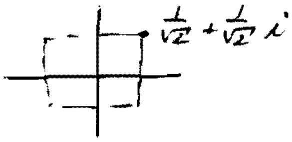
$$\int \sqrt{\tan x}\, dx \qquad u^2 = \tan x \Rightarrow x = \tan^{-1} u^2$$

$$dx = \frac{2u\,du}{1+u^4}$$

$$= \int \frac{2u^2\, du}{1+u^4}$$

$$u^4 = -1$$



$$= \int \frac{2u^2\, du}{(u^2+\sqrt2 u+1)(u^2-\sqrt2 u+1)}$$

$$(Au+B)(u^2-\sqrt2 u+1) + (Cu+D)(u^2+\sqrt2 u+1) = 2u^2$$

$\underline{u=0}$ $\quad$ $B+D=0$ $\qquad$ $\underline{\text{equate } u^4}$ $\quad$ $A+C=0$

$\underline{u=i}$

$$A\sqrt2 - B\sqrt2 i - C\sqrt2 + D\sqrt2 i = -2$$

$$\begin{array}{ll} -B+D=0 & A-C=-\sqrt2 \\ \underline{B+D=0} & \underline{A+C=0} \\ 2D=0 & 2A=-\sqrt2 \\ D=0, B=0 & A=-\frac{1}{\sqrt2}, B=\frac{1}{\sqrt2} \end{array}$$

$$= \frac{1}{\sqrt2}\int \left[\frac{u}{u^2-\sqrt2 u+1} - \frac{u}{u^2+\sqrt2 u+1}\right] du$$

$$= \frac{1}{2\sqrt2}\int \left[\frac{2u-\sqrt2}{u^2-\sqrt2 u+1} + \frac{\sqrt2}{(u-\frac{1}{\sqrt2})^2+\frac12} - \frac{2u+\sqrt2}{u^2+\sqrt2 u+1} + \frac{\sqrt2}{(u+\frac{1}{2})^2+\frac12}\right] du$$

$$= \frac{1}{2\sqrt2}\left\{\log\left(\frac{u^2-\sqrt2 u+1}{u^2+\sqrt2 u+1}\right) + 2\tan^{-1}(\sqrt2 u - 1) + 2\tan^{-1}(\sqrt2 u+1)\right\} + c$$

$$= \frac{1}{2\sqrt2}\left\{\log\left(\frac{\tan x - \sqrt{2\tan x}+1}{\tan x + \sqrt{2\tan x}+1}\right) + 2\tan^{-1}(\sqrt{2\tan x} - 1) + 2\tan^{-1}(\sqrt{2\tan x}+1)\right\} + c$$