sqrt{1 - e^{2x}}} dx.$$

- (d) (i) Write down an expression for $\tan 2x$ in terms of $\tan x$.

1

- (ii) Hence show that if $f(x) = x \cot x$, then $f(2x) = (1 - \tan^2 x)f(x)$.

3

- (e) Find the coefficient of x^3 in the expansion of $(2 - 5x)^6$.

2

QUESTION THREE (12 marks) Use a separate writing booklet.

Marks

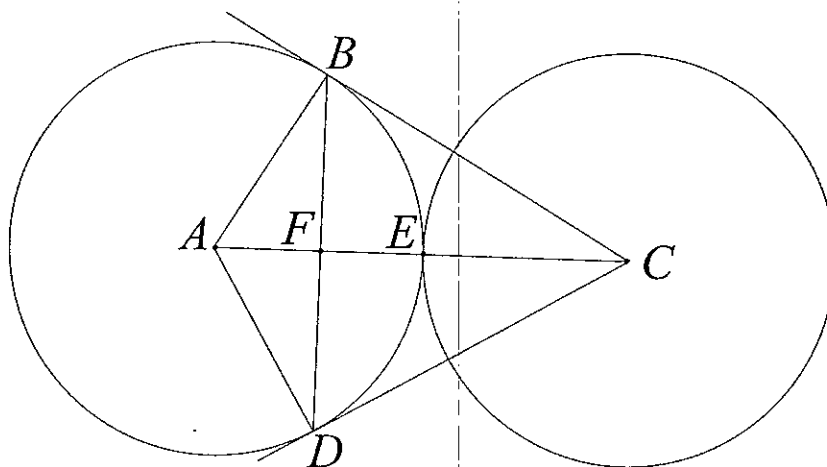
- (a) Differentiate $\cos^{-1} x^2$.

2

- (b) Show that $\int_0^{\frac{\pi}{2}} \cos^2 x dx = \frac{\pi}{4}$.

2

(c)



Two circles with equal radii and centres A and C touch externally at E as shown in the diagram. The lines BC and DC are tangents from C to the circle with centre A .

- (i) Explain why $ABCD$ is a cyclic quadrilateral. 2
- (ii) Show that E is the centre of the circle that passes through A, B, C and D . 2
- (iii) Show that $\angle BCA = \angle DCA = 30^\circ$. 2
- (iv) Deduce that $\triangle BCD$ is equilateral. 2

QUESTION FOUR (12 marks) Use a separate writing booklet.

Marks

- (a) The region between the curve $y = \frac{1}{\sqrt{1+4x^2}}$ and the x -axis is rotated about the x -axis. Find the volume of the solid enclosed between $x = \frac{2}{\sqrt{3}}$ and $x = 2\sqrt{3}$. 3

- (b) Use the substitution $u = x - 3$ to evaluate $\int_3^4 x\sqrt{x-3} dx$. 4

- (c) A metal rod is taken from a freezer at -8°C into a room where the air temperature is 22°C . The rate at which the rod warms follows Newton's law, that is

$$\frac{dT}{dt} = -k(T - 22)$$

where k is a positive integer, time t is measured in minutes, and temperature T is measured in degrees Celsius.

- (i) Show that $T = 22 - Ae^{-kt}$ is a solution of the equation $\frac{dT}{dt} = -k(T - 22)$, and find the value of A . 2
- (ii) The temperature of the rod reaches 4°C in 90 minutes. Find the exact value of k . 2
- (iii) Find the temperature of the rod after another 90 minutes. 1

Exam continues overleaf ...

QUESTION FIVE (12 marks) Use a separate writing booklet.

Marks

- (a) A particle is moving in a straight line so that its displacement x at time t seconds is given by $x = \sqrt{3} \cos 2t - \sin 2t$ metres.
- (i) Write $x = \sqrt{3} \cos 2t - \sin 2t$ in the form $x = R \cos(2t + \alpha)$, where $R > 0$ and $0 \leq \alpha < 2\pi$. 2
 - (ii) When is the particle first at $x = 1$? 1
 - (iii) What is the maximum velocity of the particle and when does it first occur? 2
- (b) (i) Show that $x^3 - x - 2 = 0$ has a root between $x = 1$ and $x = 2$. 1
- (ii) Given that $x = 1.5$ is your first approximation to a root of $x^3 - x - 2 = 0$, use one application of Newton's method to find another approximation. Give your answer correct to one decimal place. 2
- (c) A particle is moving in simple harmonic motion on a straight line. Its velocity v is given by $v^2 = 4(2x - x^2)$, where x is its displacement from a fixed point O on the line.
- (i) Show that its acceleration is given by $\ddot{x} = -4(x - 1)$. 1
 - (ii) Find the centre of the motion. 1
 - (iii) Find the displacement of the particle when its speed is half the maximum speed. 2

QUESTION SIX (12 marks) Use a separate writing booklet.

Marks

- (a) The length of a rectangle is increasing at 6 cm s^{-1} , while the breadth is decreasing so that the area of the rectangle remains constant at 50 cm^2 . Find the rate of change of the breadth when the length is 10 cm. 3
- (b) (i) Use the method of mathematical induction to show that if x is a positive integer, then $(1 + x)^n - 1$ is divisible by x , for all positive integers $n \geq 1$. 3
- (ii) Write $12^n - 4^n - 3^n + 1$ as a product of two factors. 1
- (iii) Use parts (i) and (ii) to deduce that $12^n - 4^n - 3^n + 1$ is divisible by 6 for all integers $n \geq 1$. 1
- (c) The quadratic equation $ax^2 + bx + c = 0$ has roots $x = \tan \alpha$ and $x = \tan \beta$.
- (i) Show that $\tan(\alpha + \beta) = \frac{b}{c - a}$. 2
 - (ii) Show that $\tan^2(\alpha - \beta) = \frac{b^2 - 4ac}{(a + c)^2}$. 2

QUESTION SEVEN (12 marks) Use a separate writing booklet.

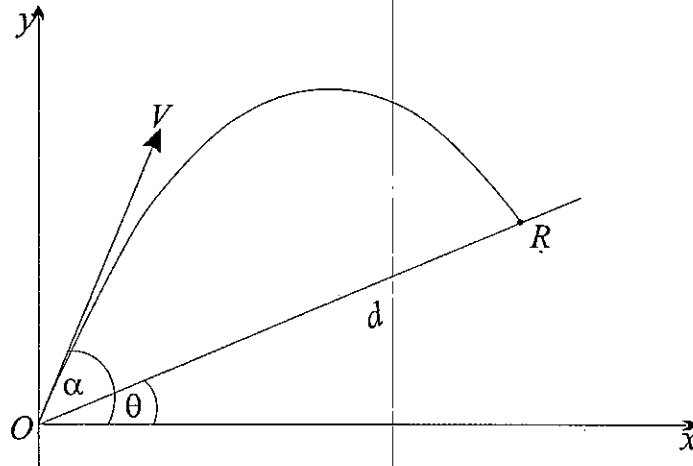
Marks

- (a) Find the value of n if

2

$${}^n C_2 + {}^n C_1 + {}^n C_0 = 37.$$

- (b)



In the diagram above, a particle is projected at an angle of elevation α with velocity V from a point O which is at the bottom of an inclined plane. The plane is inclined to the horizontal at an angle θ , where $\theta < \alpha$. The particle meets the inclined plane again at R . The acceleration due to gravity is g , and $0^\circ < \alpha < 90^\circ$. Let $OR = d$.

- (i) Given that $x = Vt \cos \alpha$ and $y = Vt \sin \alpha - \frac{1}{2}gt^2$, where t is the time elapsed, show that the Cartesian equation of the path of the particle is

1

$$y = x \tan \alpha - \frac{gx^2 \sec^2 \alpha}{2V^2}.$$

- (ii) Find expressions for the coordinates of R in terms of θ and d .

1

- (iii) Show that the range of the particle up the inclined plane is given by

3

$$d = \frac{2V^2 \cos \alpha \sin(\alpha - \theta)}{g \cos^2 \theta}.$$

- (c) Consider the identity $(1 + x)^n = 1 + {}^n C_1 x + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots + {}^n C_n x^n$, where n is a positive integer.

- (i) Use the formula for the sum of a GP to simplify

1

$$1 + (1 + x) + (1 + x)^2 + (1 + x)^3 + \dots + (1 + x)^{n-1}.$$

- (ii) Use part(i) to show that

1

$$1 + (1 + x) + (1 + x)^2 + (1 + x)^3 + \dots + (1 + x)^{n-1} = {}^n C_1 + {}^n C_2 x + {}^n C_3 x^2 + \dots + {}^n C_n x^{n-1}.$$

- (iii) Find $\int_{-1}^0 ({}^n C_1 + {}^n C_2 x + {}^n C_3 x^2 + \dots + {}^n C_n x^{n-1}) dx$.

1

- (iv) Hence show that $\sum_{r=1}^n \frac{(-1)^{r+1}}{r} {}^n C_r = \sum_{r=1}^n \frac{1}{r}$.

2

END OF EXAMINATION

Solutions Trial Ex+1 Maths

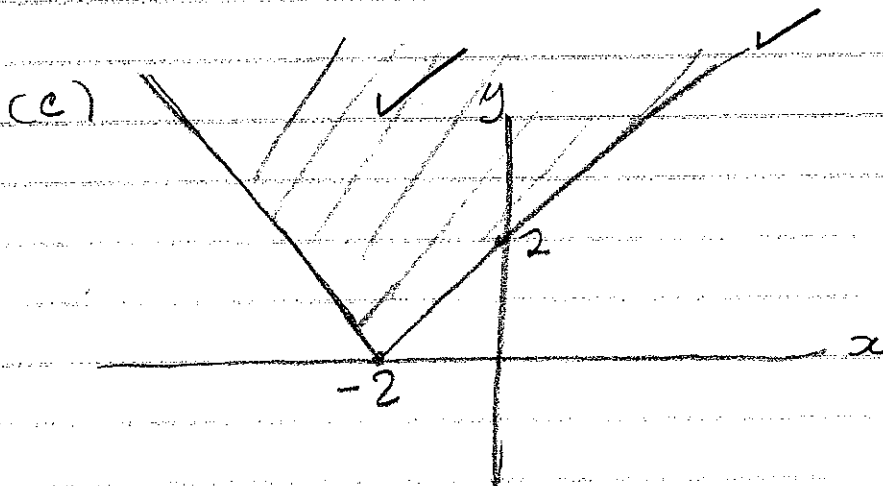
2007.

Q1.

(a) $P(x) = 2x^3 + x + a$
 $P(2) = 16 + 2 + a = 0$

$a = -18$ ✓

(b) $\log \frac{b}{c} = \log b - \log c$
 $= 2.8 - 4.1$
 $= -1.3$

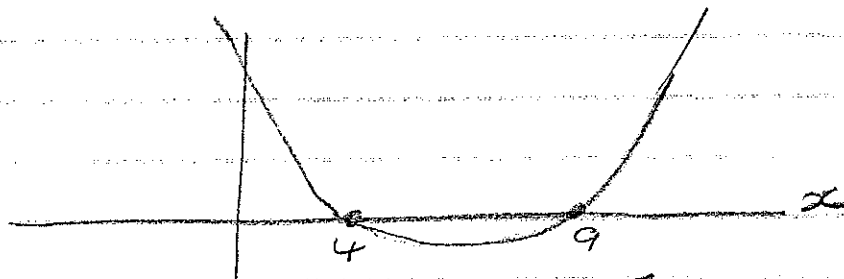


(d) $\frac{5}{x-4} \geq 1$, $x \neq 4$ ✓

$5(x-4) \geq (x-4)^2$
 $(x-4)^2 - 5(x-4) \leq 0$

$(x-4)(x-4-5) \leq 0$

$(x-4)(x-9) \leq 0$ ✓



$4 < x \leq 9$ ✓

(a)

Range: $0 \leq y \leq \pi$ ✓

Domain $-1 \leq \frac{x}{4} \leq 1$

$-4 \leq x \leq 4$ ✓

$$f. \int_0^3 \frac{dx}{9+x^2} = \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_0^3$$
$$= \frac{1}{3} (\tan^{-1} 1 - \tan^{-1} 0)$$

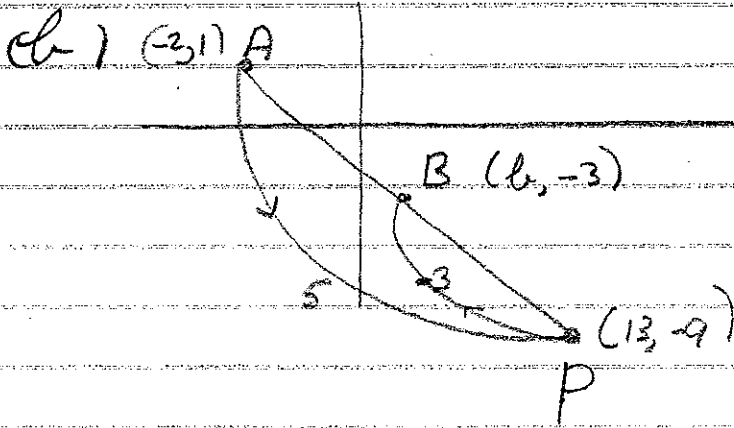
$$= \frac{1}{3} \left(\frac{\pi}{4} - 0 \right)$$

$$= \frac{\pi}{12}$$
 ✓

Q2.

$$(a) \lim_{x \rightarrow 0} \frac{\sin \frac{x}{3}}{2x} = \frac{1}{6} \lim_{x \rightarrow 0} \frac{\sin \frac{x}{3}}{\frac{x}{3}} = \frac{1}{6}$$

✓ correct intermediate step is necessary here.



$$\frac{(-3) \times (-2) + 5b}{5-3} = 13 \quad \checkmark$$

$$\frac{6+5b}{2} = 13$$

$$6+5b = 26$$

$$5b = 20$$

$$b = 4 \quad \checkmark$$

(c) $u = e^x$
 $du = e^x dx$

$$\int \frac{e^x}{\sqrt{1-e^{2x}}} dx = \int \frac{du}{\sqrt{1-u^2}} \quad \checkmark$$

$$= \sin^{-1} u + C \quad \checkmark$$

$$= \sin^{-1} e^x + C \quad \checkmark$$

(ignore +c)

(d)

(i) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \quad \checkmark$

(ii) $f(x) = x \cot 2x$

$$f(2x) = 2x \cot 2x \quad \checkmark$$

$$= 2x \times \frac{1 - \tan^2 x}{2 \tan x} \quad \checkmark$$

$$= \frac{x(1 - \tan^2 x)}{\tan x}$$

$$= x(1 - \tan^2 x) \cot x \quad \checkmark$$

$$= (1 - \tan^2 x) f(x)$$

(e) $(2-5x)^6$

general term is ${}^6C_r 2^r (-5)^{6-r}$

need $6-r = 3$ so

so coefficient is ${}^6C_3 2^3 (-5)^3 = -20000 \quad \checkmark \checkmark$

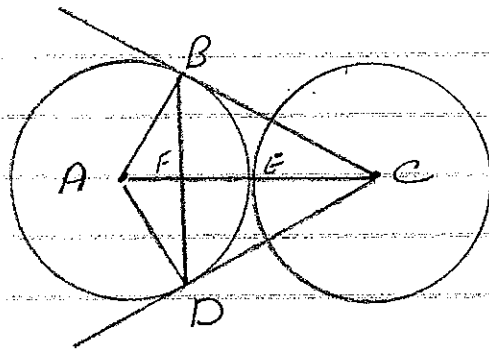
\checkmark for 6C_3 , \checkmark for $2^3(-5)^3$, \checkmark for $r=3$,

Q3.

(a) $y = \cos^{-1} x^2$
 $\frac{dy}{dx} = \frac{-2x \checkmark}{\sqrt{1-x^4} \checkmark}$ ($\frac{-2x}{\sqrt{1-x^4}}$ is worth \checkmark one mark.)

(b) $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 + \cos 2x) \, dx \checkmark$
 $= \frac{1}{2} \left[x + \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{2}} \checkmark$
 $= \frac{1}{2} \left[\left(\frac{\pi}{2} + \frac{1}{2} \sin \pi \right) - \left(0 + \frac{1}{2} \sin 0 \right) \right]$
 $= \frac{\pi}{4}$

(c)



- (i) The angle between a tangent and chord is a right angle }
so $\angle ABC$ and $\angle ADC$ are right angles }
so the opposite angles of $ABCD$ are supplementary }
so $ABCD$ is cyclic } \checkmark

(iii) Right angles are subtended at the circumference by diameters
So AC is a diameter of ABCD ✓
But AE = EC, since these are radii of equal circles ✓
So E is the centre of the circle

$$(iii) \sin \angle BCA = \frac{AB}{AC} = \frac{1}{2}$$

$$\text{so } \angle BCA = 30^\circ \quad \checkmark$$

$$\text{Similarly } \sin \angle ACD = \frac{AD}{AC} = \frac{1}{2}$$

$$\text{so } \angle ACD = 30^\circ \quad \checkmark$$

(iv) Tangents from an external point are equal

So BC = DC and $\triangle BCD$ is isosceles ✓

$$\text{So } \angle CBD = \angle BDC$$

$$\angle CBD + \angle BDC = 180^\circ - 60^\circ, \text{ angle sum of triangle } \checkmark$$
$$= 120^\circ$$

$$\text{So } \angle CBD = \angle BDC = 60^\circ$$

and $\triangle BCD$ is equilateral

(There are many other correct ways to do this question)

Q4.

$$(a) \quad y = \frac{1}{\sqrt{1+4x^2}} = \frac{1}{\sqrt{4(\frac{1}{4}+x^2)}} = \frac{\frac{1}{2}}{\sqrt{\frac{1}{4}+x^2}}$$

$$V = \frac{\pi}{2} \int_{\frac{2}{\sqrt{3}}}^{2\sqrt{3}} \frac{1}{\frac{1}{4}+x^2} dx \quad \checkmark \quad a = \frac{1}{2}$$

$$= \frac{\pi}{2} \left[\tan^{-1} 2x \right]_{\frac{2}{\sqrt{3}}}^{2\sqrt{3}} \quad \checkmark$$

$$= \frac{\pi}{2} \left(\tan^{-1} 4\sqrt{3} - \tan^{-1} \frac{4}{\sqrt{3}} \right) \quad \checkmark$$

(b) $u = x - 3$

$$du = dx$$

limits: when $x = 4$, $u = 1$

$x = 3$, $u = 0$

$$\int_3^4 2\sqrt{x-3} dx = \int_0^1 (u+3)\sqrt{u} du \quad \checkmark \quad \checkmark$$

$$= \int_0^1 (u^{3/2} + 3u^{1/2}) du \quad \checkmark$$

$$= \left[\frac{2u^{5/2}}{5} + 3u^{3/2} \times \frac{2}{3} \right]_0^1$$

$$= \left[\frac{2}{5}u^{5/2} + 2u^{3/2} \right]_0^1$$

$$= 2\frac{2}{5} \quad \checkmark$$

c.

$$(i) \quad T = 22 - Ae^{-kt}$$
$$\frac{dT}{dt} = kAe^{-kt}$$
$$= k(22 - T)$$
$$= -k(T - 22)$$

when $t=0$, $T=-8$

$$-8 = 22 - A$$

$$A = 30$$

ii) $T=4$ when $t=90$

$$4 = 22 - 30e^{-90k}$$

$$30e^{-90k} = 18$$

$$e^{-90k} = \frac{18}{30}$$

$$-90k = \ln \frac{3}{5}$$

$$k = -\frac{1}{90} \ln \frac{3}{5}$$

iii) Find T when $t = 180$

$$T = 22 - 30e^{-180k}$$

$$= 22 - 30e^{-180 \times (-\frac{1}{90} \ln \frac{3}{5})}$$

$$= 22 - 30e^{2 \ln \frac{3}{5}}$$

$$= 22 - 30 \times \frac{9}{25}$$

$$= 22 - \frac{54}{5}$$

$$= 11 \frac{1}{5}$$

Q5

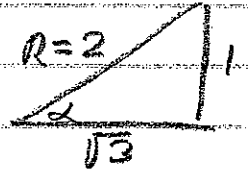
(a)

$$(i) \quad x = \sqrt{3} \cos 2t - \sin 2t = R \cos(2t + \alpha)$$
$$= R \cos 2t \cos \alpha - R \sin 2t \sin \alpha$$

$$R \cos \alpha = \sqrt{3} \quad \text{and} \quad R \sin \alpha = 1$$

$$\cos \alpha = \frac{\sqrt{3}}{R}$$

$$\sin \alpha = \frac{1}{R}$$



$$\text{so } R = \sqrt{3+1} = 2 \quad \checkmark$$

$$\text{and } \alpha = \frac{\pi}{6} \quad \checkmark$$

$$\text{So } x = 2 \cos\left(2t + \frac{\pi}{6}\right)$$

(ii) Find t when $x=1$

$$2 \cos\left(2t + \frac{\pi}{6}\right) = 1$$

$$\cos\left(2t + \frac{\pi}{6}\right) = \frac{1}{2}$$

$$2t + \frac{\pi}{6} = \frac{\pi}{3}$$

$$2t = \frac{\pi}{6}$$

$$t = \frac{\pi}{12} \text{ seconds} \quad \checkmark$$

$$(iii) \quad x = 2 \cos\left(2t + \frac{\pi}{6}\right)$$

$$\dot{x} = -4 \sin\left(2t + \frac{\pi}{6}\right)$$

maximum value of \dot{x} is 4 m s^{-1} , it occurs when

$$\sin\left(2t + \frac{\pi}{6}\right) = -1$$

$$2t + \frac{\pi}{6} = \frac{3\pi}{2}$$

$$2t = \frac{3\pi}{2} - \frac{\pi}{6}$$

$$= \frac{4\pi}{3}$$

$$t = \frac{2\pi}{3} \quad \checkmark$$

it reaches maximum velocity after $\frac{2\pi}{3} \text{ s}$.

(b) (i)

$$f(x) = x^3 - x - 2$$

$$f(1) = 1 - 1 - 2 = -2$$

$$f(2) = 8 - 2 - 2 = 4$$

} ✓

$f(x)$ changes sign between $x=1$ and 2 , it is continuous, so there is a root between $x=1$ and 2 .

(ii) $f'(x) = 3x^2 - 1$

$$f(1.5) = 1.5^3 - 1.5 - 2$$

$$f'(1.5) = 3(1.5)^2 - 1$$

$$x_1 = 1.5 - \frac{1.5^3 - 1.5 - 2}{3(1.5)^2 - 1} \quad \checkmark$$

$$\approx 1.5 \quad \checkmark$$

$\left(\frac{35}{23} \text{ is exact}\right)$
also accept

(c) i) $\frac{1}{2}v^2 = 2(2x - x^2) = 4x - 2x^2$

$$\ddot{x} = \frac{d(\frac{1}{2}v^2)}{dx}$$

$$= 4 - 4x$$

$$= -4(x-1)$$

} ✓

ii) Centre is when $\ddot{x} = 0$, $x=1$ ✓

iii) Maximum speed is when $\dot{x} = 0$, $x=1$.

$$x=1, \quad v^2 = 4(2-1) = 4$$

so maximum speed is 2, and half is 1. ✓

$$\text{so } 2\sqrt{2x-x^2} = 1$$

$$2x-x^2 = \frac{1}{4}$$

$$4x^2 - 8x + 1 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 16}}{8}$$

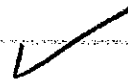
$$= \frac{8 \pm \sqrt{48}}{8}$$

$$48 = 16 \times 3$$

$$= \frac{8 \pm 4\sqrt{3}}{8}$$

$$= \frac{2 \pm \sqrt{3}}{2}$$

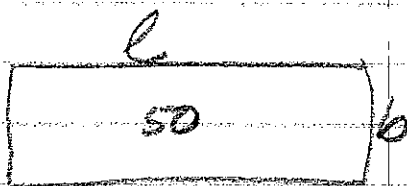
$$= 1 \pm \frac{\sqrt{3}}{2} \text{ m}$$



So when the particle is $1 + \frac{\sqrt{3}}{2}$ or $1 - \frac{\sqrt{3}}{2}$ m from 0 it has half the maximum speed.

6.

(a)



$$lb = 50 \quad \text{so} \quad b = \frac{50}{l} = 50l^{-1}$$

$$\frac{db}{dl} = -50l^{-2} = -\frac{50}{l^2} \checkmark$$

$$\frac{db}{dt} = \frac{db}{dl} \frac{dl}{dt}$$

$$= -\frac{50}{l^2} \times 6 \checkmark$$

$$= -\frac{50}{100} \times 6 \quad \text{when } l=10$$

$$= -3 \text{ cm s}^{-1} \checkmark$$

The breadth is decreasing at 3 cm s^{-1}

(b)

A. For $n=1$, we have $(1+x)^1 - 1 = x$ which is divisible by x . \checkmark

B. Assume the expression is divisible by x for some integer k .

$$\text{then } (1+x)^k - 1 = xM, \quad M \text{ an integer} \checkmark$$

Now prove that $(1+x)^{k+1} - 1$ is divisible by x .

$$\begin{aligned} \text{Now, } (1+x)^{k+1} - 1 &= (1+x)(1+x)^k - 1 \\ &= (1+x)(xM+1) - 1, \quad \text{using the induction hypothesis} \\ &= xM+1 + xM+x - 1 \checkmark \\ &= xM + xM + x \end{aligned}$$

$$\begin{aligned}
 &= xM + x^2M + xL \\
 &= x(M + xM + L) \quad \text{which is} \\
 &\text{divisible by } x
 \end{aligned}$$

So, from parts A + B using the principle of mathematical induction the expression is divisible by x for all $n \geq 1$.

$$\begin{aligned}
 \text{(ii)} \quad &12^n - 4^n - 3^n + 1 \\
 &= 4^n(3^n - 1) - 1(3^n - 1) \\
 &= (3^n - 1)(4^n - 1) \quad \checkmark
 \end{aligned}$$

(iii) From (i), $3^n - 1 = (1+2)^n - 1$ is divisible by 2
 $4^n - 1 = (1+3)^n - 1$ is divisible by 3
 so the product is divisible by 6 for $n \geq 1$.

c)

$$\begin{aligned}
 \text{(i)} \quad &\text{Sum of roots} = \tan \alpha + \tan \beta = -\frac{b}{a} \\
 &\text{product of roots} = \tan \alpha \tan \beta = \frac{c}{a} \quad \checkmark
 \end{aligned}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$= \frac{-\frac{b}{a}}{1 - \frac{c}{a}} \quad \checkmark$$

$$= \frac{-b}{a-c}$$

$$= \frac{b}{c-a}$$

ii)

$$\tan^2(\alpha - \beta) = \left(\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \right)^2$$

$$= \frac{\tan^2 \alpha - 2 \tan \alpha \tan \beta + \tan^2 \beta}{(1 + \tan \alpha \tan \beta)^2}$$

$$= \frac{(\tan \alpha + \tan \beta)^2 - 4 \tan \alpha \tan \beta}{(1 + \tan \alpha \tan \beta)^2} \checkmark$$

$$= \frac{b^2}{a^2} - \frac{4c}{a}$$

$$\left(1 + \frac{c}{a} \right)^2 \checkmark$$

$$= \frac{b^2 - 4ac}{(a+c)^2}$$

Q7.

$$a) \quad {}^nC_2 + {}^nC_1 + {}^nC_0 = 37$$

$$\frac{n(n-1)}{2} + n + 1 = 37 \quad \checkmark$$

$$n^2 - n + 2n + 2 = 74$$

$$n^2 + n - 72 = 0$$

$$(n+8)(n-9) = 0, \quad n \geq 2$$

$$\text{so } n = 8 \quad \checkmark$$

$$b) \text{ (i) } t = \frac{x}{V \cos \alpha}$$

$$y = V \frac{x}{V \cos \alpha} \sin \alpha - \frac{1}{2} g \frac{x^2}{V^2 \cos^2 \alpha} \quad \checkmark$$

$$= x \tan \alpha - \frac{1}{2} \frac{g x^2}{V^2} \sec^2 \alpha$$

$$\text{ii) } R = (d \cos \theta, d \sin \theta) \quad \checkmark$$

iii) from (i) and (ii)

$$d \sin \theta = d \cos \theta \tan \alpha - \frac{1}{2} \frac{g d^2 \cos^2 \theta \sec^2 \alpha}{V^2} \quad \checkmark$$

$$\sin \theta = \cos \theta \tan \alpha - \frac{g d \cos^2 \theta \sec^2 \alpha}{2V^2}$$

$$\frac{d g \cos^2 \theta}{2V^2 \cos^2 \alpha} = \cos \theta \tan \alpha - \sin \theta$$

$$d = \left(\frac{\cos \theta \sin \alpha}{\cos \alpha} - \sin \theta \right) \frac{2V^2 \cos^2 \alpha}{g \cos^2 \theta} \quad \checkmark$$

$$= \frac{2V^2 (\cos \theta \sin \alpha \cos \alpha - \sin \theta \cos^2 \alpha)}{g \cos^2 \theta}$$

$$= \frac{2V^2 \cos \alpha (\cos \theta \sin \alpha - \sin \theta \cos \alpha)}{g \cos^2 \theta}$$

\checkmark
recognising
 $\sin(\alpha - \theta)$

$$= \frac{2V^2 \cos^2 \alpha \sin(\alpha - \theta)}{g \cos \theta}$$

c) next page

C

$$\begin{aligned}
 \text{i)} \quad 1 + (1+x) + (1+x)^2 + \dots + (1+x)^{n-1} &= \frac{1((1+x)^n - 1)}{(1+x) - 1} \\
 &= \frac{(1+x)^n - 1}{x} \quad \checkmark
 \end{aligned}$$

ii)

$$\begin{aligned}
 1 + (1+x) + (1+x)^2 + \dots + (1+x)^{n-1} &= \frac{(1+x)^n - 1}{x} \\
 &= \frac{(1 + {}^n C_1 x + {}^n C_2 x^2 + \dots + {}^n C_n x^n) - 1}{x} \quad \checkmark \\
 &= \frac{{}^n C_1 x + {}^n C_2 x^2 + \dots + {}^n C_n x^n}{x} \\
 &= {}^n C_1 + {}^n C_2 x + \dots + {}^n C_n x^{n-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{iii)} \quad \int_{-1}^0 ({}^n C_1 + {}^n C_2 x + \dots + {}^n C_n x^{n-1}) dx \\
 &= \left[{}^n C_1 x + \frac{1}{2} {}^n C_2 x^2 + \frac{1}{3} {}^n C_3 x^3 + \dots + \frac{1}{n} {}^n C_n x^n \right]_{-1}^0 \\
 &= {}^n C_1 - \frac{1}{2} {}^n C_2 + \frac{1}{3} {}^n C_3 - \dots - \frac{(-1)^{n+1}}{n} {}^n C_n \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 \text{iv)} \quad \text{Now } \sum_{r=1}^n \frac{(-1)^{r+1}}{r} {}^n C_r &= {}^n C_1 - \frac{1}{2} {}^n C_2 + \frac{1}{3} {}^n C_3 - \dots - \frac{(-1)^{n+1}}{n} {}^n C_n \\
 &= \int_{-1}^0 ({}^n C_1 + {}^n C_2 x + \dots + {}^n C_n x^{n-1}) dx \quad (\text{over})
 \end{aligned}$$

$$= \int_{-1}^0 1 + (1+x) + (1+x)^2 + \dots + (1+x)^{n-1} dx \quad \checkmark$$

$$= \left[x + \frac{(1+x)^2}{2} + \frac{(1+x)^3}{3} + \dots + \frac{(1+x)^n}{n} \right]_{-1}^0$$

$$= \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right) - (-1)$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

$$= \sum_{r=1}^n \frac{1}{r}$$

