De Moivre's Theorem

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

for all integers n

this extends to;

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

e.g. $(1-i)^5$

$$= \left[\sqrt{2}cis\left(-\frac{\pi}{4}\right)\right]^5$$

$$= \left(\sqrt{2}\right)^5 cis\left(-\frac{5\pi}{4}\right)$$

$$= 4\sqrt{2}cis\left(\frac{3\pi}{4}\right)$$

$$|z| = \sqrt{1^2 + (-1)^2}$$

$$= \sqrt{2}$$

$$\arg z = \tan^{-1}\left(\frac{-1}{1}\right)$$

$$= -\frac{\pi}{4}$$

$$(1-i)^5$$

$$= 4\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$= 4\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$$

$$= -4 + 4i$$

Finding Roots

If $z^n = x + iy$

$$z^n = r\operatorname{cis}\theta$$

$$z = \sqrt[n]{r}\operatorname{cis}\left[\frac{2\pi k + \theta}{n}\right] \qquad k = 0, 1, \ldots, n-1$$

e.g. (i) $z^2 = 4i$

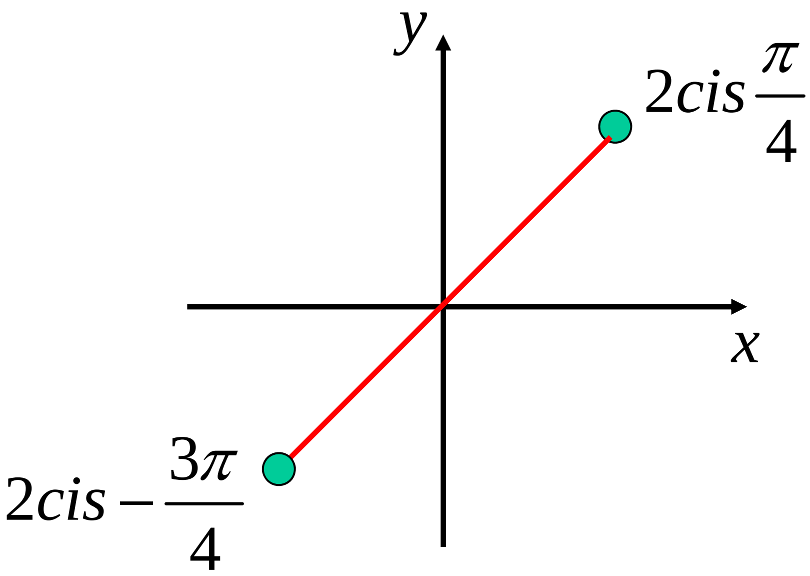
$$z^2 = 4\operatorname{cis}\frac{\pi}{2}$$

$$z = 2\operatorname{cis}\left[\frac{2\pi k + \frac{\pi}{2}}{2}\right] \qquad k = 0, 1$$

$$z = 2\operatorname{cis}\frac{\pi}{4}, 2\operatorname{cis}\frac{5\pi}{4}$$

$$z = 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right), 2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$$

OR



$$z = \sqrt{2} + \sqrt{2}i, -\sqrt{2} - \sqrt{2}i$$

$$(ii)\quad x^4 - 16 = 0$$

$$x^4 = 16$$

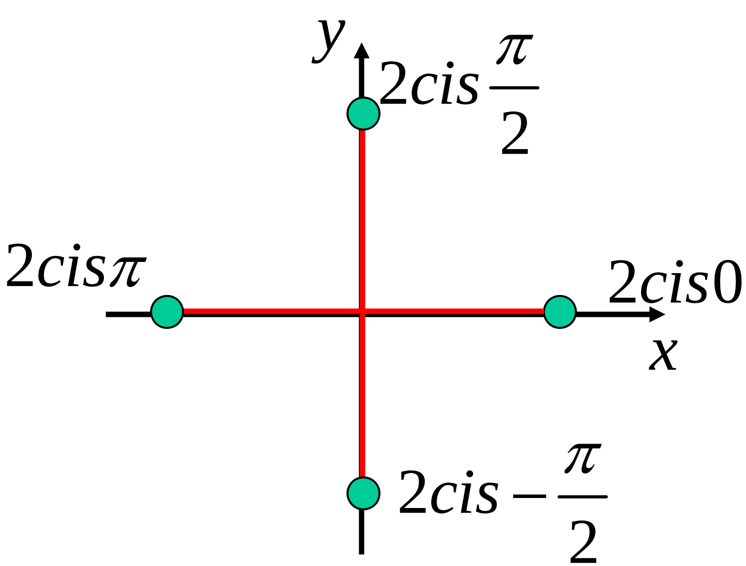
$$x^4 = 16cis0$$

$$x = 2cis\left[\frac{2\pi k}{4}\right] \qquad k = 0, 1, 2, 3$$

$$x = 2cis0, 2cis\frac{\pi}{2}, 2cis\pi, 2cis\frac{3\pi}{2}$$

$$\underline{x = 2, 2i, -2, -2i}$$

OR



Cambridge: **Exercise 7A;**
1, 2, 3 abef, 5, 6, 7,
9 to 14, 16 to 18

Patel: **Exercise 4E;**
1 to 4 ac