

If $a+b+c=0$

$$\text{show } (2a-b)^3 + (2b-c)^3 + (2c-a)^3 \\ = 3(2a-b)(2b-c)(2c-a)$$

$$\begin{aligned} & (2a-b)^3 + (2b-c)^3 + (2c-a)^3 \\ &= (2a-b+2b-c) \left[(2a-b)^2 - (2a-b)(2b-c) + (2b-c)^2 \right] + (2c-a)^3 \\ &= (2a+b-c) \left[(2a-b)^2 - (2a-b)(2b-c) + (2b-c)^2 \right] + (2c-a)^3 \\ &= (-a+2c) \left[(2a-b)^2 - (2a-b)(2b-c) + (2b-c)^2 \right] + (2c-a)^3 \end{aligned}$$

$$\begin{aligned}
&= (2c-a) \left[-(2a-b)^2 + (2a-b)(2b-c) - (2b-c)^2 + (2c-a)^2 \right] \\
&= (2c-a) \left[-(2a-b)^2 + (2a-b)(2b-c) + (2c-a+2b-c) \right. \\
&= (2c-a) \left[-(2a-b)^2 + (2a-b)(2b-c) + (b-2a)(4c-b) \right] \quad (2c-a-2b+c) \\
&= (2c-a)(2a-b) \left[-(2a-b) + (2b-c) - (4c-b) \right] \\
&= (2c-a)(2a-b)(-2a+4b-5c) \\
&= (2c-a)(2a-b)(2b+2c+4b-5c) \\
&= (2c-a)(2a-b)(6b-3c) \\
&= \underline{3(2c-a)(2a-b)(2b-c)}
\end{aligned}$$

