Using Matrices to Solve Simultaneous Equations

2×2 matrix: $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Determinant of A: $\det A=|A|=ad-bc$

Adjoint matrix: $\text{adj}A=\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

swap

change signs

Inverse of A: $A^{-1}=\dfrac{\text{adj}A}{|A|}$

If $|A|=0$, then lines are parallel

i.e. no solutions

$$A\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} m \\ n \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}=A^{-1}\begin{pmatrix} m \\ n \end{pmatrix}$$

e.g. (i) $2x+3y=21 \ldots(1)$

$5x+2y=3 \ \ \ldots(2)$

$$\begin{pmatrix} 2 & 3 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 21 \\ 3 \end{pmatrix}$$

$|A| = 2\times 2 - 5\times 3$

$$\begin{pmatrix} 2 & 5 \\ 3 & 2 \end{pmatrix}$$

$$\text{adj}A=\begin{pmatrix} 2 & -3 \\ -5 & 2 \end{pmatrix}$$

1. swap

2.change signs

$$\begin{pmatrix} x \\ y \end{pmatrix}=\frac{1}{-11}\begin{pmatrix} 2 & -3 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 21 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}=\frac{1}{-11}\begin{pmatrix} 2\times 21-3\times 3 \\ -5\times 21+2\times 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}=\frac{1}{-11}\begin{pmatrix} 33 \\ -99 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -3 \\ 9 \end{pmatrix}$$

Multiply the row of the matrix with the column of the vector

$\therefore x=-3, y=9$

$(ii)\ 4x-5y=3 \quad \ldots(1)$

$3x+7y=13 \quad \ldots(2)$

$$\begin{pmatrix} 4 & -5 \\ 3 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 3 \\ 13 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}=\frac{1}{43}\begin{pmatrix} 7 & 5 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 3 \\ 13 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}=\frac{1}{43}\begin{pmatrix} 7\times 3+5\times 13 \\ -3\times 3+4\times 13 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}=\frac{1}{43}\begin{pmatrix} 86 \\ 43 \end{pmatrix}$$

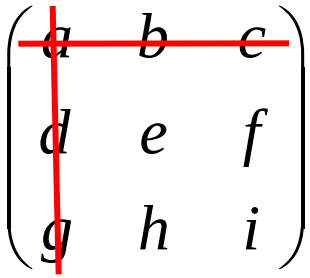
$$\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\therefore x=2, y=1$$

3×3 matrix: $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

Determinant of A: $\det A = |A| = aei + bfg + cdh - gec - hfa - idb$

Adjoint matrix: $\mathrm{adj}A = \begin{pmatrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} & -\begin{vmatrix} d & f \\ g & i \end{vmatrix} & \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ -\begin{vmatrix} b & c \\ h & i \end{vmatrix} & \begin{vmatrix} a & c \\ g & i \end{vmatrix} & -\begin{vmatrix} a & b \\ g & h \end{vmatrix} \\ \begin{vmatrix} b & c \\ e & f \end{vmatrix} & -\begin{vmatrix} a & c \\ d & f \end{vmatrix} & \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{pmatrix}^t$



1. cover up row and column of the position looking for
2. find determinant of the remaining matrix

t = transpose rows and columns

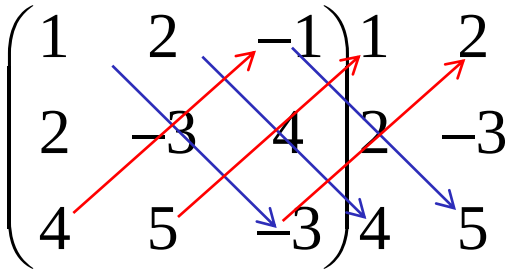
e.g $x+2y-z=-5 \quad \ldots(1)$

$2x-3y+4z=28 \quad \ldots(2)$

$4x+5y-3z=-10 \ldots(3)$

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & -3 & 4 \\ 4 & 5 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} -5 \\ 28 \\ -10 \end{pmatrix}$$

$$|A|=\begin{vmatrix} 1 & 2 & -1 \\ 2 & -3 & 4 \\ 4 & 5 & -3 \end{vmatrix}$$



$$=1\times-3\times-3+2\times4\times4+(-1)\times2\times5\ -4\times-3\times-1-5\times4\times1-(-3)\times2\times2$$

$$=9+32-10\ -12-20+12$$

$$=11$$

$$A=\begin{pmatrix}1 & 2 & -1\\ 2 & -3 & 4\\ 4 & 5 & -3\end{pmatrix}$$

$$\mathrm{adj}A=\begin{pmatrix}\begin{vmatrix}-3 & 4\\ 5 & -3\end{vmatrix} & -\begin{vmatrix}2 & 4\\ 4 & -3\end{vmatrix} & \begin{vmatrix}2 & -3\\ 4 & 5\end{vmatrix}\\ -\begin{vmatrix}2 & -1\\ 5 & -3\end{vmatrix} & \begin{vmatrix}1 & -1\\ 4 & -3\end{vmatrix} & -\begin{vmatrix}1 & 2\\ 4 & 5\end{vmatrix}\\ \begin{vmatrix}2 & -1\\ -3 & 4\end{vmatrix} & -\begin{vmatrix}1 & -1\\ 2 & 4\end{vmatrix} & \begin{vmatrix}1 & 2\\ 2 & -3\end{vmatrix}\end{pmatrix}^t$$

$$=\begin{pmatrix}-11 & 22 & 22\\ 1 & 1 & 3\\ 5 & -6 & -7\end{pmatrix}^t$$

$$=\begin{pmatrix}-11 & 1 & 5\\ 22 & 1 & -6\\ 22 & 3 & -7\end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -1 \\ 2 & -3 & 4 \\ 4 & 5 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 28 \\ -10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{11}\begin{pmatrix} -11 & 1 & 5 \\ 22 & 1 & -6 \\ 22 & 3 & -7 \end{pmatrix}\begin{pmatrix} -5 \\ 28 \\ -10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{11}\begin{pmatrix} -11\times -5+1\times 28+5\times -10 \\ 22\times -5+1\times 28-6\times -10 \\ 22\times -5+3\times 28-7\times -10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 33 \\ -22 \\ 44 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$$

$\therefore x = 3, y = -2, z = 4$

Worksheet
+
try some of Exercise 1H; 1, 2, 6