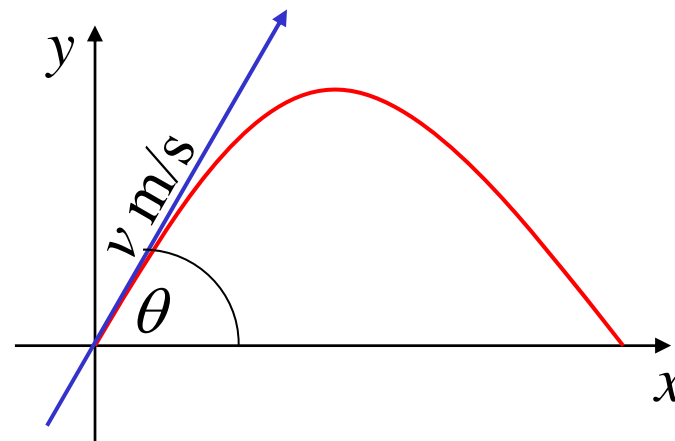


Projectile Motion

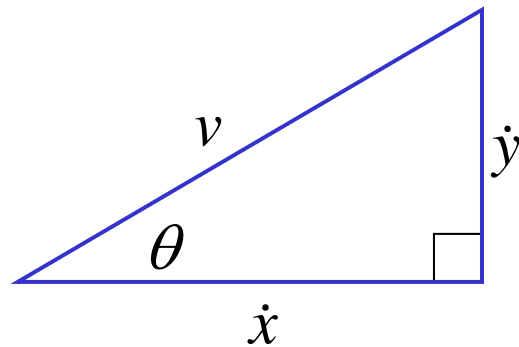


maximum range

$$\theta = 45^\circ$$

Initial conditions

when $t = 0$



$$\frac{\dot{x}}{v} = \cos \theta$$

$$\dot{x} = v \cos \theta$$

$$x = 0$$

$$\frac{\dot{y}}{v} = \sin \theta$$

$$\dot{y} = v \sin \theta$$

$$y = 0$$

$$\underline{\ddot{x} = 0}$$

$$\dot{x} = c_1$$

when $t = 0, \dot{x} = v \cos \theta$

$$c_1 = v \cos \theta$$

$$\underline{\dot{x} = v \cos \theta}$$

$$x = vt \cos \theta + c_3$$

when $t = 0, x = 0$

$$c_3 = 0$$

$$\underline{x = vt \cos \theta}$$

$$\underline{\ddot{y} = -g}$$

$$\dot{y} = -gt + c_2$$

$$\dot{y} = v \sin \theta$$

$$c_2 = v \sin \theta$$

$$\underline{\dot{y} = -gt + v \sin \theta}$$

$$y = -\frac{1}{2}gt^2 + vt \sin \theta + c_4$$

$$y = 0$$

$$c_4 = 0$$

$$\underline{y = -\frac{1}{2}gt^2 + vt \sin \theta}$$

Note: parametric coordinates of a parabola

$$t = \frac{x}{v \cos \theta} \quad y = -\frac{gx^2}{2v^2 \cos^2 \theta} + \frac{x \sin \theta}{\cos \theta}$$

$$y = -\frac{gx^2}{2v^2} \sec^2 \theta + x \tan \theta$$

$$y = -\frac{gx^2}{2v^2} (\tan^2 \theta + 1) + x \tan \theta$$

Common Questions

(1) When does the particle hit the ground?

Particle hits the ground when $y = 0$

(2) What is the range of the particle?

(i) find when $y = 0$

roots of the quadratic

(ii) substitute into x

(3) What is the greatest height of the particle?

(i) find when $\dot{y} = 0$

vertex of the parabola

(ii) substitute into y

(4) What angle does the particle make with the ground?

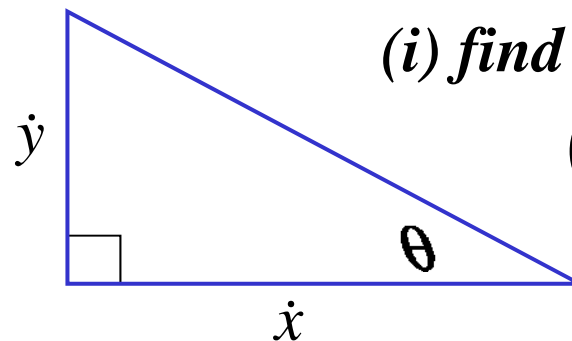
(i) find when $y = 0$

(i) find slope of the tangent

(ii) substitute into \dot{y}

(ii) $m = \tan \theta$

(iii) $\tan \theta = \frac{\dot{y}}{\dot{x}}$



Summary

A particle undergoing projectile motion obeys

$$\ddot{x} = 0 \quad \text{and} \quad \ddot{y} = -g$$

with initial conditions

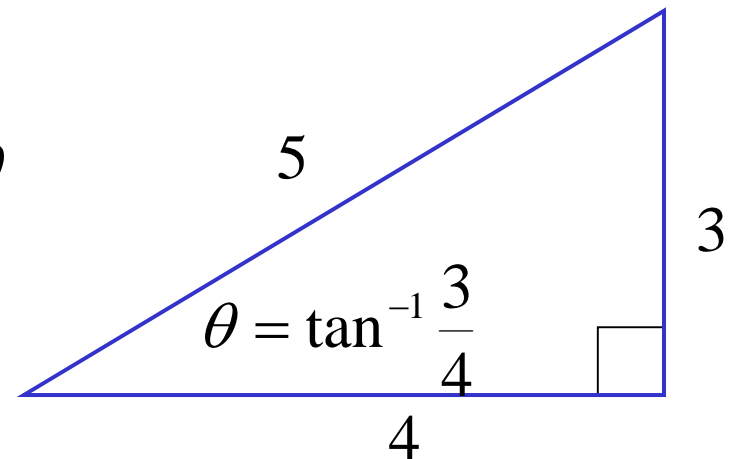
$$\dot{x} = v \cos \theta \quad \text{and} \quad \dot{y} = v \sin \theta$$

e.g. A ball is thrown with an initial velocity of 25 m/s at an angle of $\theta = \tan^{-1} \frac{3}{4}$ to the ground. Determine;

a) greatest height obtained

Initial conditions

$$\begin{aligned} \dot{x} &= v \cos \theta & \dot{y} &= v \sin \theta \\ \dot{x} &= 25 \left(\frac{4}{5} \right) & \dot{y} &= 25 \left(\frac{3}{5} \right) \\ &= 20 \text{m/s} & &= 15 \text{m/s} \end{aligned}$$



$$\underline{\ddot{x} = 0}$$

$$\dot{x} = c_1$$

when $t = 0, \dot{x} = 20$

$$c_1 = 20$$

$$\underline{\dot{x} = 20}$$

$$x = 20t + c_3$$

when $t = 0, x = 0$

$$c_3 = 0$$

$$\underline{x = 20t}$$

greatest height occurs when $\dot{y} = 0$

$$-10t + 15 = 0$$

$$t = \frac{3}{2}$$

\therefore greatest height is $11\frac{1}{4}$ m above the ground

$$\underline{\ddot{y} = -10}$$

$$\dot{y} = -10t + c_2$$

$$\dot{y} = 15$$

$$c_2 = 15$$

$$\underline{\dot{y} = -10t + 15}$$

$$y = -5t^2 + 15t + c_4$$

$$y = 0$$

$$c_4 = 0$$

$$\underline{y = -5t^2 + 15t}$$

$$\text{when } t = \frac{3}{2}, y = -5\left(\frac{3}{2}\right)^2 + 15\left(\frac{3}{2}\right)$$

$$= \frac{45}{4}$$

b) range

time of flight is 3 seconds

$$\begin{aligned}\text{when } t = 3, x &= 20(3) \\ &= 60\end{aligned}$$

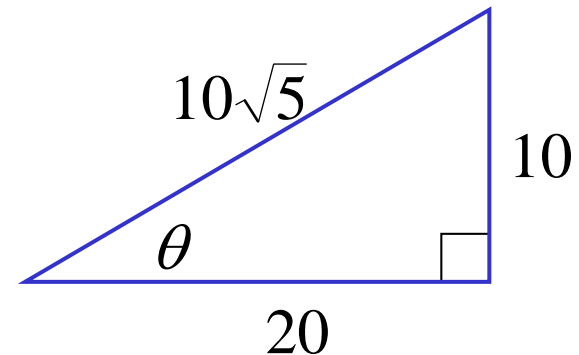
\therefore range is 60m

c) velocity and direction of the ball after $\frac{1}{2}$ second

$$\begin{aligned}\text{when } t = \frac{1}{2}, \dot{x} &= 20 & \dot{y} &= -10\left(\frac{1}{2}\right) + 15 \\ & & &= 10\end{aligned}$$

$$\tan \theta = \frac{1}{2}$$

$$\theta = 26^\circ 34'$$



\therefore after $\frac{1}{2}$ second, velocity = $10\sqrt{5}$ m/s and it is traveling at

an angle of $26^\circ 34'$ to the horizontal

d) cartesian equation of the path

$$x = 20t$$

$$y = -5t^2 + 15t$$

$$t = \frac{x}{20}$$

$$y = -5\left(\frac{x}{20}\right)^2 + 15\left(\frac{x}{20}\right)$$

$$y = \frac{-x^2}{80} + \frac{3x}{4}$$

Using the cartesian equation to solve the problem

a) greatest height is y value of the vertex

$$\begin{aligned}\Delta &= \left(\frac{3}{4}\right)^2 - 4\left(-\frac{1}{80}\right)(0) \\ &= \frac{9}{16}\end{aligned}$$

$$\begin{aligned}y &= -\frac{\Delta}{4a} \\ &= -\frac{9}{16} \times -\frac{20}{1} \\ &= \frac{45}{4}\end{aligned}$$

\therefore greatest height is $11\frac{1}{4}$ m above the ground

b) range

$$y = \frac{-x^2}{80} + \frac{3x}{4}$$
$$= \frac{x}{4} \left(3 - \frac{x}{20} \right)$$

roots are 0 and 60

\therefore range is 60m

**Exercise 3G; 1ac, 2ac,
4, 6, 8, 9, 11, 13, 16, 18**

**Exercise 3H; 2, 4, 6,
7, 10, 11**

c) velocity and direction of the ball after $\frac{1}{2}$ second

when $t = \frac{1}{2}$, $x = 10$

$$y = \frac{-x^2}{80} + \frac{3x}{4}$$

$$\frac{dy}{dx} = \frac{-x}{40} + \frac{3}{4}$$

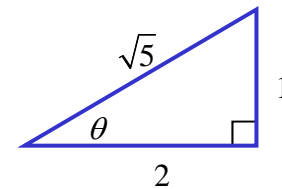
when $x = 10$, $\frac{dy}{dx} = \frac{1}{2}$

$$\tan \theta = \frac{1}{2}$$

$$\theta = 26^\circ 34'$$

$$v = \frac{x}{t \cos \theta}$$
$$= \frac{10}{\frac{1}{2} \times \frac{2}{\sqrt{5}}}$$

$$= 10\sqrt{5}$$



\therefore after $\frac{1}{2}$ second, velocity = $10\sqrt{5}$ m/s and it is

traveling at an angle of $26^\circ 34'$ to the horizontal