to $\displaystyle\int \operatorname{cosec}^3 x\,dx$

$u = \operatorname{cosec} x$ $\quad$ $v = -\cot x$

$du = -\operatorname{cosec} x \cot x\,dx$ $\quad$ $dv = \operatorname{cosec}^2 x\,dx$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x \cot^2 x\,dx$$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x(\operatorname{cosec}^2 x - 1)\,dx$$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x\,dx + \int \operatorname{cosec} x\,dx$$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x\,dx - \log(\operatorname{cosec} x + \cot x)$$

$$\therefore 2\int \operatorname{cosec}^3 x\,dx = -\operatorname{cosec} x \cot x - \log(\operatorname{cosec} x + \cot x) + c$$

$$\underline{\int \operatorname{cosec}^3 x\,dx = -\frac{1}{2}\operatorname{cosec} x \cot x - \frac{1}{2}\log(\operatorname{cosec} x + \cot x) + c}$$

24 $\displaystyle\int \frac{\sin x}{2+\cos x}\,dx$

$$= -\log(2+\cos x) + c$$

$u = t^2$

$du = 2t\,dt$

$$\int \frac{\frac{2t}{1+t^2}\cdot\frac{2\,dt}{1+t^2}}{2+\frac{1-t^2}{1+t^2}}$$

$$= \int \frac{4t\,dt}{2(1+t^2)^2 + (1-t^2)(1+t^2)}$$

$$= \int \frac{4t\,dt}{t^4+4t^2+3}$$

$$= \int \frac{2\,du}{u^2+4u+3}$$

$$= \int \frac{2\,du}{(u+2)^2-1}$$

$$= \log\left(\frac{u+2-1}{u+2+1}\right) + c$$

$$= \log\left(\frac{t^2+1}{t^2+3}\right) + c$$

$$= \log\left(\frac{\tan^2\frac{x}{2}+1}{\tan^2\frac{x}{2}+3}\right) + c$$