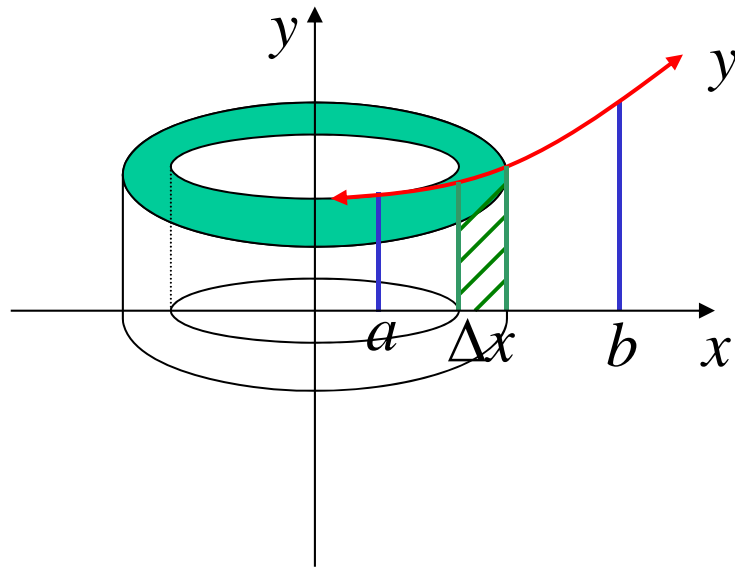
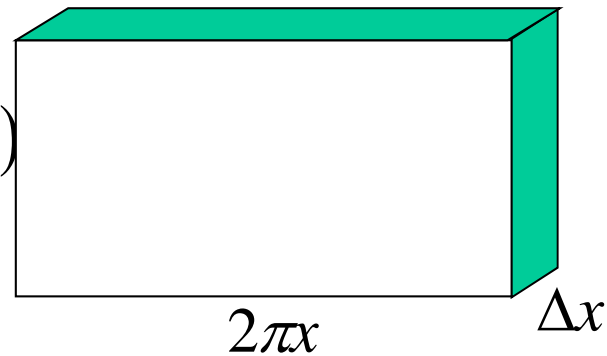


Volumes By Cylindrical Shells

(slice || rotation)



$$y = f(x)$$

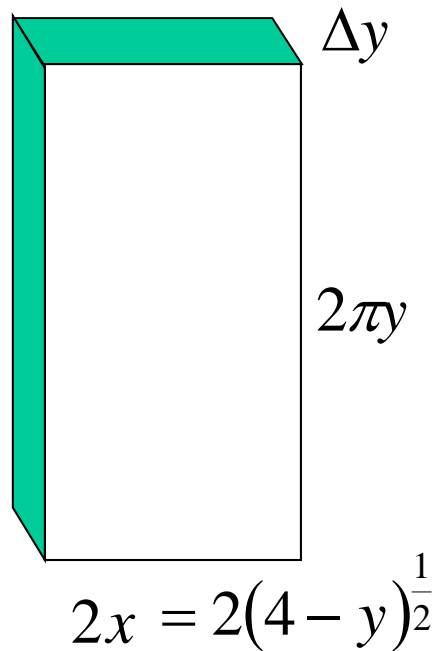
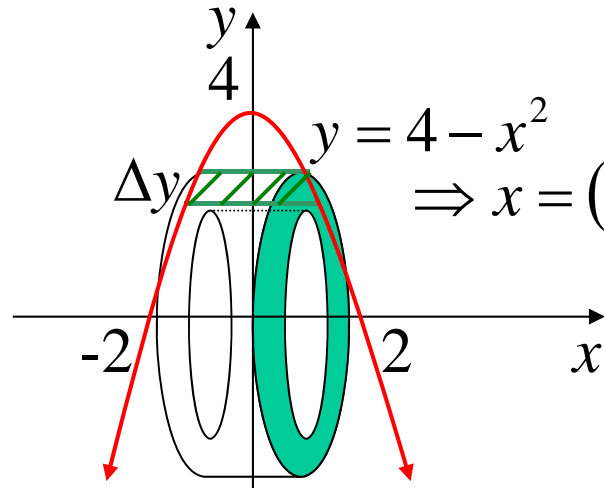


$$A(x) = 2\pi x \cdot f(x)$$

$$\Delta V = 2\pi x \cdot f(x) \cdot \Delta x$$

$$\begin{aligned} V &= \lim_{\Delta x \rightarrow 0} \sum_{x=a}^b 2\pi x \cdot f(x) \cdot \Delta x \\ &= 2\pi \int_a^b x f(x) dx \end{aligned}$$

e.g. (i) Find the volume generated when the area between $y = 4 - x^2$ and the x axis is rotated about the x axis

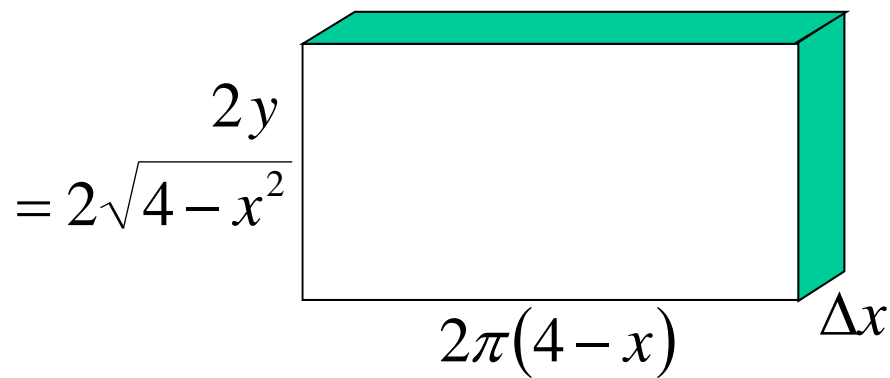
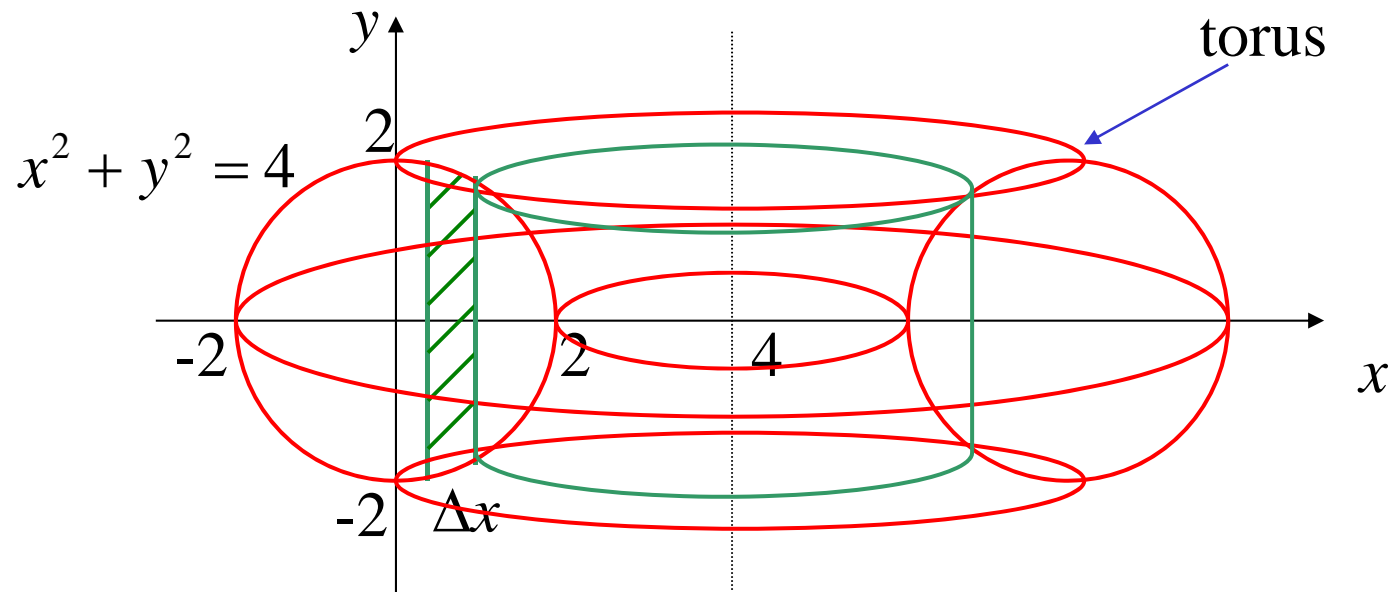


$$A(y) = 2\pi y \left[2(4 - y)^{\frac{1}{2}} \right]$$

$$\Delta V = 4\pi y (4 - y)^{\frac{1}{2}} \cdot \Delta y$$

$$\begin{aligned}
 V &= \lim_{\Delta y \rightarrow 0} \sum_{x=0}^4 4\pi y (4 - y)^{\frac{1}{2}} \cdot \Delta y \\
 &= 4\pi \int_0^4 y (4 - y)^{\frac{1}{2}} dy \\
 &= 4\pi \int_0^4 (4 - y) y^{\frac{1}{2}} dy \\
 &= 4\pi \int_0^4 \left(4y^{\frac{1}{2}} - y^{\frac{3}{2}} \right) dy \\
 &= 4\pi \left[\frac{8}{3} y^{\frac{3}{2}} - \frac{2}{5} y^{\frac{5}{2}} \right]_0^4 \\
 &= 4\pi \left\{ \frac{8}{3} (8) - \frac{2}{5} (32) - 0 \right\} \\
 &= \frac{512\pi}{15} \text{ units}^3
 \end{aligned}$$

(ii) Find the volume generated when $x^2 + y^2 = 4$ is rotated about the line $x = 4$



$$A(x) = 2\sqrt{4 - x^2} \cdot 2\pi(4 - x)$$

$$= 4\pi(4 - x)\sqrt{4 - x^2}$$

$$\Delta V = 4\pi(4 - x)\sqrt{4 - x^2} \cdot \Delta x$$

$$\begin{aligned}
V &= \lim_{\Delta x \rightarrow 0} \sum_{x=-2}^2 4\pi(4-x)\sqrt{4-x^2} \cdot \Delta x \\
&= 4\pi \int_{-2}^2 (4-x)\sqrt{4-x^2} dx \\
&= 4\pi \int_{-2}^2 4\sqrt{4-x^2} dx - 4\pi \int_{-2}^2 x\sqrt{4-x^2} dx \\
&= 16\pi \int_{-2}^2 \sqrt{4-x^2} dx - 4\pi \int_{-2}^2 x\sqrt{4-x^2} dx
\end{aligned}$$

semi-circle

odd \times even = odd function

$$= 16\pi \left[\frac{1}{2} \pi (2)^2 \right] - 0$$

$$= \underline{32\pi^2 \text{ units}^3}$$

“Cambridge” Exercise 6B

**1, 2, 3a, 4 to 7, 9, 10, 11,
13, 14, 15, 17, 18**

***Patel* Exercise 3B;
3, 4, 6, 8**

***Patel* Exercise 3C;
1, 3, 9, 12, 20**