

# *Solving Quadratics*

e.g.  $x^2 + 3x + 7 = 0$

$$x = \frac{-3 \pm \sqrt{9 - 28}}{2}$$

$$= \frac{-3 \pm \sqrt{-19}}{2}$$

$$= \frac{-3 \pm \sqrt{19}i}{2}$$

$$x = \frac{-3 + \sqrt{19}i}{2} \text{ or } x = \frac{-3 - \sqrt{19}i}{2}$$

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**OR**

*by completing the square*

$$x^2 + 3x + 7 = 0$$

$$\left(x + \frac{3}{2}\right)^2 + \frac{19}{4} = 0$$

$$\left(x + \frac{3}{2}\right)^2 - \frac{19}{4}i^2 = 0$$

$$\left(x + \frac{3}{2} + \frac{\sqrt{19}}{2}i\right)\left(x + \frac{3}{2} - \frac{\sqrt{19}}{2}i\right) = 0$$

$$x = -\frac{3}{2} + \frac{\sqrt{19}}{2}i \text{ or } x = -\frac{3}{2} - \frac{\sqrt{19}}{2}i$$

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# *Creating Quadratics*

All quadratics with real coefficients have conjugate pairs as solutions

$$\begin{aligned}(x - z)(x - \bar{z}) &= x^2 - (z + \bar{z})x + z\bar{z} \\ &= x^2 - 2\operatorname{Re}(z)x + z\bar{z}\end{aligned}$$

(iii) Find the quadratic equation with roots  $4 + i$  and  $4 - i$

$$\alpha + \beta = 8 \qquad \alpha\beta = 17$$

$\therefore$  equation is  $x^2 - 8x + 17 = 0$

# Finding Square Roots

$$a + ib = \sqrt{x + iy}$$

$$(a + ib)^2 = x + iy$$

$$a^2 + 2abi - b^2 = x + iy$$

$$\text{If } \sqrt{x + iy} = a + ib$$

$$\text{then } a^2 - b^2 = x$$

$$2ab = y$$

e.g. Find  $\sqrt{-12 + 16i}$

$$a^2 - b^2 = -12$$

$$2ab = 16$$

$$a^2 - \frac{64}{a^2} = -12$$

$$b = \frac{8}{a}$$

$$a^4 + 12a^2 - 64 = 0$$

$$(a^2 - 4)(a^2 + 16) = 0$$

$$a^2 = 4 \quad \text{or} \quad a^2 = -16$$

$$a = \pm 2 \quad \text{no real solutions}$$

$$\therefore b = \pm 4$$

$$\underline{\sqrt{-12 + 16i} = \pm(2 + 4i)}$$

**OR**

$$\begin{aligned} a + ib &= \sqrt{x + iy} \\ a^2 - b^2 &= x \\ 2ab &= y \end{aligned} \quad \Rightarrow \quad \begin{aligned} x^2 + y^2 &= (a^2 - b^2)^2 + 4a^2b^2 \\ &= a^4 - 2a^2b^2 + b^4 + 4a^2b^2 \\ &= a^4 + 2a^2b^2 + b^4 \end{aligned}$$

If  $\sqrt{x + iy} = a + ib$

then  $a^2 - b^2 = x$

$$a^2 + b^2 = \sqrt{x^2 + y^2}$$

*Note:*  $y > 0$ , then  $a$  and  $b$  have same sign

$y < 0$ , then  $a$  and  $b$  have different sign

e.g. Find  $\sqrt{-12 + 16i}$

$$a^2 - b^2 = -12$$

$$a^2 + b^2 = 20$$

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$$2a^2 = 8$$

$$a^2 = 4$$

$$a = 2 \quad \therefore b = 4$$

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$$\sqrt{-12 + 16i} = \pm(2 + 4i)$$

(ii) Find  $\sqrt{5-4i}$

$$a^2 - b^2 = 5$$

$$a^2 + b^2 = \sqrt{41}$$

$$2a^2 = 5 + \sqrt{41}$$

$$a^2 = \frac{1}{2}(5 + \sqrt{41})$$

$$a = \frac{\sqrt{5 + \sqrt{41}}}{\sqrt{2}}$$

$$a = \frac{1}{2}\sqrt{10 + 2\sqrt{41}}$$

$$\therefore b^2 = \frac{1}{2}(5 + \sqrt{41}) - 5$$

$$= \frac{1}{2}(-5 + \sqrt{41})$$

$$b = \frac{\sqrt{-5 + \sqrt{41}}}{\sqrt{2}}$$

$$b = \frac{1}{2}\sqrt{-10 + 2\sqrt{41}}$$

$$\sqrt{5-4i} = \pm \frac{1}{2} \left( \sqrt{10 + 2\sqrt{41}} - i\sqrt{-10 + 2\sqrt{41}} \right)$$

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(iii) Solve  $z^2 + (2+i)z + (2-2i) = 0$

$$z = \frac{-(2+i) \pm \sqrt{(2+i)^2 - 4(2-2i)}}{2}$$

$$= \frac{-(2+i) \pm \sqrt{4+4i-1-8+8i}}{2}$$

$$= \frac{-(2+i) \pm \sqrt{-5+12i}}{2}$$

$$= \frac{-(2+i) \pm (2+3i)}{2}$$

$$= \frac{2i}{2} \quad \text{or} \quad \frac{-4-4i}{2}$$

$$= \underline{i} \quad \text{or} \quad \underline{-2-2i}$$

$$\sqrt{-5+12i}$$

$$a^2 - b^2 = -5$$

$$a^2 + b^2 = 13$$

$$\underline{2a^2} = 8$$

$$a^2 = 4$$

$$a = 2 \quad \therefore b = 3$$

***Cambridge: Exercise 1B; 1 to 4 ace etc, 5, 6, 7 cf,  
9, 10, 16 ac***