Properties of Complex Conjugates

$$(1)\ |z| = |\bar{z}|$$

$$(2)\ \arg z = -\arg \bar{z}$$

$$(3)\ z\bar{z} = x^2 + y^2$$

$$= |z|^2$$

$$(4)\ \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

$$(5)\ \overline{z_1 z_2} = \overline{z_1} \times \overline{z_2}$$

$$(6)\ \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$$

$$(7)\ \frac{1}{z} = \frac{\bar{z}}{|z|^2}$$

$e.g.$ If $x+iy=\sqrt{\frac{6+2i}{3-i}}$, show that $x^2+y^2=2$

$$x+iy=\sqrt{\frac{6+2i}{3-i}}$$

$$(x+iy)^2=\frac{6+2i}{3-i}\ldots(1)$$

$$\overline{(x+iy)^2}=\overline{\left(\frac{6+2i}{3-i}\right)}$$

$$=\frac{\overline{6+2i}}{\overline{3-i}}$$

$$(x-iy)^2=\frac{6-2i}{3+i}\ldots(2)$$

Multiply $(1)\times(2)$

$$(x+iy)^2(x-iy)^2=\frac{6+2i}{3-i}\times\frac{6-2i}{3+i}$$

$$(x^2+y^2)^2=\frac{36+4}{9+1}$$

$$=4$$

$$x^2+y^2=2$$

Exercise 4H; 1 to 6