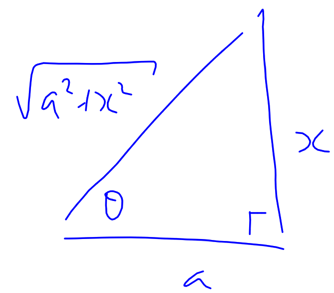
17.

$$\int \frac{dx}{x\sqrt{a^2+x^2}}$$

$x = a\tan\theta$

$dx = a\sec^2\theta \, d\theta$



$$= \int \frac{a\sec^2\theta \, d\theta}{a\tan\theta \cdot a\sec\theta}$$

$$= \frac{1}{a}\int \frac{\sec\theta}{\tan\theta} \, d\theta$$

$$= \frac{1}{a}\int \operatorname{cosec}\theta \, d\theta$$

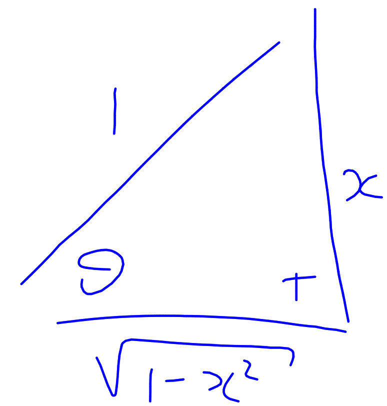
$$= -\frac{1}{a}\log\left(\operatorname{cosec}\theta + \cot\theta\right) + c$$

$$= \frac{1}{a}\log\left(\frac{\sqrt{a^2+x^2}}{x} + \frac{a}{x}\right) + c$$

16) $$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$$

$x = \sin\theta$

$dx = \cos\theta \, d\theta$



$$= \int \frac{\cos\theta \, d\theta}{\sin^2\theta \cdot \cos\theta}$$

$$= \int \operatorname{cosec}^2\theta \, d\theta$$

$$= -\cot\theta + c$$

$$= -\frac{\sqrt{1-x^2}}{x} + c$$

2c)

$$\int \frac{x}{\sqrt{x}+1}\,dx$$

$x = u^2$

$dx = 2u\,du$

$$= \int \frac{u^2 \cdot 2u\,du}{u+1}$$

$$= \int \frac{2u^3}{u+1}\,du$$

$=$

27/ $$\int \frac{dx}{(1+x^2)^2}$$

$x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$$= \int \frac{\sec^2\theta \, d\theta}{\sec^4\theta}$$

$$= \int \frac{1}{\sec^2\theta} \, d\theta$$

$$= \int \cos^2\theta \, d\theta$$

7) $\int \log\left(x+\sqrt{x^2-1}\right)dx$

$u = \log\left(x+\sqrt{x^2-1}\right)$ $\qquad v = x$

$= x\log\left(x+\sqrt{x^2-1}\right) - \int \frac{x\,dx}{\sqrt{x^2-1}}$

$= x\log\left(x+\sqrt{x^2-1}\right) - \sqrt{x^2-1} + c$

$du = \frac{1+\frac{x}{\sqrt{x^2-1}}}{x+\sqrt{x^2-1}}\,dx$ $\qquad dv = dx$

$= \frac{x+\sqrt{x^2-1}}{x\sqrt{x^2-1}+x^2-1}\,dx$

$= \frac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}\left(\sqrt{x^2-1}+x\right)}\,dx$

$= \frac{dx}{\sqrt{x^2-1}}$

42/

$$\int \frac{dx}{e^{x}+e^{-x}}$$

$$= \int \frac{dx}{e^{x}+\frac{1}{e^{x}}}$$

$$= \int \frac{e^{x}\,dx}{e^{2x}+1}$$

$$= \int \frac{du}{u^{2}+1}$$

$$= \tan^{-1} u + c$$

$$= \tan^{-1} e^{x} + c$$

$u = e^{x}$

$du = e^{x}\,dx$

55)

$$\int \frac{dx}{e^{x}-1}$$

$$x = \log u$$

$$dx = \frac{du}{u}$$

$$= \int \frac{\frac{du}{u}}{u-1}$$

$$= \int \frac{du}{u(u-1)}$$

$$\int \frac{dx}{e^x - 1} = \int \left[\frac{1 - e^x}{e^x - 1} + \frac{e^x}{e^x - 1} \right] dx$$

$$= \int \left(-1 + \frac{e^x}{e^x - 1} \right) dx$$

$$= -x + \log(e^x - 1) + c$$

7b)

$$\int \frac{dx}{(x+1)^{\frac{1}{2}} + (x+1)}$$

$u^2 = x+1$

$2u\,du = dx$

$$= \int \frac{2u\,du}{u+u^2}$$

$$= 2\int \frac{du}{1+u}$$

$$= 2\log(1+u) + c$$

$$= 2\log\left(1+\sqrt{x+1}\right) + c$$