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$$\int_0^{\frac{\pi}{2}} \sin^n\theta \, d\theta = \frac{n-1}{n} \times \frac{n-3}{n-2} \times \cdots \times \frac{2}{3} I_1$$

If n is odd:

$$= \frac{(n-1)(n-3) \times \cdots \times 2}{n(n-2) \times \cdots \times 3} \int_0^{\frac{\pi}{2}} \sin\theta \, d\theta$$

$$= \frac{(n-1)(n-3) \times \cdots \times 2}{n(n-2) \times \cdots \times 3} \Big[-\cos\theta \Big]_0^{\frac{\pi}{2}}$$

$$= \frac{(n-1)(n-3) \times \cdots \times 2}{n(n-2) \times \cdots \times 3} \times 1$$

$$= \frac{(n-1)(n-3) \times \cdots \times 2}{n(n-2) \times \cdots \times 3 \times 1}$$

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$$\int x^n \sin^{-1}x\,dx = \frac{x^{n+1}}{n+1}\sin^{-1}x - \frac{1}{n+1}\int \frac{x^{n+1}}{\sqrt{1-x^2}}\,dx$$

$$\int_0^1 x\sin^{-1}x = \left[\frac{x^2}{2}\sin^{-1}x\right]_0^1 - \frac{1}{2}\int_0^1 \frac{x^2}{\sqrt{1-x^2}}\,dx$$

$$= \frac{\pi}{4} + \frac{1}{2}\int_0^1 \frac{1-x^2}{\sqrt{1-x^2}}\,dx - \frac{1}{2}\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$= \frac{\pi}{4} + \frac{1}{2}\int_0^1 \sqrt{1-x^2}\,dx - \frac{1}{2}\left[\sin^{-1}x\right]_0^1$$

$$= \frac{\pi}{4} - \frac{\pi}{4} + \frac{1}{2}\times\frac{1}{4}\pi(1)^2$$

$$= \underline{\underline{\frac{\pi}{8}}}$$