

Complex Numbers

Solving Quadratics

$$x^2 + 1 = 0$$

$$x^2 = -1$$

no real solutions

In order to solve this equation we define a new number

$$i = \sqrt{-1} \quad \text{or} \quad i^2 = -1$$

i is an imaginary number

$$x^2 = -1$$

$$x = \pm\sqrt{-1}$$

$$x = \pm i$$

All complex numbers (z) can be written as; $z = x + iy$

Definitions;

(1) All complex numbers contain a **real** and an **imaginary** part

$$\begin{aligned}\operatorname{Re}(z) &= x \\ \operatorname{Im}(z) &= y\end{aligned}$$

e.g. $z = 3 + 5i$

$$\operatorname{Re}(z) = 3$$

$$\operatorname{Im}(z) = 5$$

(2) If $\operatorname{Re}(z) = 0$, then z is a **pure imaginary number**

e.g. $\sqrt{3}i, -6i$

(3) If $\operatorname{Im}(z) = 0$, then z is a **real number**

e.g. $\frac{3}{4}, \pi, e, -4$

(4) Every complex number $z = x + iy$, has a **complex conjugate**

$$\bar{z} = x - iy$$

e.g. $z = -2 - \sqrt{7}i$

$$\bar{z} = -2 + \sqrt{7}i$$

Complex Numbers

$$x + iy$$

Imaginary Numbers

$$y \neq 0$$

Pure
Imaginary Numbers

$$x = 0, y \neq 0$$

Real Numbers

$$y = 0$$

Rational Numbers

Fractions

Integers

Naturals

Zero

Negatives

Irrational Numbers

NOTE: Imaginary numbers
cannot be ordered

Basic Operations

As i a surd, the operations with complex numbers are the same as surds

Addition

$$\begin{aligned}(4 - 3i) + (-8 + 2i) \\ = \underline{-4 - i}\end{aligned}$$

Subtraction

$$\begin{aligned}(4 - 3i) - (-8 + 2i) \\ = \underline{12 - 5i}\end{aligned}$$

Multiplication

$$\begin{aligned}(4 - 3i)(-8 + 2i) \\ = -32 + 8i + 24i - 6i^2 \\ = -32 + 32i + 6 \\ = \underline{-26 + 32i}\end{aligned}$$

Division (Realising The Denominator)

$$\begin{aligned}\frac{(4 - 3i)}{(-8 + 2i)} \times \frac{(-8 - 2i)}{(-8 - 2i)} \\ = \frac{-32 - 8i + 24i - 6}{64 + 4} \\ = \frac{-38 + 16i}{68} \\ = \underline{\underline{\frac{-19}{34} + \frac{8}{34}i}}\end{aligned}$$

Conjugate Basics

$$(1) \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

$$(4) z\bar{z} = x^2 + y^2$$

$$(2) \overline{z_1 z_2} = \overline{z_1} \times \overline{z_2}$$

$$(5) \frac{1}{z} = \frac{\bar{z}}{z\bar{z}}$$

$$(3) \overline{\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}} = \begin{pmatrix} \overline{z_1} \\ \overline{z_2} \end{pmatrix}$$

$$\begin{aligned} \text{e.g. } \frac{1}{(4-3i)} &= \frac{4+3i}{16+9} \\ &= \frac{4}{25} + \frac{3}{25}i \end{aligned}$$

Complex Equations

If two complex numbers are equal, then their real parts are equal and their imaginary parts are equal

$$\text{i.e. If } a_1 + b_1i = a_2 + b_2i$$

then

$$a_1 = a_2$$

$$b_1 = b_2$$

$$\text{e.g. (i) } (2 + 3i)(x + iy) = (4 - 2i)$$

$$2x + 2iy + 3ix - 3y = 4 - 2i$$

$$\therefore 2x - 3y = 4 \quad \text{and} \quad 3x - 2y = -2$$

$$\therefore x = -\frac{14}{5}, \quad y = -\frac{16}{5}$$

$$\begin{aligned} & \Rightarrow \begin{array}{r} 4x - 6y = 8 \\ 9x - 6y = -6 \\ \hline 5x = -14 \end{array} \end{aligned}$$

$$x = -\frac{14}{5}$$

$$(ii) \quad z + 2iw = 4 + 3i$$

$$2z + iw = 3 + 4i$$

$$\begin{array}{r} z + 2iw = 4 + 3i \\ \Rightarrow \\ 4z + 2iw = 6 + 8i \\ \hline 3z \quad \quad = 2 + 5i \end{array}$$

$$z = \frac{2}{3} + \frac{5}{3}i$$

$$\therefore \frac{2}{3} + \frac{5}{3}i + 2iw = 4 + 3i$$

$$2iw = \frac{10}{3} + \frac{4}{3}i$$

$$w = \frac{10}{6i} + \frac{4}{6}$$

$$= \frac{2}{3} - \frac{5}{3}i$$

$$w = \frac{2}{3} - \frac{5}{3}i, \quad z = \frac{2}{3} + \frac{5}{3}i$$

Cambridge: Exercise 1A; 1 to 10 ace etc, 11bd, 13 to 20

Patel: Exercise 4A; 28 ac