(6) t results

If $t = \tan\frac{x}{2}$

then $\sin x = \frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$

$$\frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$$

$$= \frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)$$

$$= \frac{1}{2}\left(1+t^2\right)$$

$$\frac{dx}{dt} = \frac{2}{1+t^2}$$

$$dx = \frac{2dt}{1+t^2}$$

e.g. $\int \frac{dx}{7+6\cos x}$

$$t = \tan\frac{x}{2}$$

$$dx = \frac{2dt}{1+t^2}$$

$$= \int \frac{\frac{2dt}{1+t^2}}{7+6\left(\frac{1-t^2}{1+t^2}\right)}$$

$$= \int \frac{2dt}{7+7t^2+6-6t^2}$$

$$= \int \frac{2dt}{13+t^2}$$

$$= \frac{2}{\sqrt{13}}\tan^{-1}\frac{t}{\sqrt{13}} + c$$

$$= \frac{2}{\sqrt{13}}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{13}}\right) + c$$

If $t = \tan x$

$$\frac{dt}{dx} = \sec^2 x$$

$$= 1 + \tan^2 x$$

$$= 1 + t^2$$

$$\frac{dx}{dt} = \frac{1}{1+t^2}$$

$$dx = \frac{dt}{1+t^2}$$

In General :

If $t = \tan\dfrac{x}{a}$

$$\frac{dt}{dx} = \frac{1}{a}\sec^2\frac{x}{a}$$

$$= \frac{1}{a}\left(1 + \tan^2\frac{x}{a}\right)$$

$$= \frac{1}{a}\left(1 + t^2\right)$$

$$\frac{dx}{dt} = \frac{a}{1+t^2}$$

$$dx = \frac{a\,dt}{1+t^2}$$

Patel **Exercise 2C; 20 21, 24, 25**

Cambridge **Exercise 2F; 9bc, 17bc**