



Name: \_\_\_\_\_

Teacher: \_\_\_\_\_

Class: \_\_\_\_\_

FORT STREET HIGH SCHOOL

2011  
HIGHER SCHOOL CERTIFICATE COURSE  
ASSESSMENT TASK 3: TRIAL HSC

Mathematics Extension 1

**Time allowed: 2 hours**

(plus 5 minutes reading time)

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively.	1, 2	
Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry and polynomials.	3, 4	
Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as rates, kinematics and growth and decay.	5,6	
Synthesises mathematical solutions to harder problems such as projectiles and 3D trigonometry and communicates them in appropriate form.	7	

Question	1	2	3	4	5	6	7	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/84	

**Directions to candidates:**

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- **Each new question is to be started in a new booklet**

## STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

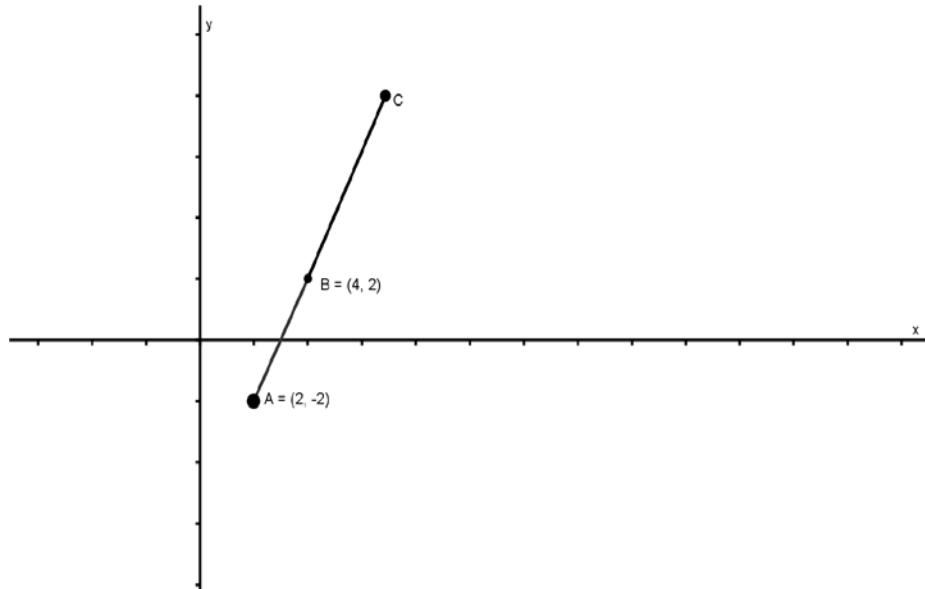
NOTE :  $\ln x = \log_e x, \quad x > 0$

Question 1 (12 marks)

Use a **SEPARATE** writing booklet

- a) If  $A(2,-2)$  and  $B(4,2)$ , find the co-ordinates of the point  $C(x, y)$ , as shown in the diagram below, given that  $AC : CB = 7 : 5$

2



- b) Find the perpendicular distance from the point  $(1,2)$  to the line  $y = 3x - 5$   
(Express the answer in exact rationalised form)

2

c) Solve  $\frac{x+2}{x+1} \geq 3$

3

d) Differentiate  $e^x \tan^{-1} \frac{x}{2}$

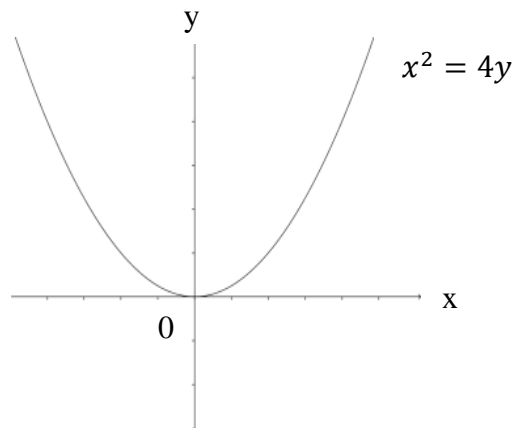
3

e) Evaluate  $\lim_{x \rightarrow 0} \frac{2 \sin 2x}{x}$

2

**Question 2** (12 marks)      **Use a SEPARATE writing booklet**

- a) T  $(2t, t^2)$  is a point on the parabola  $x^2 = 4y$  with focus F. The tangent to the parabola at T makes an acute angle  $\theta$  with the line FT.



- i) Show that the tangent to the parabola at T has gradient  $t$ . 1
- ii) Find  $\tan \theta$  in simplest form in terms of  $t$ . 3
- b) Evaluate  $2 \int_0^{\frac{\pi}{4}} \cos^2 4x \, dx$  3
- c) Without using calculus, sketch  $f(x) = \frac{x^2}{x^2 - 4}$  3
- Showing all the important features.
- d) Consider the function  $f(x) = x - e^{-2x}$ .
- Use one application of Newton's Method with an initial approximation of  $x = 0.5$  to find the value of the  $x$  intercept on the graph of  $y = f(x)$ , giving the answer correct to two decimal places. 2

**Question 3** (12 marks)      **Use a SEPARATE writing booklet**

a) A, B, C and D are points on the circumference of a circle.  
AB produced intersects DC produced at point P. AB = 12cm, BP = 3cm  
and CD = 4cm.

i) Draw a clear sketch showing the above information.

1

ii) Find the length of CP.

2

b) The equation  $8x^3 - 36x^2 + 22x + 21 = 0$

5

has roots which form an arithmetic progression. Find the roots.

c) Find the area enclosed between the curves  $y = \sin 2x$  and  $y = 2\sin^2 x$ .

$0 \leq x \leq \frac{\pi}{4}$ . (Answer correct to 2 decimal places).

4

**Question 4** (12 marks)      **Use a SEPARATE writing booklet**

a) i) Express  $\sqrt{3} \cos x - \sin x$  in the form of  $R \cos (x + a)$   
where  $0 < a < \frac{\pi}{2}$ , and  $R > 0$

2

ii) Hence, solve  $\sqrt{3} \cos x - \sin x = \sqrt{2}$  for  $0 \leq x \leq \pi$

2

(Answer in terms of  $\pi$ ).

b) Show that  $\frac{d}{dx} (\sin^{-1} x + \sqrt{1-x^2}) = \sqrt{\frac{1-x}{1+x}}$ ,

hence evaluate  $\int_0^{\frac{1}{2}} \sqrt{\frac{1-x}{1+x}} dx$  (Answer in exact form)

4

c) Use Mathematical Induction to prove the following result for positive integral  
values of n:

4

$$\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{1}{1.3} + \frac{1}{3.5} + \dots \dots \dots \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

**Question 5** (12 marks)      **Use a SEPARATE writing booklet**

a) A particle P, initially at rest at  $x = 2$  metres from the origin is moving along a straight line with an acceleration given by:

$$\frac{d^2x}{dt^2} = -4 \left( x + \frac{16}{x^3} \right).$$

i) Show that if the velocity is  $v$  m/s at any given time, then

3

$$v^2 = \frac{64}{x^2} - 4x^2$$

ii) Hence, calculate the velocity when P is **halfway to** the origin.

1

iii) Calculate the **time** taken for the particle to reach the origin, given that

$$\frac{d}{dx} \left( \frac{1}{2} \cos^{-1} \left( \frac{x}{2} \right)^2 \right) = \frac{-x}{\sqrt{16-x^4}} \quad (\text{Answer in terms of } \pi). \quad 4$$

b) Laura placed a cup of noodle soup with a temperature  $95^\circ\text{C}$  in her room which has a temperature of  $20^\circ\text{C}$ . In 5 minutes the cup of noodle soup cools to  $60^\circ\text{C}$ . Assuming the rate of heat loss is proportional to the excess of its temperature above room temperature, that is:

$$\frac{dT}{dt} = -k(T - 20),$$

i) Show that  $T = 20 + Ae^{-kt}$  is a solution of

1

$$\frac{dT}{dt} = -k(T - 20),$$

ii) If Laura likes to drink her noodle soup at  $50^\circ\text{C}$ . Calculate the **extra** minutes she has to leave it to cool down.

(Answer to 1 decimal place).

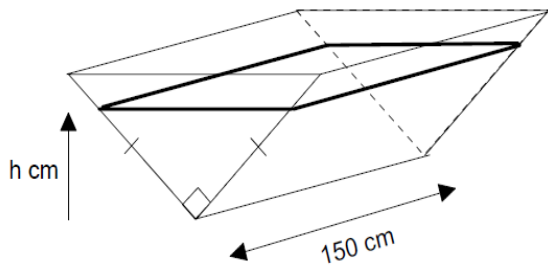
3

**Question 6** (12 marks)      **Use a SEPARATE writing booklet**

a) The displacement,  $x$  cm, of an object from the origin is given by  
 $x = 2\sin t - 3\cos t$ ,  $t \geq 0$ , where time  $t$ , is measured in seconds.

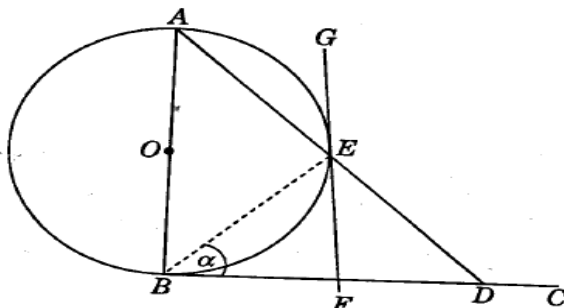
- i) Show that the object is moving in Simple Harmonic Motion. 1
- ii) At what time does the object **first** reach its maximum velocity? 2  
 (Answer correct to 2 decimal places).

b) The diagram below shows a water trough 150cm long that has a cross section of a right angled isosceles triangle. Water is poured in at a constant rate of 3 litres per minute.



- i) Show that when the depth of water is  $h$  cm, the volume of water in the tank is  $150h^2 \text{ cm}^3$ . 2
- ii) Find the rate at which the water is rising when the depth is 5 cm. 3

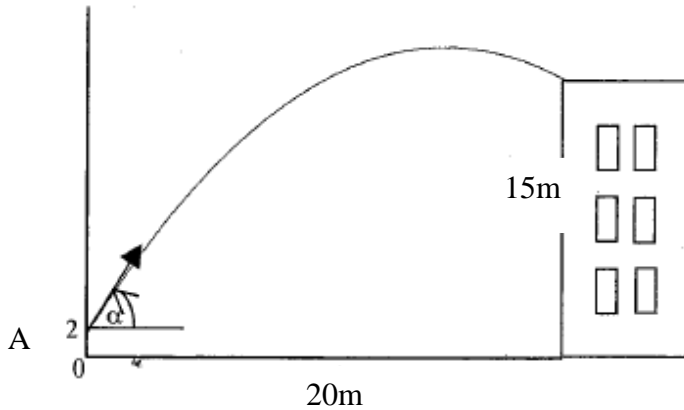
c) In the diagram, AB is a diameter of the circle, centre **O**, and BC is a tangent to the circle at B. The line AED intersects the circle at E and BC at D. The tangent to the circle at E intersects BC at F, Let  $\angle EBF = \alpha$ .



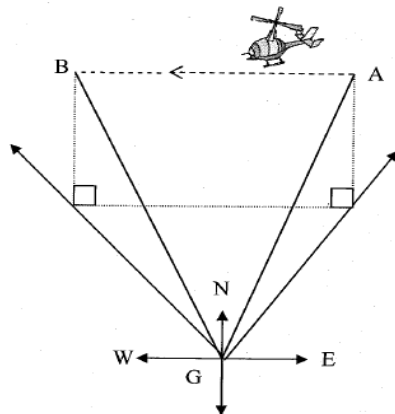
- i) Copy the diagram into your Writing Booklet with all the relevant information. 1
- ii) Prove that  $\angle FED = \frac{\pi}{2} - \alpha$ . 3

**Question 7** (12 marks)      **Use a SEPARATE writing booklet**

- a) Andrew whose height is **2 metres** throws a ball from area A to the roof of the Cohen building which is **15metres** high. He throws the ball at an initial velocity of **25m/s**, and he is **20 metres** from the base of the building. (Assume  $\ddot{x} = 0$  and  $\ddot{y} = -10\text{m/s}^2$ )



- i) Show that  $y = x \tan \alpha - \frac{5x^2}{v^2} (1 + \tan^2 \alpha) + 2$ , at any time  $t$ . 3
- ii) Hence, find between which two angles of projection must he throw the ball to ensure that it lands on the roof of the building?  
(Answer to the nearest degrees). 3
- b) A helicopter flies due west from A to B at a constant speed of **420km/h**. From a point G on the ground the bearing of the helicopter when it is at A is  **$079^\circ T$**  with an angle of elevation  $\beta$ . Four minutes later the helicopter is at B with a bearing from G being  **$302^\circ T$**  and an angle of elevation  **$32^\circ$** . The altitude of the helicopter is  **$h \text{ km}$** .



- i) Calculate the height of the plane to the nearest metre. 4
- ii) Calculate the value of  $\beta$  to the nearest degree. 2



Extra

- a) i) Show that  $\frac{d}{dx} \log (\operatorname{cosec} x + \cot x) = -\operatorname{cosec} x$  1
- ii) Determine the volume generated when  $y = -\operatorname{cosec} x$ , 2  
is rotated about the  $x$ -axis, and the ordinates  $x = \frac{\pi}{3}$  and  $x = \frac{\pi}{2}$ .  
(Leave the answer in terms of  $\pi$ ).

**END**

# Solutions

1)

a)  $A(2, -2)$        $B(4, 2)$

X  
7    -3

C is external

Point C:

$$x = \frac{7(4) - (5 \times 2)}{2} \quad \checkmark$$

$$y = \frac{7(2) + 10}{2}$$

$$\therefore (9, 12) \quad \checkmark$$

① working  
① point.

Many did not realise this was external

b)  $d = \frac{ax + by + c}{\sqrt{a^2 + b^2}}$

Point (1, 2)

$$a = 3, b = -1, c = -5$$

division and still many errors in formula.

$$= \frac{3 - 2 - 5}{\sqrt{10}} \quad \checkmark$$

① working  
① Answer

$$= \frac{-4}{\sqrt{10}} = \frac{4}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$

$$= \frac{2\sqrt{10}}{5} \quad \checkmark$$

c)  $\frac{x+2}{x+1} \geq 3$       ( $x \neq -1$ )

(x b.s by  $(x+1)^2$ )

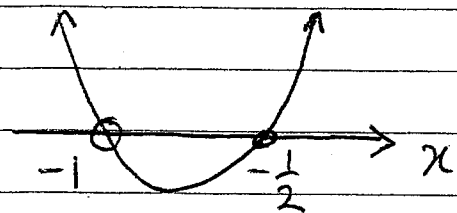
$$(x+2)(x+1) \geq 3(x+1)^2 \quad \checkmark$$

$$3(x+1)^2 - (x+1)(x+2) \leq 0$$

$$(x+1)[3x+3 - (x+2)] \leq 0$$

$$(x+1)(2x+1) \leq 0$$

$$-1 < x \leq -\frac{1}{2} \quad \checkmark$$



① working  
(any method)

① values

① correct inequality signs      ①

$x = -1$  so need to take care not to put it in final answer.

$$e) \frac{d}{dx} (e^x \cdot \tan^{-1} \frac{x}{2}) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$= e^x \cdot \frac{2}{4+x^2} + \tan^{-1} \frac{x}{2} \cdot e^x$$

$$= \frac{2e^x}{4+x^2} + e^x \cdot \tan^{-1} \frac{x}{2}$$

$$u = e^x$$

$$\frac{du}{dx} = e^x$$

$$v = \tan^{-1} \frac{x}{2}$$

$$\frac{dv}{dx} = \frac{2}{1+(\frac{x}{2})^2}$$

$$= \frac{2}{4+x^2}$$

① Product rule

① correct derivative of  $\tan^{-1} \frac{x}{2}$

① Answer

f) Evaluate:

$$\lim_{x \rightarrow 0} \frac{2 \sin 2x}{x}$$

$$= 4 \lim_{x \rightarrow 0} \frac{\sin 2x}{2x}$$

$$= 4$$

## Question 2

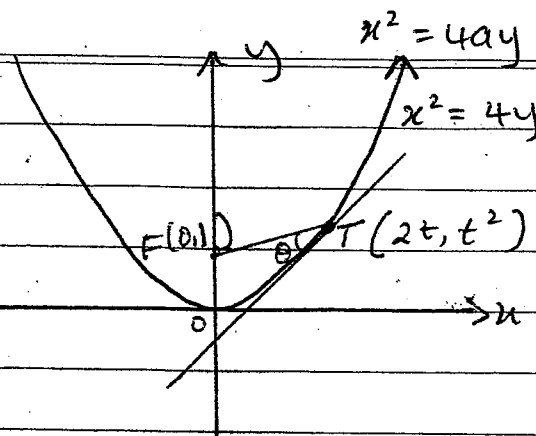
a) i)

$$y = \frac{x^2}{4}$$

$$\frac{dy}{dx} = \frac{x}{2}, T(2t, t^2)$$

$$\frac{dy}{dx} = \frac{2t}{2}$$

$$= t \quad \text{--- ①}$$



$$4a = 4$$

$$a = 1$$

Some did not calculate the value of  $a$ .

ii) from ①

$$m_T = t$$

$$m_{FT} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{t^2 - 1}{2t - 0}$$

$$= \frac{t^2 - 1}{2t} \quad \text{--- ②}$$

$$m_1 - m_2$$

$$= t - \left( \frac{t^2 - 1}{2t} \right)$$

$$= \frac{2t^2 - t^2 + 1}{2t}$$

$$= \frac{t^2 + 1}{2t}$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$1 + m_1 m_2$$

$$= 1 + t \cdot \frac{t^2 - 1}{2t}$$

$$= 1 + \frac{t^2 - 1}{2}$$

$$= \left| \frac{(t^2 + 1) \times 2}{2t (t^2 + 1)} \right|$$

$$= \frac{t^2 + 1}{2}$$

$$= \frac{1}{t} \quad \checkmark$$

many made mistakes in the algebraic manipulation of  $\frac{m_1 - m_2}{1 + m_1 m_2}$

Some left the answer for  $\tan \theta$  as an obtuse angle. (Did not read the question carefully)

③

$$b) \int_0^{\frac{\pi}{4}} 2 \cos^2 4x \, dx$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = \cos^2 x - (1 - \cos^2 x)$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos^2 x = \frac{1}{2} (\cos 2x + 1)$$

$$= 2 \int_0^{\frac{\pi}{4}} \frac{1}{2} (\cos 8x + 1) \, dx$$

$$= \int_0^{\frac{\pi}{4}} (\cos 8x + 1) \, dx$$

$$= \left[ \frac{1}{8} \sin 8x + x \right]_0^{\frac{\pi}{4}}$$

mostly well done.

$$= \left[ \frac{\pi}{4} + \frac{1}{8} \sin \left( 8 \times \frac{\pi}{4} \right) \right] - [0]$$

$$= \frac{\pi}{4}$$

$$c) f(x) = \frac{x^2}{x^2 - 4} \quad (x \neq \pm 2)$$

$$y = \frac{x^2}{(x-2)(x+2)}$$

$$\begin{array}{r} x^2 - 4 \overline{) x^2} \\ \underline{-(x^2 - 4)} \\ 4 \end{array}$$

$$\therefore y = 1 + \frac{4}{x^2 - 4}$$

$$(y-1) = \frac{4}{(x-2)(x+2)}$$

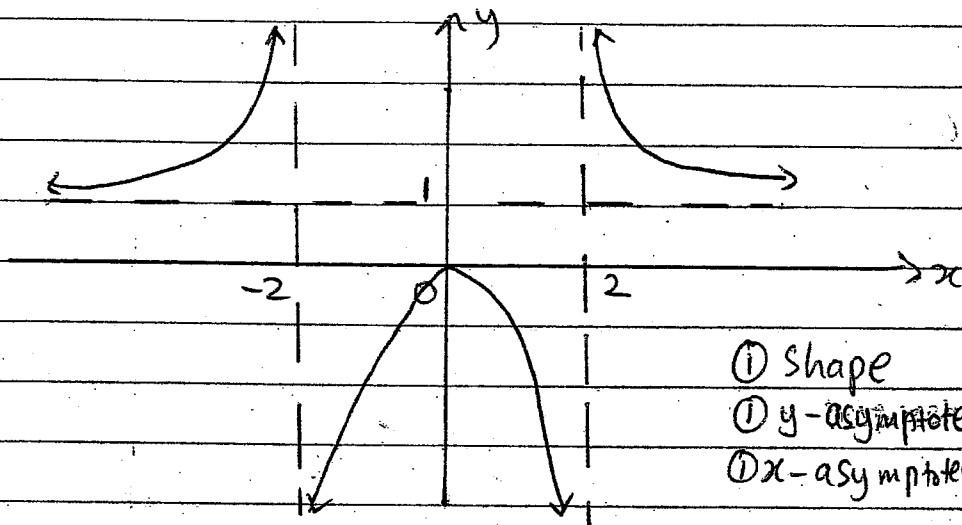
Intercepts:

$$x=0, y=0$$

Critical Points:

$$(x-2)(x+2)(y-1) = 4$$

$$x \neq \pm 2, y \neq 1$$



many did not find the horizontal asymptote.

- ① Shape
- ① y-asymptote
- ① x-asymptotes

d)  $f(x) = x - e^{-2x} \Rightarrow f(0.5) = 0.5 - e^{-1} = 0.132\dots$

$$f'(x) = 1 + 2e^{-2x}$$

$$f'(0.5) = 1 + 2e^{-1} = 1.7358\dots$$

well done.

$$x = 0.5 - \frac{f(x)}{f'(x)}$$

$$= 0.5 - \frac{f(0.5)}{f'(0.5)} \checkmark$$

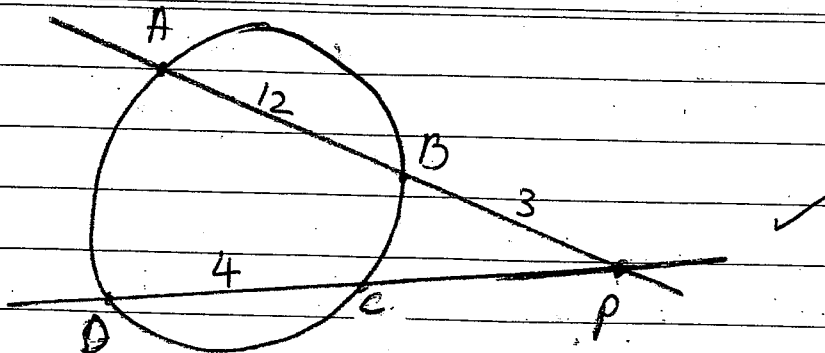
- ① Working
- ① Answer

$$= 0.5 - \frac{0.132\dots}{1.7368\dots}$$

$$= 0.42 \checkmark$$

### Question 3

3)ai)



Very poorly done

Students need to learn this rule.

ii) let  $CP = x$

$$x(x+4) = 3(15) \quad \checkmark$$

$$x^2 + 4x - 45 = 0$$

$$(x+9)(x-5) = 0$$

$$x = 5 \text{ cm}, (x \neq -9)$$

✓

Also points are always cyclically named.

DC - produced means to extend DC in that direction

b)  $8x^3 - 36x^2 + 22x + 21 = 0$

$$x + \beta + \gamma = \frac{36}{8} = \frac{9}{2} \quad \text{--- (1)} \quad \checkmark$$

$$x\beta + x\gamma + \beta\gamma = \frac{22}{8} = \frac{11}{4} \quad \text{--- (2)}$$

$$x\beta\gamma = \frac{-21}{8} \quad \text{--- (3)} \quad \checkmark$$

For AP:  $x, \beta, \gamma$

$$\boxed{\beta = \frac{x + \gamma}{2}} \quad \text{or} \quad \boxed{2\beta = x + \gamma}$$

(4)

Madly made a reasonably simple question very complicated & too many simple arithmetic errors.



Sub ④ into ①

$$3\beta = \frac{9}{2}$$

$$\beta = \frac{3}{2} \checkmark$$

∴ from ①

$$\alpha + \gamma = 3$$

$$\alpha = 3 - \gamma$$

from ③

$$\alpha\gamma \cdot \frac{3}{2} = -\frac{21}{8}$$

$$\alpha\gamma = -\frac{7}{4} \quad \text{--- ⑤}$$

$$\therefore \alpha(3-\alpha) = -\frac{7}{4} \checkmark$$

$$4\alpha^2 - 12\alpha = 7$$

$$4\alpha^2 - 12\alpha - 7 = 0$$

$$(2\alpha + 1)(2\alpha - 7) = 0$$

$$\alpha = -\frac{1}{2}, \frac{7}{2} \checkmark$$

$$\begin{aligned} \gamma &= 3 - \alpha \\ &= \frac{7}{2} \end{aligned}$$

∴ The roots are:  $-\frac{1}{2}, \frac{3}{2}, \frac{7}{2}$

difference = 2

c)  $\int_0^{\frac{\pi}{4}} \sin 2x - 2\sin^2 x \, dx$

$$= \int_0^{\frac{\pi}{4}} (\sin 2x + \cos 2x - 1) \, dx \checkmark$$

$$= \left[ -\frac{\cos 2x}{2} + \frac{\sin 2x}{2} - x \right]_0^{\frac{\pi}{4}}$$

$$= \left[ -0 + \frac{1}{2} - \frac{\pi}{4} \right] - (-1 + 0 - 0)$$

$$= 0.21 \text{ cm}^2 \checkmark$$

$$2\sin^2 x = 1 - \cos 2x$$

② Working  
any method

① 1 root solved

① working to  
find the other  
roots

① Answers

Need to  
always take  
absolute  
value in  
case wrong  
curve or  
dep. Also

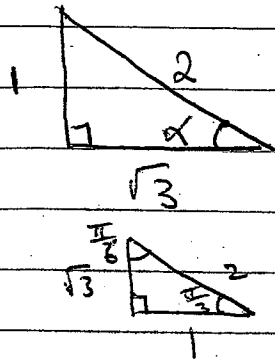
- ① substitution
- ① integration
- ① values
- ① Answer

⑦

# Question 4

a) i)  $\sqrt{3} \cos x - \sin x = R \cos(x + \alpha)$

mostly well executed



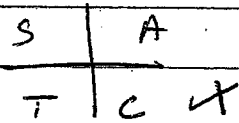
$$\begin{aligned} \tan \alpha &= \frac{b}{a} \\ &= \frac{1}{\sqrt{3}} \\ &= \frac{\pi}{6} \end{aligned}$$

$\therefore \sqrt{3} \cos x - \sin x = 2 \cos(x + \frac{\pi}{6})$

- ① The amplitude
- ① The acute angle

ii) hence:

$$2 \cos(x + \frac{\pi}{6}) = \sqrt{2}$$



Full marks for answer

$$\cos(x + \frac{\pi}{6}) = \frac{\sqrt{2}}{2} \quad (0 \leq x \leq \pi)$$

of  $\frac{\pi}{12}$  without showing elimination of  $\frac{\pi}{12}$

$$x + \frac{\pi}{6} = \frac{\pi}{4}, \frac{7\pi}{4}$$

$$x = \frac{\pi}{12}, \frac{19\pi}{12}$$

$\frac{\pi}{12}$

$x = \frac{\pi}{12}$  is the only soln.

$$b) \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{2} (1-x^2)^{-\frac{1}{2}} (-2x)$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} \quad \checkmark$$

$$= \frac{1-x}{\sqrt{1-x^2}}$$

$$= \frac{(1-x)}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}$$

$$= \frac{(1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}}} \quad \checkmark$$

$$= \sqrt{\frac{(1-x)}{(1+x)}}$$

$$\therefore \int_0^{\frac{1}{2}} \sqrt{\frac{1-x}{1+x}} dx$$



$$= \left[ \sin^{-1} x + \sqrt{1-x^2} \right]_0^{\frac{1}{2}}$$

$$= \left[ \frac{\pi}{6} + \sqrt{\frac{3}{4}} \right] - [0+1]$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{2} - 1 \quad \checkmark$$

straight forward  
for most  
students,  
some making  
numerical  
errors.

9

$$c) S(n) = \sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \dots + \frac{1}{(2n-1)(2n+1)}$$

$$= \frac{n}{2n+1}$$

errors included  
assumption  
containing

Step 1:

let  $n=1$

$$\text{LHS} = \frac{1}{(1)(3)}$$

$$\text{RHS} = \frac{1}{3}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore S(1)$  is true

$$\frac{1}{(2k-1)(2k+1)}$$

on LHS only,

ie not

Step 2:

Assume  $S(k)$  is also true ✓

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1} \text{ --- ①}$$

whole sum,  
working on bs

in proof,  
insufficient

conclusion

(not linking the  
previous steps)

Step 3:

Prove  $S(k+1)$

LHS : from ①

RHS :

$$\frac{k}{2k+1} + \frac{1}{(2(k+1)-1)(2(k+1)+1)}$$

$$= \frac{k+1}{2(k+1)+1}$$

$$\frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{k+1}{2k+3}$$

$$\text{LHS} = \frac{k(2k+3) + 1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2 + 3k + 1}{(2k+1)(2k+3)}$$

$$= \frac{(2k+1)(k+1)}{(2k+1)(2k+3)} = \frac{k+1}{2k+3}$$

$$= \text{RHS}$$

① assumption

① using the  
assumption to  
prove for  $k+1$

① proving

① conclusion

$\therefore$  Since it is true for  $n=1, n=k, n=k+1$  ( $n=1, n=2, n=$   
then it is true for  $n \geq 1$  ✓

⑩

# Question 5

a) i)  $\frac{d^2x}{dt^2} = -4\left(x + \frac{16}{x^3}\right)$

$$\frac{d}{dx}\left(\frac{v^2}{2}\right) = -4\left(x + 16x^{-3}\right)$$

$$\frac{v^2}{2} = -4 \int \left(x + 16x^{-3}\right) dx \quad \checkmark$$

$$\frac{v^2}{2} = -4\left(\frac{x^2}{2} + \frac{16x^{-2}}{-2}\right) + C$$

$$\frac{v^2}{2} = -2x^2 + 64x^{-2} + C \quad \checkmark$$

$$v^2 = \frac{64}{x^2} - 4x^2 + C$$

Initially at rest,  $\therefore C = 0 \quad \checkmark$   
 ie  $\therefore v = 0, x = 2$

$$v^2 = \frac{64}{x^2} - 4x^2 \quad \text{as required.}$$

ii)  $v^2 = \frac{64}{x^2} - 4x^2 \quad (x = 1)$

$$v^2 = 64 - 4$$

$$v^2 = 60$$

$$v = -2\sqrt{15} \text{ m/s } (-7.75 \text{ m/s})$$

\* negative velocity  $\checkmark$

Some tried to integrate (cannot integrate  $x$  with respect to  $t$ )!!

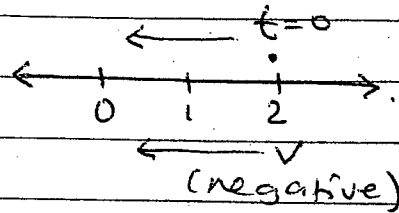
some ignored  $C$  + therefore did not evaluate it

Many did not realise velocity was negative (moving in a -ve direction)

No mark awarded

$$\text{iii) } V = - \left[ \frac{64 - 4x^4}{x^2} \right]^{\frac{1}{2}}$$

$$\frac{dx}{dt} = \left[ \frac{4(16 - x^4)}{x^2} \right]^{\frac{1}{2}}$$



$$= -2 \left( \frac{16 - x^4}{x^2} \right)^{\frac{1}{2}}$$

$$\therefore \frac{dt}{dx} = - \frac{x}{2 \sqrt{16 - x^4}} \quad \checkmark$$

- negative sign was omitted after.

$$t = \frac{1}{2} \int \frac{-x}{\sqrt{16 - x^4}} dx \quad \checkmark$$

$$= \frac{1}{2} \left[ \frac{1}{2} \cos^{-1} \left( \frac{x^2}{2} \right) \right]_2^0 \quad \checkmark$$

$$= \frac{1}{4} \left[ \frac{\pi}{2} - 0 \right]$$

$$= \frac{\pi}{8} \text{ seconds. } \checkmark$$

Some used '0' + used  $t=0, x=2$  to evaluate it

Many had trouble using the integral given correctly.

$\therefore$  It takes  $\frac{\pi}{8}$  s to get to the origin.

① expression for  $\frac{dt}{dx}$

① expression for  $t$

① Integration.  $\int$  / finding  $c$

① Answer

b) i)  $T = 20 + Ae^{-kt}$  ( $Ae^{-kt} = T - 20$ )

$$\frac{dT}{dt} = -kAe^{-kt}$$

$$= -k(T - 20)$$

$\therefore$  It is the solution.

ii)  $t = 0$

$$95 = 20 + Ae^{-0}$$

$$\therefore A = 75$$

$$T = 20 + 75e^{-kt}$$

$t = 5$   $60 = 20 + 75e^{-5k}$

$$75e^{-5k} = 40$$

$$-5k = \ln\left(\frac{8}{15}\right)$$

$$-k = \frac{1}{5} \ln\left(\frac{8}{15}\right)$$

$$T = 50^\circ$$

$$75e^{-kt} = 30$$

$$\frac{1}{5} \ln\left(\frac{8}{15}\right) t = \ln\left(\frac{2}{5}\right)$$

$$t = \ln\left(\frac{2}{5}\right) \div \frac{1}{5} \ln\left(\frac{8}{15}\right)$$

$$= 7.3 \text{ minutes}$$

$$\therefore \text{Extra time} = 2.3 \text{ mins}$$

need to state this. If it is a sto question, you can't just write down what is req'd. (No longer need to integrate to show T)

Some did not find A first which caused further errors

① expression/ value for A/k

Method executed quite well

or  $k =$

① Value for t

Some did not give the extra time

① Extra time

(Some had answers under 5 minutes  $\Rightarrow$  so should have realised there was an error)

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Accuracy of answer 1 dp!!

# Question 6

a) i)  $x = 2\sin t - 3\cos t$   
 $\dot{x} = 2\cos t + 3\sin t$

$\ddot{x} = -2\sin t + 3\cos t$   
 $= -(2\sin t - 3\cos t) \quad \text{--- ①}$

$\ddot{x} = -x \quad \checkmark \quad (n=1)$

$\therefore$  motion is in SHM.

Need to demonstrate  $\ddot{x} = -n^2x$  not just rewrite  $x$ .

ii) at maximum velocity:

$a = \ddot{x} = \frac{dv}{dt} = 0$

from ①  $3\cos t = 2\sin t$  S | ①

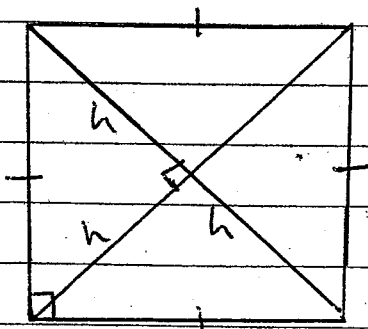
$\tan t = \frac{3}{2} \quad \checkmark$  ① | c

$\tan t = \frac{3}{2} \quad \checkmark$

$t = 0.98 \text{ sec.}$

~~Must use~~ changing ~~bracket~~ because need to have calculator in radian measure.

b) i)



Area =  $\frac{1}{2} (h \times 2h)$

$= h^2$

$\therefore V = Ah$

$= h^2 (150)$

$= 150 h^2 \text{ cm}^3$

All units must be same so 3L needs to be converted to  $\text{cm}^3$ .

ii)  $\frac{dv}{dt} = 3 \text{ l/min}, \frac{dh}{dt} = ?, h = 5 \text{ cm}$

$\frac{dv}{dt} = \frac{dv}{dh} \left( \frac{dh}{dt} \right) = ?$

$= 3000 \div 300 h \quad \checkmark$

$= \frac{30}{(3 \times 5)}$

$\frac{dh}{dt} = \frac{dv}{dt} \div \frac{dv}{dh}$

$= 2 \text{ cm/min} \quad \checkmark$

$\checkmark \rightarrow$

$\therefore \frac{dh}{dt} = 2 \text{ cm/min}$

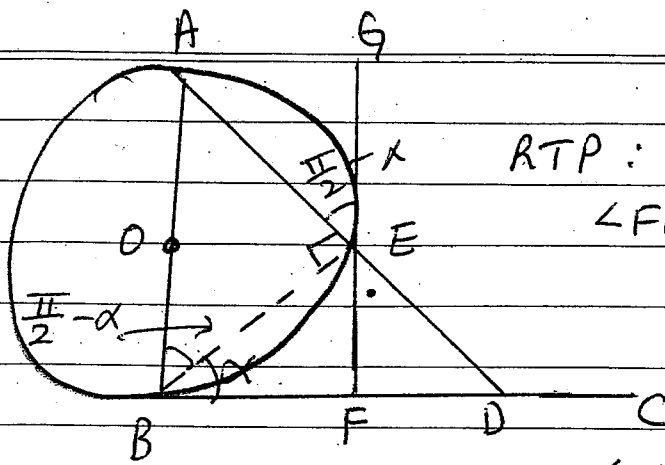
① expression for  $\frac{dh}{dt}$

① working

① Answer



c)  
i)



RTP:

$$\angle FED = \frac{\pi}{2} - \alpha$$

marked on the diagram ✓

ii)  $\angle AEB = \frac{\pi}{2}$  (AB is a diameter)

$$\angle ABF = \frac{\pi}{2} \text{ (tangent } \perp \text{ diameter)}$$

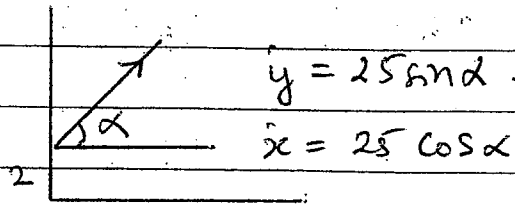
$$\therefore \angle ABE = \frac{\pi}{2} - \alpha \quad \checkmark$$

$$\begin{aligned} \angle ABE &= \angle AEG \quad (\angle \text{ in alt. segment}) \\ &= \frac{\pi}{2} - \alpha \quad \checkmark \end{aligned}$$

$$\begin{aligned} \angle AEG &= \angle DEF \quad (\text{vertically opp}) \\ &= \frac{\pi}{2} - \alpha \quad \checkmark \end{aligned}$$

## Question 7

a) i)  $\ddot{y} = -10$



$$\dot{y} = \int -10 dt$$

$$\dot{y} = -10t + C, \quad t=0, \quad \dot{y} = 25 \sin \alpha$$

$$\therefore \dot{y} = 25 \sin \alpha - 10t$$

OR  $\dot{y} = v \sin \alpha - 10t$

$$y = \int (25 \sin \alpha - 10t) dt$$

$$= 25t \sin \alpha - 5t^2 + C \quad \checkmark$$

$$t=0, \quad y=2 \quad \therefore C=2$$

$$\therefore y = 25t \sin \alpha - 5t^2 + 2$$

OR

$$y = vt \sin \alpha - 5t^2 + 2 \quad \text{--- (1)}$$

Horizontal:

$$x = vt \cos \alpha$$

$$\therefore t = \frac{x}{v \cos \alpha} \quad \text{--- (2)}$$

Sub (2) into (1)

$$y = v \left( \frac{x}{v \cos \alpha} \right) \sin \alpha - 5 \left( \frac{x^2}{v^2 \cos^2 \alpha} \right) + 2$$

$$y = x \tan \alpha - \frac{5x^2}{v^2 \cos^2 \alpha} + 2$$

$$\therefore y = x \tan \alpha - \frac{5x^2}{v^2} (1 + \tan^2 \alpha) + 2$$

Many students  
did not  
derive equations  
of motion.

- ① Integration of  $y, x, \dot{y}$  and  $\dot{x}$
- ① Substitution
- ① expression for  $y$ .

$$\frac{1}{\cos^2 \alpha} = \sec^2 \alpha$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\tan^2 \alpha + 1 = \sec^2 \alpha$$

ii)

$$y = x \tan \alpha - \frac{5x^2}{v^2} (1 + \tan^2 \alpha) + 2$$

$$15 = 20 \tan \alpha - \frac{16}{5} (1 + \tan^2 \alpha) + 2 \quad \checkmark$$

$$75 = 100 \tan \alpha - 16 - 16 \tan^2 \alpha + 10$$

$$16 \tan^2 \alpha - 100 \tan \alpha + 81 = 0$$

$$\tan \alpha = \frac{100 \pm \sqrt{100^2 - 4(16)(81)}}{32} \quad \checkmark$$

$$= \frac{100 \pm 69.40}{32}$$

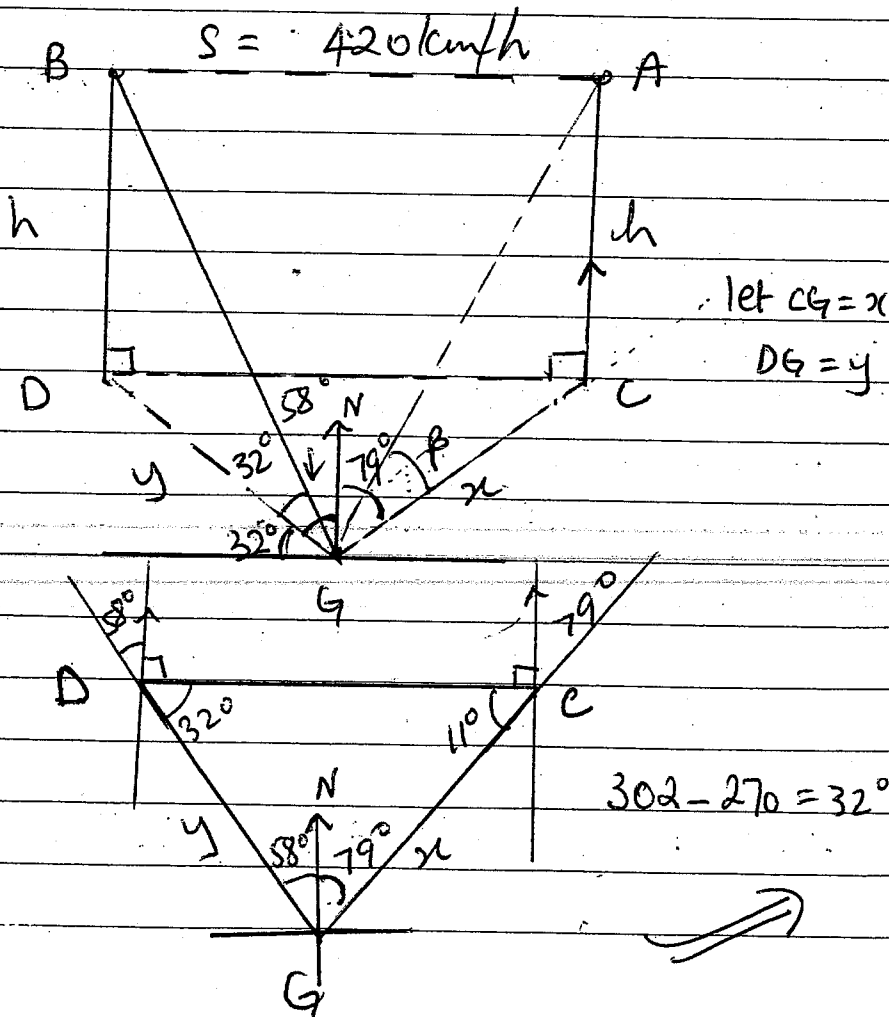
$$\alpha = 44^\circ \text{ and } 79^\circ$$

$$\therefore 44^\circ \leq \alpha \leq 79^\circ \quad \checkmark$$

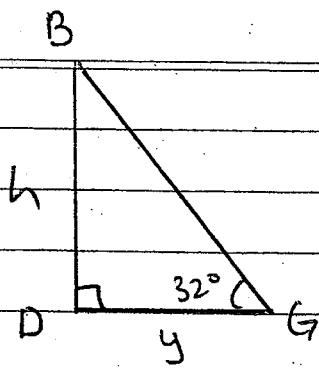
More care needed to be taken with these steps.

- ① Substitution
- ① quadratic formula
- ① Answer.

b) i)



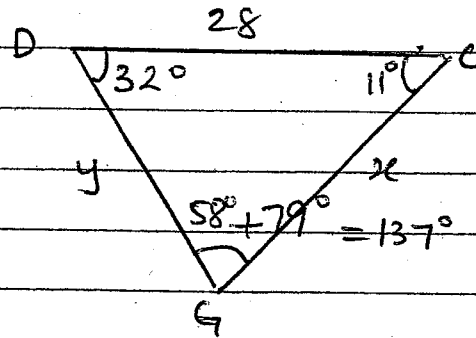
many student did not draw diagrams or confused bearings with oblique angles.



$$\tan 32^\circ = \frac{h}{y}$$

$$y = \frac{h}{\tan 32^\circ} \quad \text{--- (1)}$$

$$\begin{aligned} DC &= \text{speed} \times \text{time} \\ &= 420 \times \frac{4}{60} \\ &= 28 \text{ km} \end{aligned}$$



$$\frac{y}{\sin 11^\circ} = \frac{28}{\sin 137^\circ} \quad \checkmark$$

$$y = \frac{28 \sin 11^\circ}{\sin 137^\circ} \quad \text{--- (2)}$$

Sub (1) into (2)

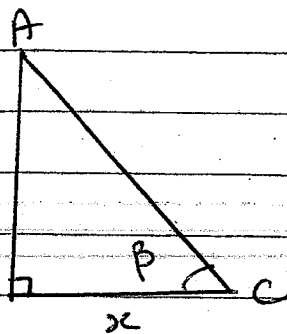
$$\frac{h}{\tan 32^\circ} = \frac{28 \sin 11^\circ}{\sin 137^\circ} \quad \checkmark$$

$$h = \tan 32^\circ \times \frac{28 \sin 11^\circ}{\sin 137^\circ}$$

$$= 4.895 \text{ km} \quad \checkmark \quad (4895 \text{ m})$$

ii)

$$h = 4895$$



from  $\triangle CDG$

$$\frac{x}{\sin 32^\circ} = \frac{28000}{\sin 137^\circ}$$

$$x = 21756 \quad \checkmark$$

$$\therefore \tan \beta = \frac{4895}{21756}$$

$$\beta = 13^\circ \quad \checkmark$$

END

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