# *Factorising Complex Expressions*

If a polynomial's coefficients are all real then the roots will appear in complex conjugate pairs.

Every polynomial of degree *n* can be;

- factorised as a mixture of quadratic and linear factors over the real field
- factorised to *n* linear factors over the complex field

***NOTE:*** odd ordered polynomials must have a real root

$$e.g.\ (i)\ x^2+2x+2 = (x+1)^2+1$$

$$= (x+1+i)(x+1-i)$$

$$(ii)\ z^4 + z^2 - 12 = 0$$

$$(z^2 - 3)(z^2 + 4) = 0$$

$$(z + \sqrt{3})(z - \sqrt{3})(z^2 + 4) = 0 \qquad \text{(factorised over Real numbers)}$$

$$(z + \sqrt{3})(z - \sqrt{3})(z + 2i)(z - 2i) = 0 \qquad \text{(factorised over Complex numbers)}$$

$$\underline{z = \pm\sqrt{3} \quad \text{or} \quad z = \pm 2i}$$

If $(x - a)$ is a factor of $P(x)$, then $P(a) = 0$

If $(ax - b)$ is a factor of $P(x)$, then $P\left(\frac{b}{a}\right) = 0$

$(iii)$ Factorise $2x^3 - 3x^2 + 8x + 5$

as it is a cubic it must have a real factor

$$P\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 8\left(-\frac{1}{2}\right) + 5$$

$$= -\frac{1}{4} - \frac{3}{4} - 4 + 5$$

$$= 0$$

$$\therefore 2x^3 - 3x^2 + 8x + 5$$

$$= (2x + 1)(x^2 - 2x + 5)$$

$$= (2x + 1)\left[(x - 1)^2 + 4\right]$$

$$\underline{= (2x + 1)(x - 1 - 2i)(x - 1 + 2i)}$$

$(iv)$ Given that $P(x) = 4x^4 + 8x^3 + 5x^2 + x - 3$ has two rational zeros, find these zeros and factorise $P(x)$ over the complex field.

$$P\left(\frac{1}{2}\right) = 4\left(\frac{1}{16}\right) + 8\left(\frac{1}{8}\right) + 5\left(\frac{1}{4}\right) + \frac{1}{2} - 3$$

$$= 0$$

$$\therefore (2x-1) \text{ is a factor}$$

$$P(x) = (2x-1)(2x^3 + 5x^2 + 5x + 3)$$

$$P\left(-\frac{3}{2}\right) = 2\left(\frac{-27}{8}\right) + 5\left(\frac{9}{4}\right) + 5\left(-\frac{3}{2}\right) + 3$$

$$= 0$$

$$\therefore (2x+3) \text{ is a factor}$$

$$\therefore \text{rational zeros are } \frac{1}{2} \text{ and } -\frac{3}{2}$$

$$P(x) = 4x^4 + 8x^3 + 5x^2 + x - 3$$

$$= (2x-1)(2x+3)(x^2 + x \qquad +1)$$

$$= (2x-1)(2x+3)\left[\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}\right]$$

$$= (2x-1)(2x+3)\left(x+\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\left(x+\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)$$

$$1 \times 3 \times 2x = 6x$$

$$1 \times 2x \times -1 = -2x$$

$$4x$$

$$-1 \times 3 \times ?\,x = -3x$$

***Cambridge:*** **Exercise 5A; 1b, 2, 3, 5, 6, 7b, 8, 9, 10, 12 to 15**