Inequations & Inequalities

An **inequation** is a problem where we are trying to find possible values, solved the same as an equation, ending up with a pronumeral as the subject of the inequation.

e.g. Solve $5x+3<2$

$$5x+3<2$$

$$5x<-1$$

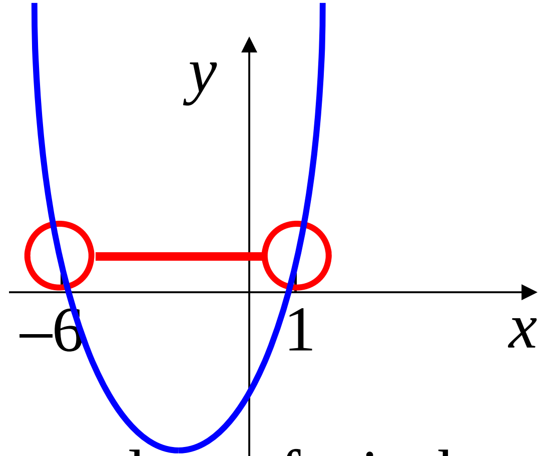
$$x<-\frac{1}{5}$$

1. Quadratic Inequations

e.g. $(i)\ x^2 + 5x - 6 < 0$

$$(x+6)(x-1) < 0$$

$$\underline{-6 < x < 1}$$



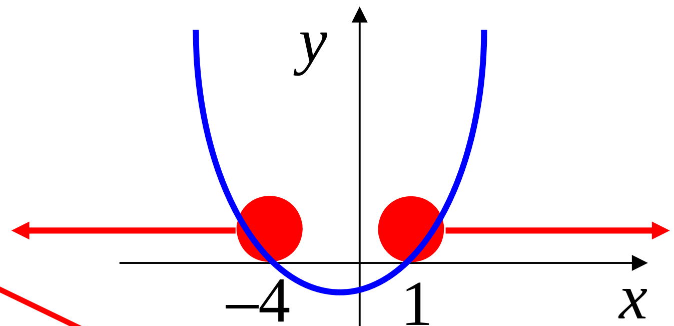
Q: for what values of x is the parabola below the x axis?

$(ii)\ -x^2 - 3x + 4 \le 0$

$$x^2 + 3x - 4 \ge 0$$

$$(x+4)(x-1) \ge 0$$

$$\underline{x \le -4 \text{ or } x \ge 1}$$



Q: for what values of x is the parabola above the x axis?

Note: *quadratic inequalities always have solutions in the form* *$\underline{?< x < ?}$ OR $\underline{x < ? \text{ or } x > ?}$*

2. Inequations with Pronumerals in the Denominator

e.g. (*i*) $\frac{1}{x} \geq 3$

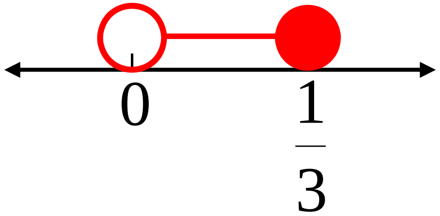
1) Find the value where the denominator is zero $\quad x \neq 0$

2) Solve the “equation”

$$\frac{1}{x} = 3$$

$$x = \frac{1}{3}$$

3) Plot these values on a number line



4) Test regions

Test $x = -1$ $\quad \frac{1}{-1} \geq 3$ ✗

Test $x = \frac{1}{4}$ $\quad \frac{1}{\frac{1}{4}} \geq 3$ ✓

Test $x = 1$ $\quad \frac{1}{1} \geq 3$ ✗

$$\therefore 0 < x \leq \frac{1}{3}$$

(*ii*) $\frac{2}{x+3} < 5$

$$x + 3 \neq 0$$

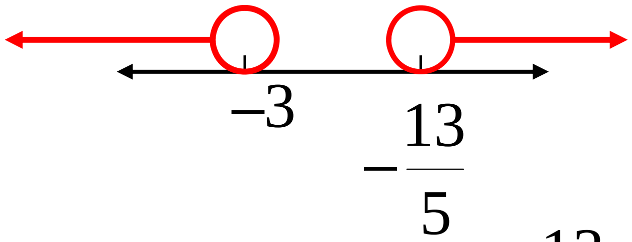
$$x \neq -3$$

$$\frac{2}{x+3} = 5$$

$$2 = 5x + 15$$

$$5x = -13$$

$$x = -\frac{13}{5}$$



$$\therefore x < -3 \text{ or } x > -\frac{13}{5}$$

An **inequality** is a problem that you are proving to be true for a known set of values, ending up with what you are trying to prove, solved the same as an identity proof.

After stating what you are trying to prove (or the question might do it for you) you start with an algebraic expression (NOT an equation/inequation) and manipulate it into another expression.

In order to prove LHS < RHS, you might;

1) start with the LHS and manipulate it into < RHS

2) start with LHS – RHS and manipulate it into < 0

3) start with the LHS and manipulate it into a new LHS expression then independently start with the RHS and manipulate it into a new RHS expression, then prove that

$$\textit{new}\ \text{LHS} < \textit{new}\ \text{RHS}$$

$$\therefore \text{LHS} < \text{RHS}$$

3. Proving Inequalities

(I) Start with a known result

If $x-y>y+z$ prove $y<\frac{x-z}{2}$

$$x-y>y+z$$
$$-2y>z-x$$
$$y<\frac{x-z}{2}$$

Note: *you can only move terms from side to side of* ***known facts***

(II) Move everything to the left

Show that if $a\geq 0, b\geq 0$ then $ab\left(a^2+b^2\right)\geq 2a^2b^2$

$$ab\left(a^2+b^2\right)-2a^2b^2=ab\left(a^2-2ab+b^2\right)$$
$$=ab\left(a-b\right)^2$$
$$\geq 0$$
$$\therefore ab\left(a^2+b^2\right)\geq 2a^2b^2$$

(III) Squares are positive or zero

Show that if a, b and c are positive, then $a^2+b^2+c^2 \geq ab+bc+ac$

$$(a-b)^2 \geq 0$$

$$a^2-2ab+b^2 \geq 0$$

$$a^2+b^2 \geq 2ab$$

$$a^2+c^2 \geq 2ac$$

$$b^2+c^2 \geq 2bc$$

$$2a^2+2b^2+2c^2 \geq 2ab+2bc+2ac$$

$$a^2+b^2+c^2 \geq ab+bc+ac$$

Exercise 3A; 4, 6ace, 7bdf, 8bdf, 9, 11, 12, 13ac, 15, 18bcd, 22, 24