

Integration By Parts

When an integral is a product of two functions and neither is the derivative of the other, we integrate by parts.

$$\int u dv = uv - \int v du$$

u should be chosen so that differentiation makes it a simpler function.

dv should be chosen so that it can be integrated

$$\begin{aligned} \text{e.g. (i)} \quad & \int x \cos x dx \\ &= x \sin x - \int \sin x dx \\ &= x \sin x + \cos x + c \end{aligned}$$

$$\begin{aligned} u &= x & v &= \sin x \\ du &= dx & dv &= \cos x dx \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & \int \log x dx \\ &= x \log x - \int dx \\ &= x \log x - x + c \end{aligned}$$

$$\begin{aligned} u &= \log x & v &= x \\ du &= \frac{dx}{x} & dv &= dx \end{aligned}$$

$$(iii) \int_0^1 x e^{-7x} dx$$

$$= \left[-\frac{1}{7} x e^{-7x} \right]_0^1 + \frac{1}{7} \int_0^1 e^{-7x} dx$$

$$= \left[-\frac{1}{7} x e^{-7x} - \frac{1}{49} e^{-7x} \right]_0^1$$

$$= \left\{ -\frac{1}{7} e^{-7} - \frac{1}{49} e^{-7} \right\} - \left\{ 0 - \frac{1}{49} \right\}$$

$$= -\frac{8}{49} e^{-7} + \frac{1}{49}$$

$$u = x$$

$$du = dx$$

$$v = -\frac{1}{7} e^{-7x}$$

$$dv = e^{-7x} dx$$

$$(iv) \int e^x \cos x dx$$

$$= e^x \sin x - \int e^x \sin x dx$$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x dx$$

$$\therefore 2 \int e^x \cos x dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x dx = \frac{1}{2} e^x \sin x + \frac{1}{2} e^x \cos x + c$$

$$u = e^x$$

$$v = \sin x$$

$$du = e^x dx$$

$$dv = \cos x dx$$

$$u = e^x$$

$$v = -\cos x$$

$$du = e^x dx$$

$$dv = \sin x dx$$

Exercise 2B; 1 to 6 ac, 7 to 11, 12 ace etc