

Locus and Complex Numbers

$\omega = f(z)$, find the locus of ω or z

given some condition for ω or z

(Make the condition the subject)

ω is purely real $\Rightarrow \text{Im}(\omega) = 0, \arg \omega = 0$ or π

ω is purely imaginary $\Rightarrow \text{Re}(\omega) = 0, \arg \omega = \pm \frac{\pi}{2}$

$\arg \left(\frac{\text{linear function}}{\text{linear function}} \right) = \theta \Rightarrow$ locus is an arc of a circle

* minor arc if $\theta > \frac{\pi}{2}$

* major arc if $\theta < \frac{\pi}{2}$

* semicircle if $\theta = \frac{\pi}{2}$

e.g.(i) Find the locus of w if $w = \frac{z+2}{2}, |z| = 4$

$$w = \frac{z+2}{2}$$

$$zw = z + 2$$

$$z(w-1) = 2$$

$$z = \frac{2}{w-1}$$

$$\therefore \left| \frac{2}{w-1} \right| = 4$$

$$\frac{2}{|w-1|} = 4$$

$$|w-1| = \frac{1}{2}$$

\therefore locus is a circle, centre $(1,0)$ and radius $\frac{1}{2}$

$$\text{i.e. } (x-1)^2 + y^2 = \frac{1}{4}$$

(ii) Find the locus of z if $w = \frac{z+1}{z-1}$ and w is purely real

$$w = \frac{(x+1)+iy}{(x-1)+iy} \times \frac{(x-1)-iy}{(x-1)-iy} \quad \text{OR} \quad \text{If } w \text{ is purely real then } \arg w = 0 \text{ or } \pi$$

$$= \frac{(x^2-1) - i(x+1)y + i(x-1)y + y^2}{(x-1)^2 + y^2}$$

$$\text{i.e. } \arg\left(\frac{z+1}{z-1}\right) = 0 \text{ or } \pi$$

If w is purely real then $\text{Im}(w) = 0$

$$\text{i.e. } -(x+1)y + (x-1)y = 0$$

$$-xy - y + xy - y = 0$$

$$-2y = 0$$

$$y = 0$$

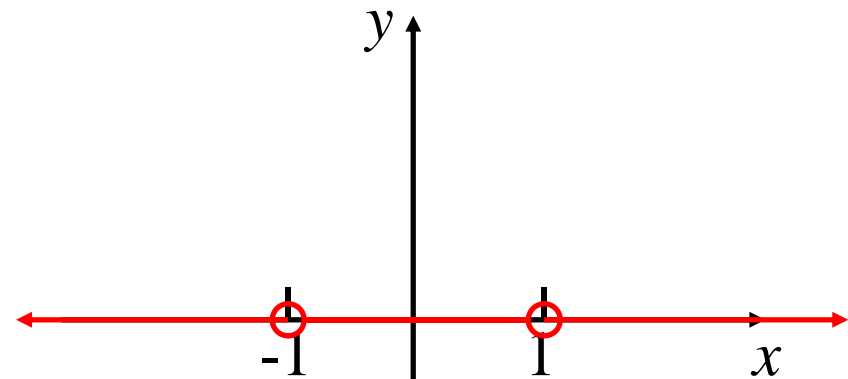
\therefore locus is $y = 0$, excluding $(1,0)$

$(z-1 \neq 0, \text{bottom of fraction } \neq 0)$

Note: locus is $y = 0$, excluding $(\pm 1, 0)$ only

i.e. answer the original question

$$\arg(z+1) - \arg(z-1) = 0 \text{ or } \pi$$



locus is $y = 0$, excluding $(\pm 1, 0)$

(iii) Find the locus of z if $\arg\left(\frac{z}{z-4}\right) = \frac{\pi}{6}$

$$\arg\left(\frac{z}{z-4}\right) = \frac{\pi}{6}$$

$$\frac{y}{2} = \tan 60$$

$$r^2 = 2^2 + (2\sqrt{3})^2$$

$$\arg z - \arg(z-4) = \frac{\pi}{6}$$

$$y = 2 \tan 60$$

$$r^2 = 16$$

$$= 2\sqrt{3}$$

$$r = 4$$

\therefore centre is $(2, -2\sqrt{3})$

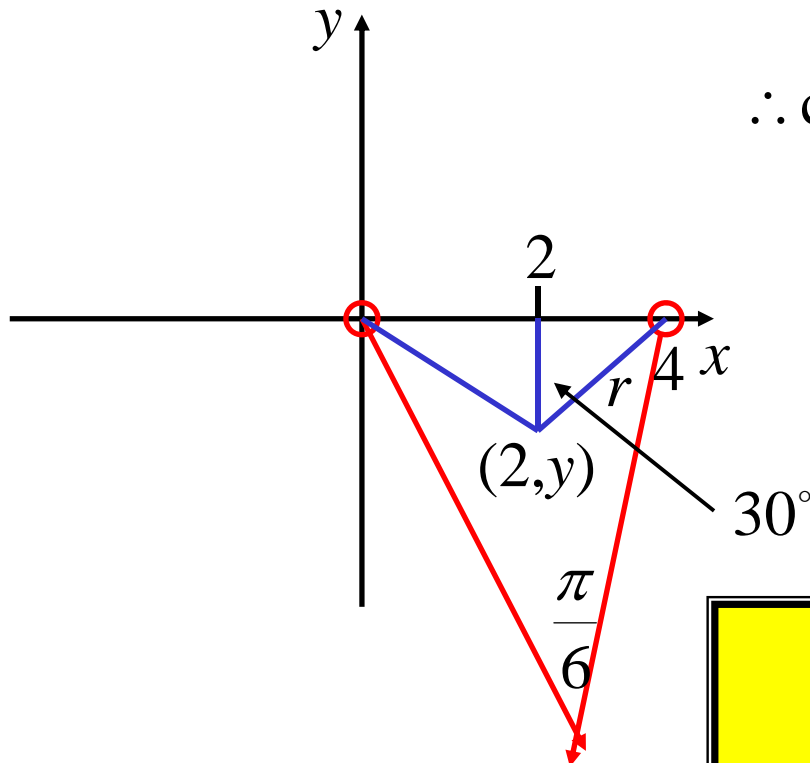
\therefore locus is the major arc of the circle

$$(x-2)^2 + (y+2\sqrt{3})^2 = 16$$

formed by the

chord joining $(0,0)$ and $(4,0)$ but not

including these points.



NOTE: $\arg z > \arg(z-4)$

\therefore below axis

Patel: Exercise 4N; 5, 6

Cambridge: Exercise 1F; 10 to 20

HSC Geometrical Complex Numbers Questions