SYDNEY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT
TRIAL EXAMINATIONS 2006

FORM VI

MATHEMATICS EXTENSION 1

Examination date

Tuesday 8th August 2006

Time allowed

2 hours (plus 5 minutes reading time)

Instructions

All seven questions may be attempted.
All seven questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Approved calculators and templates may be used.
A list of standard integrals is provided at the end of the examination paper.

Collection

Write your candidate number clearly on each booklet.
Hand in the seven questions in a single well-ordered pile.
Hand in a booklet for each question, even if it has not been attempted.
If you use a second booklet for a question, place it inside the first.
Keep the printed examination paper and bring it to your next Mathematics lesson.

Checklist

SGS booklets: 7 per boy. A total of 1000 booklets should be sufficient.
Candidature: 120 boys.

Examiner

JNC

QUESTION ONE (12 marks) Use a separate writing booklet. **Marks**

(a) Find the exact value of $\tan^{-1}(-\sqrt{3})$. $\boxed{1}$

(b) Differentiate $e^{2x}\sin x$. $\boxed{2}$

(c) Find the exact value of $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$. $\boxed{2}$

(d) Find the acute angle, correct to the nearest minute, between the lines $3x + y = 4$ and $x - y = 1$. $\boxed{2}$

(e) Given $A(2,1)$ and $B(7,3)$, find the coordinates of the point C which divides the interval AB externally in the ratio $2:3$. $\boxed{2}$

(f) Use the substitution $u = x + 1$ to find $\displaystyle\int x(x+1)^3\,dx$. $\boxed{3}$

QUESTION TWO (12 marks) Use a separate writing booklet. **Marks**

(a) Solve $\dfrac{3}{x-1} \le 2$. $\boxed{3}$

(b) Find the value of h if $x - 2$ is a factor of $P(x) = 3x^2 - 2hx + 7$. $\boxed{2}$

(c) Consider the function $f(x) = 3\cos^{-1}\dfrac{x}{2}$.

(i) Evaluate $f(0)$. $\boxed{1}$

(ii) State the domain and range of $y = f(x)$. $\boxed{2}$

(iii) Sketch the graph of $f(x)$. $\boxed{1}$

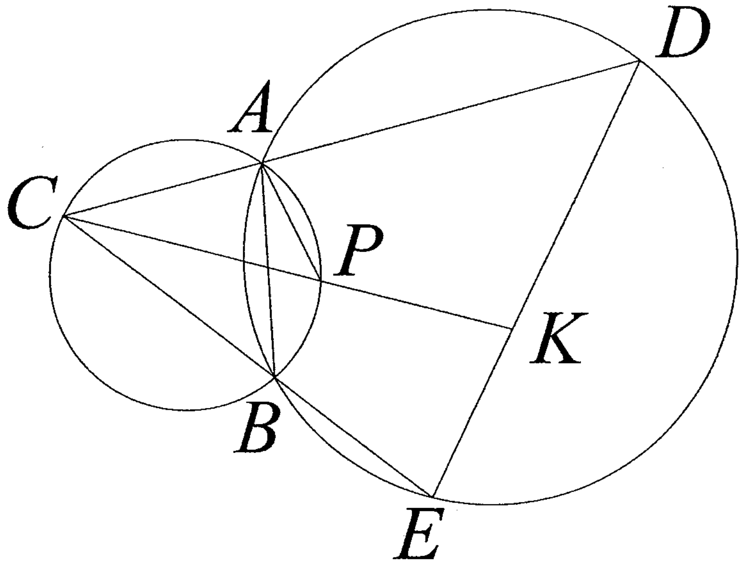
(d) A particle executes simple harmonic motion about the origin with period T seconds and amplitude A centimetres. Find its maximum speed in terms of T and A. $\boxed{3}$

Exam continues next page ...

QUESTION THREE (12 marks) Use a separate writing booklet. **Marks**

(a) Show that the equation of the normal to the parabola $x = 2at$, $y = at^2$ at the point where $t = T$ is given by $x + Ty = 2aT + aT^3$. $\boxed{3}$

(b)



In the diagram above, the two circles intersect at A and B, and CAD, CBE, CPK and DKE are straight lines.

(i) Give a reason why $\angle APC = \angle ABC$. $\boxed{1}$

(ii) Hence, or otherwise, show that $ADKP$ is a cyclic quadrilateral. $\boxed{3}$

(c) A cup of hot milk at temperature T° Celsius loses heat when placed in a cooler environment. It cools according to the law given by the differential equation

$$\frac{dT}{dt} = -k(T - S)$$

where t is the time elapsed in minutes, S is the temperature of the environment in degrees Celsius and k is a positive constant.

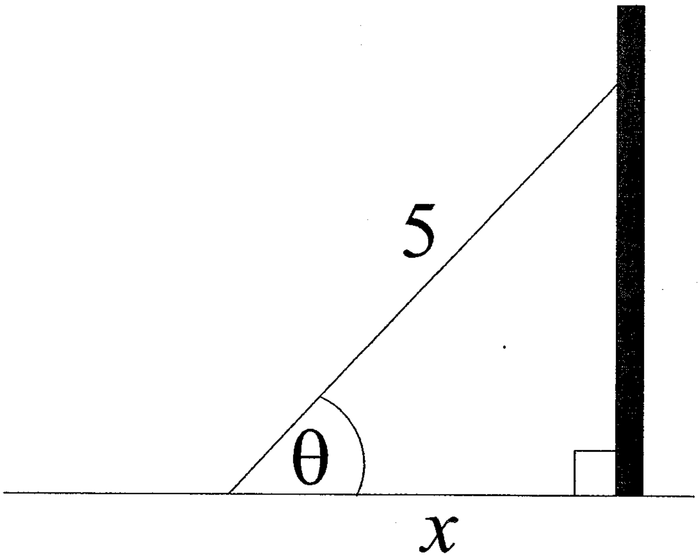
(i) Show that $T = S + Ae^{-kt}$, where A is a constant, is a solution of the differential equation. $\boxed{1}$

(ii) (α) A cup of milk at 80° C is placed in an environment at 20° C, and after ten minutes it has cooled to 40° C. Find the exact value of k. $\boxed{3}$

(β) Find the temperature of the milk after five more minutes have elapsed. Give your answer rounded to the nearest tenth of a degree. $\boxed{1}$

QUESTION FOUR (12 marks) Use a separate writing booklet. **Marks**

(a) **[3]**



The diagram above shows a 5 metre ladder leaning against a wall on level ground. The base of the ladder is sliding away from the wall at 2 centimetres per second. Find the rate at which the angle of inclination θ is changing when the foot of the ladder is 3 metres from the wall.

(b) Find the coefficient of x^2 in the expansion of $\left(x^2 + \dfrac{2}{x}\right)^{10}$. **[3]**

(c) (i) Taking about one-third of a page and on the same set of axes, draw sketches of $y = \ln x$ and $y = \sin x$ for $0 \le x \le 2\pi$. **[2]**

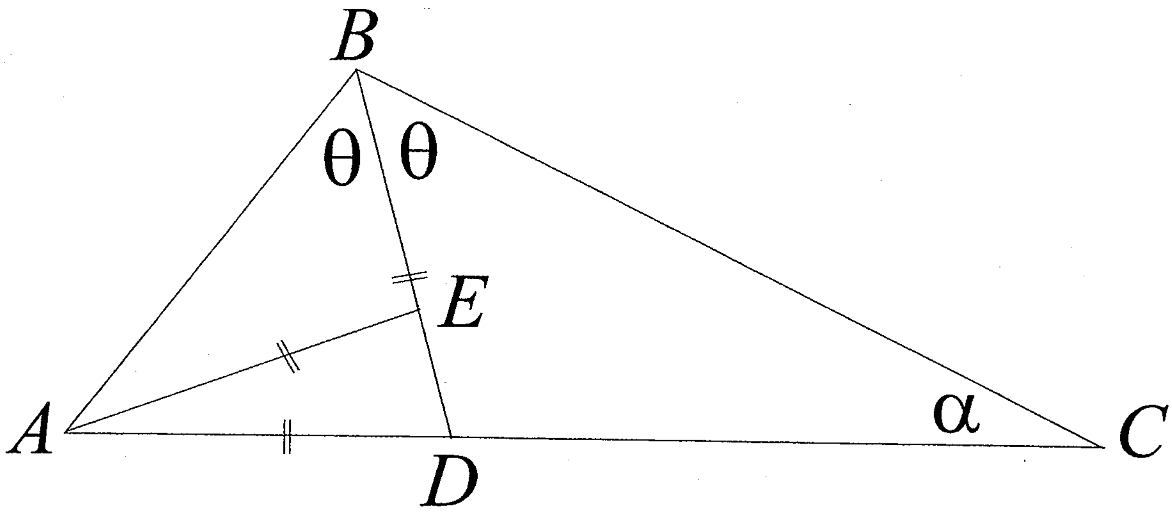
(ii) On your diagram, indicate the root α of the equation $\ln x - \sin x = 0$. **[1]**

(iii) Show that $\dfrac{\pi}{2} < \alpha < \dfrac{3\pi}{4}$. **[1]**

(iv) Use Newton's method once, with first approximation $x_1 = \frac{5\pi}{8}$, to find a better approximation for α. Give your answer correct to two decimal places. **[2]**

QUESTION FIVE (12 marks) Use a separate writing booklet. **Marks**

(a)



In the diagram above, ABD and AED are isosceles triangles with $AD = BD = AE$, and BD bisects $\angle ABC$. Let $\angle ABD = \angle CBD = \theta$ and let $\angle DCB = \alpha$.

(i) Show that $\angle EAB = \alpha$, giving reasons. **3**

(ii) Hence show that $\triangle ABE \;|||\; \triangle CBD$. **1**

(iii) Deduce that $AE^2 = BE \times CD$. **2**

(b) (i) By squaring both sides, show that $2n + 3 > 2\sqrt{(n+1)(n+2)}$ for $n > 0$. **1**

(ii) Prove by mathematical induction that **5**

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} > 2(\sqrt{n+1} - 1)$$

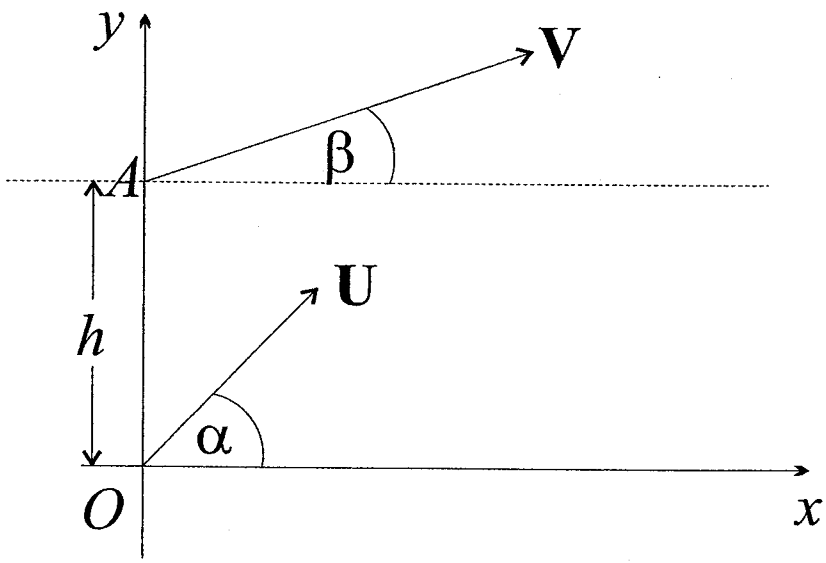
for all positive integer values of n.

QUESTION SIX (12 marks) Use a separate writing booklet. **Marks**

(a) By using the substitution $x = \tan\theta$, evaluate $\displaystyle\int_0^1 \frac{1}{(1+x^2)^{\frac{3}{2}}}\,dx$. $\boxed{4}$

(b) When $(3+2x)^n$ is expanded as a polynomial in x, the coefficients of x^5 and x^6 are equal. Find the value of n. $\boxed{4}$

(c) $\boxed{4}$



In the diagram above, a particle is projected from the origin O with speed U metres per second at an angle of elevation α. At the same instant, another particle is projected from the point A, h metres directly above O, with speed V metres per second at an angle of elevation β, where $\beta < \alpha$. The particles move freely under gravity in the same plane of motion and collide T seconds after projection.

You may assume that the horizontal and vertical components of displacement at time t seconds of the particle projected from O are given by

$$x_O = Ut\cos\alpha \text{ and } y_O = Ut\sin\alpha - \tfrac{1}{2}gt^2 \text{ respectively.}$$

You may also assume that the horizontal and vertical components of displacement at time t seconds of the particle projected from A are given by

$$x_A = Vt\cos\beta \text{ and } y_A = h + Vt\sin\beta - \tfrac{1}{2}gt^2 \text{ respectively.}$$

Show that

$$T = \frac{h\cos\beta}{U\sin(\alpha-\beta)}.$$

Exam continues next page ...

QUESTION SEVEN (12 marks) Use a separate writing booklet. **Marks**

(a) (i) Use the binomial theorem to find a simplified expansion for **1**

$$(1+x)^{10n} + (1-x)^{10n},$$

where n is a positive integer.

(ii) Hence evaluate **2**

$$1 + \binom{30}{2} + \binom{30}{4} + \cdots + \binom{30}{30}.$$

(b) Consider the function $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

(i) Show that $f(x)$ is an odd function. **1**

(ii) Examine the behaviour of $f(x)$ as $x \to \infty$ and as $x \to -\infty$. **2**

(iii) Show that the curve is increasing for all values of x. **2**

(iv) Sketch the curve $y = f(x)$. **1**

(v) If k is a positive constant, show that the area bounded by the curve $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ and the lines $x = 0$, $x = k$ and $y = 1$ is always less than $\ln 2$. **3**

END OF EXAMINATION

FORM VI - EXTENSION 1 TRIAL - 2006

Question ONE

b) $\frac{d}{dx}\left(e^{2x}\sin x\right) = 2e^{2x}\sin x + e^{2x}\cos x$ ✓ ✓

a) $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$ ✓

c) $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx = \int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\cos 2x)\, dx$ ✓

$= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\left[\left(\frac{\pi}{2}+0\right) - 0\right]$

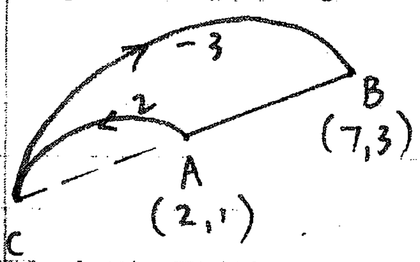
$= \frac{\pi}{4}$ ✓

d) $\left.\begin{aligned}\tan\theta &= \frac{-3-1}{1+(-3)(+1)}\end{aligned}\right\}$ ✓

$= 2$

$\therefore \theta = 63^\circ 26'$ ✓

e)



$x = \frac{-6+14}{-1}$ $\quad$ $y = \frac{-3+6}{-1}$

$= -8$ ✓ $\quad$ $= -3$ ✓

$\therefore C = (-8, -3)$

f) $u = x+1$,

$\frac{du}{dx} = 1$

$\int x(x+1)^3\, dx = \int (u-1)u^3\, du$ ✓

$= \int u^4 - u^3\, du$

$= \left[\frac{u^5}{5} - \frac{u^4}{4}\right] + c$ ✓

$= \frac{1}{5}(x+1)^5 - \frac{1}{4}(x+1)^4 + c$ ✓

QUESTION TWO

a) $\dfrac{3}{x-1} \leqslant 2 \quad , x \neq 1$

$3(x-1) \leqslant 2(x-1)^2$

$(x-1)\left(2(x-1)-3\right) \geqslant 0$

$(x-1)(2x-5) \geqslant 0$

$\therefore \quad x < 1 \quad \text{or} \quad x \geqslant \frac{5}{2}$

b) $P(2) = 12 - 4h + 7 = 0$

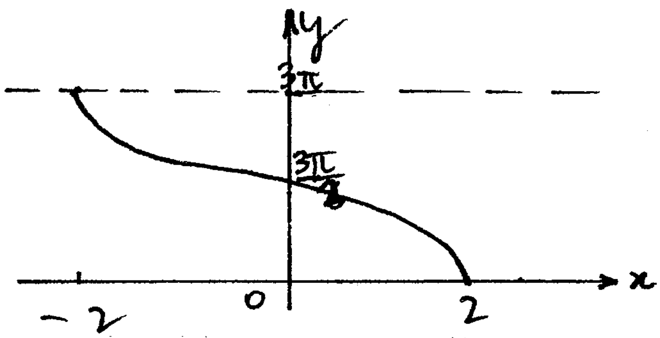
$\therefore h = \dfrac{19}{4}$

c) (i) $f(0) = \dfrac{3\pi}{2}$

(ii) $-1 \leqslant \dfrac{x}{2} \leqslant 1$ and $0 \leqslant \dfrac{y}{3} \leqslant \pi$

$-2 \leqslant x \leqslant 2$ $\quad 0 \leqslant y \leqslant 3\pi$

(iii)



d) $\dfrac{2\pi}{n} = T$

$\therefore n = \dfrac{2\pi}{T}$ and amplitude is A

$x = A\sin\left(\dfrac{2\pi}{T}t + \alpha\right)$

$\dot{x} = \dfrac{2\pi A}{T} \cancel{\sin} \cos\left(\dfrac{2\pi t}{T} + \alpha\right)$

Max velocity when

$\cos\left(\dfrac{2\pi t}{T} + \alpha\right) = +1$

So max speed $= \dfrac{2\pi A}{T}$

OR $v^2 = \left(\dfrac{2\pi}{T}\right)^2 (A^2 - x^2)$

Max at $x = 0$;

$v^2 = \left(\dfrac{2\pi A}{T}\right)^2$

$v = \pm \dfrac{2\pi A}{T}$

So max speed is $\dfrac{2\pi A}{T}$.

QUESTION THREE

(a) $\left.\begin{array}{l} x = 2at \\ y = at^2 \end{array}\right\}$ $\quad \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$= \dfrac{2at}{2a}$ ✓

$= t$

when $t = T$, $\dfrac{dy}{dx} = T$, $x = 2aT$, $y = aT^2$. ✓

gradient of normal $= -\dfrac{1}{T}$

equation of normal: $y - aT^2 = -\frac{1}{T}(x - 2aT)$

$Ty - aT^3 = -x + 2aT$ ✓

$x + Ty = 2aT + aT^3$

(b) (i) $\angle APC = \angle ABC$ (angles at circumference standing on same arc AC) ✓

(ii) Let $\angle APC = \angle ABC = \alpha$

$\angle ADE = \alpha$ (exterior $\angle$ of cyclic quad ADEB) ✓

$\angle APK = 180° - \alpha$ (straight $\angle$) ✓

$\angle APK + \angle ADE = 180° - \alpha + \alpha$

$= 180°$

$\therefore$ ADKP is a cyclic quadrilateral since interior opposite angles are supplementary. ✓

(c) (i) $T = S + Ae^{-kt}$

$T - S = Ae^{-kt}$ ——— ①

$\dfrac{dT}{dt} = \dfrac{d}{dt}\left(S + Ae^{-kt}\right)$

$= -kAe^{-kt}$ ✓

$= -k(T - S)$ from ①

(ii) (α) when $t = 0$, $S = 20$, $T = 80$.

$80 = 20 + Ae^0$

$A = 60$ ✓

when $t = 10$, $S = 20$, $T = 40$.

$40 = 20 + 60e^{-10k}$ ✓

$e^{-10k} = \dfrac{1}{3}$

$k = -\frac{1}{10}\ln\frac{1}{3}$ OR $\frac{1}{10}\ln 3$ ✓

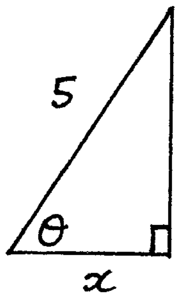
(β) when $t = 15$, $T = 20 + 60e^{-15k}$

$\doteqdot 31.5\ °C$ ✓

(nearest tenth of a degree)

QUESTION FOUR

(a)



$$\frac{dx}{dt} = 0.02 \text{ m/s}$$

$$\frac{x}{5} = \cos\theta$$

$$\therefore \theta = \cos^{-1}\frac{x}{5}$$

$$\therefore \frac{d\theta}{dx} = \frac{-1}{\sqrt{25-x^2}}$$

By the chain rule, $\frac{d\theta}{dt} = \frac{d\theta}{dx} \cdot \frac{dx}{dt}$

$$= \frac{-0.02}{\sqrt{25-x^2}}$$

When $x=3$, $\frac{d\theta}{dt} = \frac{-0.02}{4}$

$$= \frac{-1}{200} \text{ or } -0.005 \text{ rad/s}$$

(b) The general term is ${}^{10}C_r \cdot (x^2)^{10-r} \cdot \left(\frac{2}{x}\right)^r$

$$= {}^{10}C_r \cdot x^{20-2r} \cdot 2^r \cdot x^{-r}$$

$$= {}^{10}C_r \cdot 2^r \cdot x^{20-3r}$$

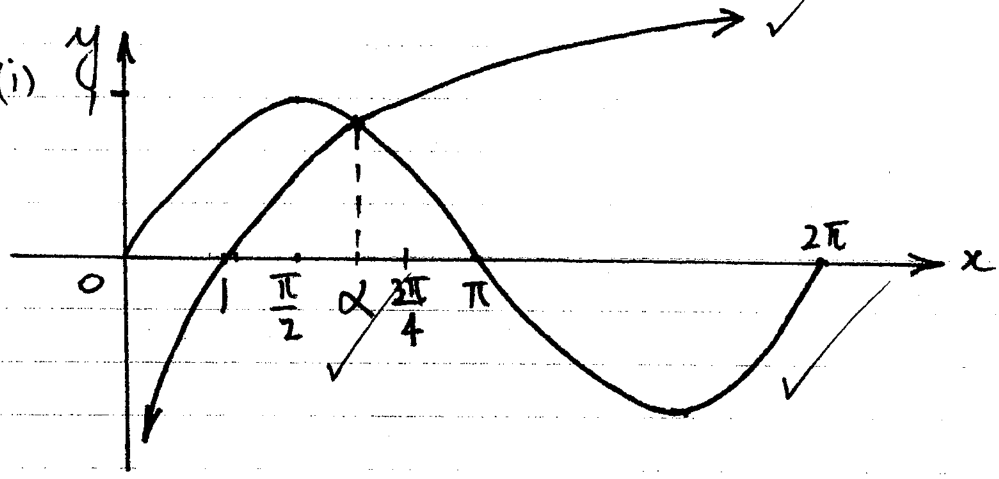
We require $20-3r = 2$

i.e. $r = 6$.

So the coefficient of x^2 is

$${}^{10}C_6 \cdot 2^6 = 13\,440.$$

c)(i)



(ii) α is marked on graph.

(iii) $\ln\frac{\pi}{2} - \sin\frac{\pi}{2} \doteqdot -0.548\ldots$

$\ln\frac{3\pi}{4} - \sin\frac{3\pi}{4} \doteqdot 0.1499\ldots$

Since $f\left(\frac{\pi}{2}\right) < 0$ and $f\left(\frac{3\pi}{4}\right) > 0$, $\frac{\pi}{2} < \alpha < \frac{3\pi}{4}$

(iv) $f'(x) = \frac{1}{x} - \cos x$

So $x_2 = \frac{5\pi}{8} - \dfrac{f\left(\frac{5\pi}{8}\right)}{f'\left(\frac{5\pi}{8}\right)}$

$\doteqdot 2.24$

QUESTION FIVE

(a)(i) $\angle ADE = \theta + \alpha$ (exterior angle of ΔBCD)

$\therefore \angle AED = \theta + \alpha$ (base angles of isosceles ΔADE)

$\therefore \angle EAB = \angle AED - \angle ABE$ (exterior angle of ΔABE)

$= (\theta + \alpha) - \theta$

$= \alpha$

(ii) In triangles ABE and CBD:

$\angle ABE = \angle CBD$ (given)

$\angle EAB = \angle DCB$ (from part (i))

$\therefore \Delta ABE \ ||| \ \Delta CBD$ (A. A.)

(iii) $\dfrac{AE}{CD} = \dfrac{BE}{BD}$ (matching sides are in the same ratio)

$\therefore AE \times BD = BE \times CD$

But $AE = BD$ (given),

so $AE^2 = BE \times CD$

(b) (i) $LHS^2 = (2n+3)^2$

$= 4n^2 + 12n + 9$

$RHS^2 = 4(n+1)(n+2)$

$= 4n^2 + 12n + 8$

$\therefore\ LHS^2 - RHS^2 = 1 > 0$

$\therefore\ LHS^2 > RHS^2$

$\therefore\ LHS > RHS$ (since LHS and RHS are both positive for $n > 0$)

(ii) When $n = 1$, $LHS = \frac{1}{\sqrt{1}} = 1$

and $RHS = 2(\sqrt{2} - 1) \doteqdot 0.8$

$\therefore$ LHS > RHS, so the result is true for $n=1$.

Suppose that the result is true for the positive integer $n = k$,

i.e. suppose that $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{k}} > 2(\sqrt{k+1} - 1)$. (*)

Prove that the result is true for $n = k+1$,

i.e. prove that $1 + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} > 2(\sqrt{k+2} - 1)$.

$$LHS = 1 + \frac{1}{\sqrt{2}} + \dots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}}$$

$$> 2(\sqrt{k+1} - 1) + \frac{1}{\sqrt{k+1}} \quad \text{(using (*))}$$

$$= \frac{2\left[(k+1) - \sqrt{k+1}\right] + 1}{\sqrt{k+1}}$$

$$= \frac{2k + 3 - 2\sqrt{k+1}}{\sqrt{k+1}}$$

$$> \frac{2\sqrt{(k+1)(k+2)} - 2\sqrt{k+1}}{\sqrt{k+1}} \quad \text{(using part (i))}$$

$$= 2(\sqrt{k+2} - 1)$$

= RHS, so the result is true for $n = k+1$ if it is true for $n = k$.

But the result is true for $n = 1$, so, by induction, it is true for all

QUESTION SIX

a) $x = \tan\theta$ When $x = 0$, $\tan\theta = 0$

$\frac{dx}{d\theta} = \sec^2\theta$ $\therefore \theta = 0$

$x = 1$, $\tan\theta = 1$

$dx = \sec^2\theta \, d\theta$ $\therefore \theta = \frac{\pi}{4}$

$$\int_0^1 \frac{1}{(1+x^2)^{3/2}}\,dx = \int_0^{\frac{\pi}{4}} \frac{\sec^2\theta \, d\theta}{(1+\tan^2\theta)^{3/2}} \quad \checkmark\checkmark$$

$$= \int_0^{\frac{\pi}{4}} \frac{d\theta}{\sec\theta} \quad \checkmark$$

$$= \left[\sin\theta\right]_0^{\frac{\pi}{4}}$$

$$= \sin\frac{\pi}{4} - 0$$

$$= \frac{1}{\sqrt{2}} \quad \checkmark$$

b) $(3+2x)^n$ has general term ${}^nC_r\, 3^{n-r} 2^r x^r$. ✓

When $r = 5$ and $r = 6$ we have:

$${}^nC_5\, 3^{n-5} 2^5 = {}^nC_6\, 3^{n-6} 2^6 \quad \checkmark$$

$$\frac{n!}{5!(n-5)!} \cdot 3 = \frac{n!}{6!(n-6)!} \cdot 2 \quad \checkmark$$

$$6 \times 3 = 2 \times (n-5)$$

$$18 = 2n - 10$$

$$\therefore n = 14. \quad \checkmark$$

c) $\left.\begin{aligned} x_o &= Ut\cos\alpha \\ y_o &= Ut\sin\alpha - \tfrac{1}{2}gt^2 \end{aligned}\right\}$ $\left.\begin{aligned} x_A &= Vt\cos\beta \\ y_A &= h + Vt\sin\beta - \tfrac{1}{2}gt^2 \end{aligned}\right\}$

Particles collide when $x_o = x_A$:

$$Ut\cos\alpha = Vt\cos\beta$$
$$U\cos\alpha = V\cos\beta \quad \text{— ①} \quad \checkmark$$

when $y_o = y_A$ and $t = T$:

$$UT\sin\alpha - \tfrac{1}{2}gT^2 = h + VT\sin\beta - \tfrac{1}{2}gT^2 \quad \text{— ②} \quad \checkmark$$

$$T(U\sin\alpha - V\sin\beta) = h$$

$$T = \frac{h}{U\sin\alpha - V\sin\beta}$$

$$= \frac{h}{U\sin\alpha - \frac{U\cos\alpha}{\cos\beta}\cdot\sin\beta} \quad \checkmark \quad \text{from ①}$$

$$\left.\begin{aligned} &= \frac{h}{\frac{U\sin\alpha\cos\beta - U\cos\alpha\sin\beta}{\cos\beta}} \\ &= \frac{h\cos\beta}{U(\sin\alpha\cos\beta - \cos\alpha\sin\beta)} \\ &= \frac{h\cos\beta}{U\sin(\alpha-\beta)} \end{aligned}\right\} \checkmark$$

7. (a) (i) $(1+x)^{10n} + (1-x)^{10n}$

$$= 1 + \binom{10n}{1}x + \binom{10n}{2}x^2 + \cdots + \binom{10n}{10n-1}x^{10n-1} + x^{10n}$$

$$+ 1 - \binom{10n}{1}x + \binom{10n}{2}x^2 - \cdots - \binom{10n}{10n-1}x^{10n-1} + x^{10n}$$

$$= 2\left[1 + \binom{10n}{2}x^2 + \cdots + \binom{10n}{10n-2}x^{10n-2} + x^{10n}\right]$$

(ii) Let $x = 1$ and $n = 3$

So $(1+1)^{30} + 0^{30} = 2\left[1 + \binom{30}{2} + \binom{30}{4} + \cdots + \binom{30}{28} + \binom{30}{30}\right]$

ie $1 + \binom{30}{2} + \binom{30}{4} + \cdots + \binom{30}{28} + \binom{30}{30} = 2^{29}$

(b) (i) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$$\Rightarrow f(-x) = \frac{e^{-x} - e^{x}}{e^{-x} + x}.$$

$$= -\frac{e^{x} + e^{-x}}{e^{x} + e^{-x}}$$

$$\Rightarrow f(-x) = -f(x)$$

Hence $f(x)$ is an odd function.

(ii) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

So $f(x) = \dfrac{1 - e^{-2x}}{1 + e^{-2x}}$

As $x \to \infty$, $e^{-2x} \to 0$

So as $x \to \infty$, $f(x) \to 1$

Furthermore as $f(x)$ is odd then as $x \to -\infty$, $f(x) \to -1$

(iii) $f(x) = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

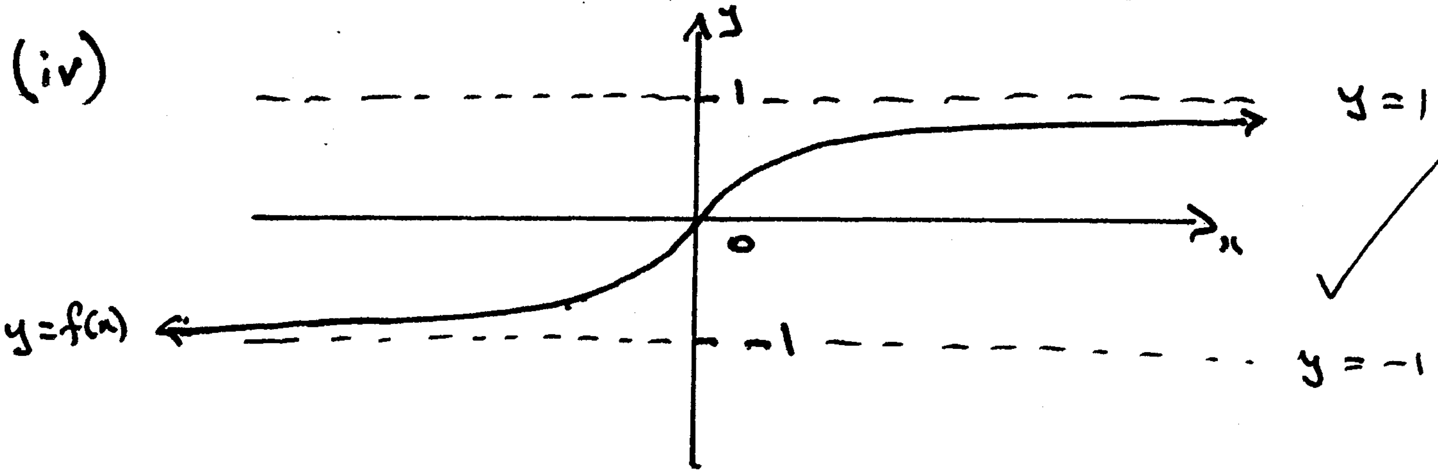
$= \dfrac{e^{2x} - 1}{e^{2x} + 1}$

$\Rightarrow f'(x) = \dfrac{2e^{2x}(e^{2x}+1) - 2e^{2x}(e^{2x}-1)}{(e^{2x}+1)^2}$ ✓

$= \dfrac{4e^{2x}}{(e^{2x}+1)^2}$.

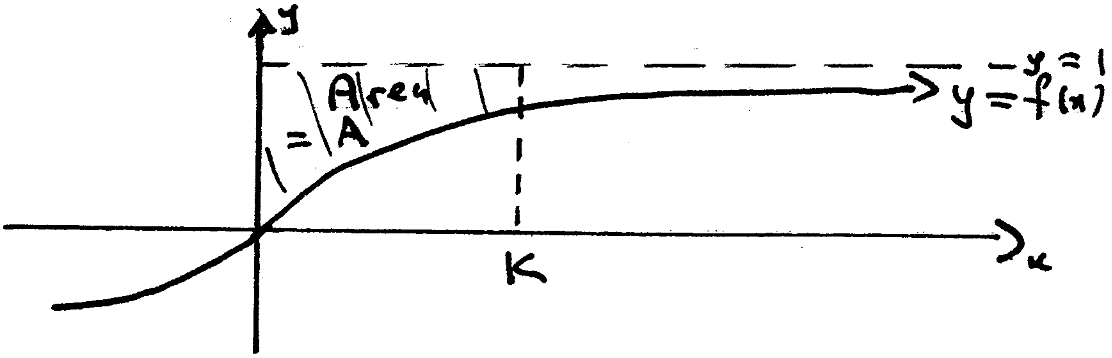
But $e^{2x} > 0$ for all x.
So $f'(x) > 0$ for all x. $\Big\}$ ✓

(iv)



✓

(v)



$A = K - \displaystyle\int_0^{K} \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\, dx$

$= K - \left[\ln(e^{x} + e^{-x}) \right]_0^{K}$ ✓

$= K - \ln\left(e^{K} + \frac{1}{e^{K}}\right) + \ln 2$

So $A = k - \ln\left(\frac{e^{2k}+1}{e^{k}}\right) + \ln 2$

$= k - \ln(e^{2k}+1) + \ln e^{k} + \ln 2$

$= 2k - \ln(e^{2k}+1) + \ln 2$

Now as both $\ln x$ and e^{2x} are increasing functions then $\ln(e^{2k}+1) > \ln e^{2k} = 2k$

Hence $2k - \ln(e^{2k}+1) < 0$ for all $k > 0$

Hence $A < \ln 2$ as required.