6/

$$1+\tan^2 x = \sec^2 x$$

$$\int (1+\tan^2 x)\,dx = \int \sec^2 x\,dx$$

$$\int \tan^2 x\,dx = \int \sec^2 x\,dx - \int 1\,dx$$

$$= \underline{\tan x - x + c}$$

b) $1 - \sin^2 x = \cos^2 x$

$$\int_0^{\frac{\pi}{3}} \frac{2}{1-\sin^2 x}\,dx = \int_0^{\frac{\pi}{3}} \frac{2}{\cos^2 x}\,dx$$

$$= 2\int_0^{\frac{\pi}{3}} \sec^2 x\,dx$$

$$= 2\Big[\tan x\Big]_0^{\frac{\pi}{3}}$$

$$= 2(\sqrt{3} - 0)$$

$$= \underline{2\sqrt{3}}$$

8a)
$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{1+\sin x}\,dx$$
$$= \Big[\log(1+\sin x)\Big]_0^{\frac{\pi}{6}}$$
$$= \log\left(\frac{1+\frac{1}{2}}{1}\right)$$
$$= \log\frac{3}{2}$$

14 f)

(i) $\displaystyle\int_0^{\frac{\pi}{3}} \cos^2 \frac{x}{2}\, dx$

$\displaystyle = \frac{1}{2}\int_0^{\frac{\pi}{3}} (1 + \cos x)\, dx$

$\displaystyle = \frac{1}{2}\Big[x + \sin x\Big]_0^{\frac{\pi}{3}}$

$\displaystyle = \frac{1}{2}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2} - 0\right)$

$\displaystyle = \frac{1}{2}\left(\frac{\pi}{3} + \frac{\sqrt{3}}{2}\right)$

15a)

$$\int e^{2x}\cos e^{2x}\,dx$$

$$= \frac{1}{2}\int 2e^{2x}\cos e^{2x}\,dx$$

$$= \frac{1}{2}\sin e^{2x} + c$$

15e

$$\int \frac{1-\cos^3 x}{1-\sin^2 x}\,dx$$

$$= \int \frac{1-\cos^3 x}{\cos^2 x}\,dx$$

$$= \int (\sec^2 x - \cos x)\,dx$$

$$= \tan x - \sin x + c$$