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$$1 + \tan^2 x = \sec^2 x$$

$$\int (1 + \tan^2 x) dx = \int \sec^2 x dx$$

$$\begin{aligned} \int \tan^2 x dx &= \int \sec^2 x dx - \int 1 dx \\ &= \underline{\tan x - x + C} \end{aligned}$$

$$b) \quad 1 - \sin^2 x = \cos^2 x$$
$$\int_0^{\frac{\pi}{3}} \frac{2}{1 - \sin^2 x} dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{2}{\cos^2 x} dx$$
$$= 2 \int_0^{\frac{\pi}{3}} \sec^2 x dx$$
$$= 2 \left[ \tan x \right]_0^{\frac{\pi}{3}}$$
$$= 2(\sqrt{3} - 0)$$
$$= \underline{2\sqrt{3}}$$

$$\begin{aligned} \text{8a)} \quad & \int_0^{\frac{\pi}{6}} \frac{\cos x}{1 + \sin x} dx \\ & = \left[ \log(1 + \sin x) \right]_0^{\frac{\pi}{6}} \\ & = \log\left(\frac{1 + \frac{1}{2}}{1}\right) \\ & = \log \frac{3}{2} \end{aligned}$$

14 f)

$$\begin{aligned} (i) \quad & \int_0^{\frac{\pi}{3}} \cos^2 \frac{x}{2} dx \\ &= \frac{1}{2} \int_0^{\frac{\pi}{3}} (1 + \cos x) dx \\ &= \frac{1}{2} \left[ x + \sin x \right]_0^{\frac{\pi}{3}} \\ &= \frac{1}{2} \left( \frac{\pi}{3} + \frac{\sqrt{3}}{2} - 0 \right) \\ &= \frac{1}{2} \left( \frac{\pi}{3} + \frac{\sqrt{3}}{2} \right) \end{aligned}$$

$$\begin{aligned} 15a) \quad & \int e^{2x} \cos e^{2x} dx \\ &= \frac{1}{2} \int 2e^{2x} \cos e^{2x} dx \\ &= \frac{1}{2} \sin e^{2x} + c \end{aligned}$$

$$\begin{aligned} 15e \quad & \int \frac{1 - \cos^3 x}{1 - \sin^2 x} dx \\ &= \int \frac{1 - \cos^3 x}{\cos^2 x} dx \\ &= \int (\sec^2 x - \cos x) dx \\ &= \underline{\tan x - \sin x + C} \end{aligned}$$