9c) $\int_0^1 \sin \pi x \, dx = \frac{2}{\pi}$

b) $\int_0^1 \sin \pi x \, dx$

	1	4	2	4	1
x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\sin \pi x$	0	$\frac{1}{\sqrt{2}}$	1	$\frac{1}{\sqrt{2}}$	0

$$\doteq \frac{1}{12}\left\{0 + 4\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) + 2(1) + 0\right\}$$

$$= \frac{1}{12}\left(\frac{8}{\sqrt{2}} + 2\right)$$

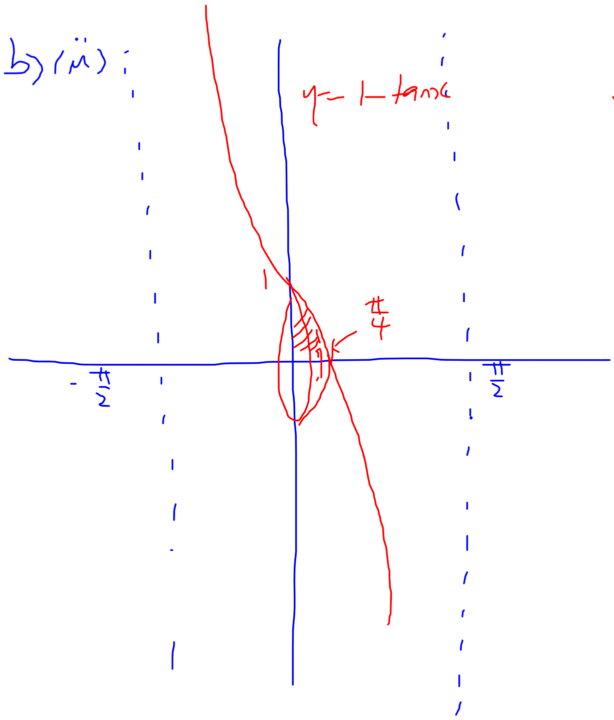
$$= \frac{1}{12}\left(4\sqrt{2} + 2\right)$$

$$= \frac{1}{6}\left(2\sqrt{2} + 1\right)$$

c) $\therefore \dfrac{2}{\pi} \doteqdot \dfrac{1}{6}(2\sqrt{2}+1)$

$$12 \doteqdot \underline{\pi(2\sqrt{2}+1)}.$$

21 b) (ii):



$$V = \pi \int_0^{\frac{\pi}{4}} (1 - \tan x)^2 \, dx$$

$$= \pi \int_0^{\frac{\pi}{4}} (1 - 2\tan x + \tan^2 x) \, dx$$

$$= \pi \int_0^{\frac{\pi}{4}} (\sec^2 x - 2\tan x) \, dx$$

$$= \pi \left[\tan x + 2\log \cos x \right]_0^{\frac{\pi}{4}}$$

$$= \pi \left(1 + 2\log \frac{1}{\sqrt{2}} - 0 - 2\log 1 \right)$$

$$= \pi (1 - \log 2) \text{ units}^3$$

24b)

a) $x^2 \sin x < x^3 < x^2 \tan x \qquad 0 < x < \frac{\pi}{2}$.

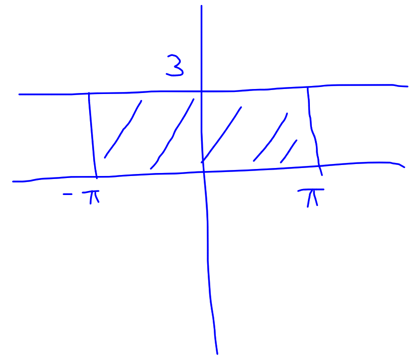
$$\int_0^{\frac{\pi}{4}} x^2 \sin x \, dx < \int_0^{\frac{\pi}{4}} x^3 \, dx < \int_0^{\frac{\pi}{4}} x^2 \tan x \, dx$$

$$\int_0^{\frac{\pi}{4}} x^2 \sin x \, dx < \frac{1}{4}\left[x^4\right]_0^{\frac{\pi}{4}} < \int_0^{\frac{\pi}{4}} x^2 \tan x \, dx$$

$$\int_0^{\frac{\pi}{4}} x^2 \sin x \, dx < \frac{\pi^4}{1024} < \int_0^{\frac{\pi}{4}} x^2 \tan x \, dx$$

26/ b) $\displaystyle\int_{-2\pi}^{2\pi} \cos^2 x \sin^3 x \, dx = \underline{0}$ (odd function)

e) $\displaystyle\int_{-\pi}^{\pi} \left(\overbrace{3}^{\text{even}} + \overbrace{2x}^{\text{odd}} + \overbrace{\sin x}^{\text{odd}}\right) dx$



$$= 2\int_0^{\pi} 3 \, dx$$

$$= 6\big[x\big]_0^{\pi}$$

$$= \underline{\underline{6\pi}}$$

26f)

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\sin 2x + \cos 3x + 3x^2\right) dx$$

$$= 2\int_{0}^{\frac{\pi}{2}} \left(\cos 3x + 3x^2\right) dx$$

$$= 2\left[\frac{1}{3}\sin 3x + x^3\right]_0^{\frac{\pi}{2}}$$

$$= 2\left(-\frac{1}{3} + \frac{\pi^3}{8} - 0\right)$$

$$= \frac{3\pi^3 - 8}{12}$$