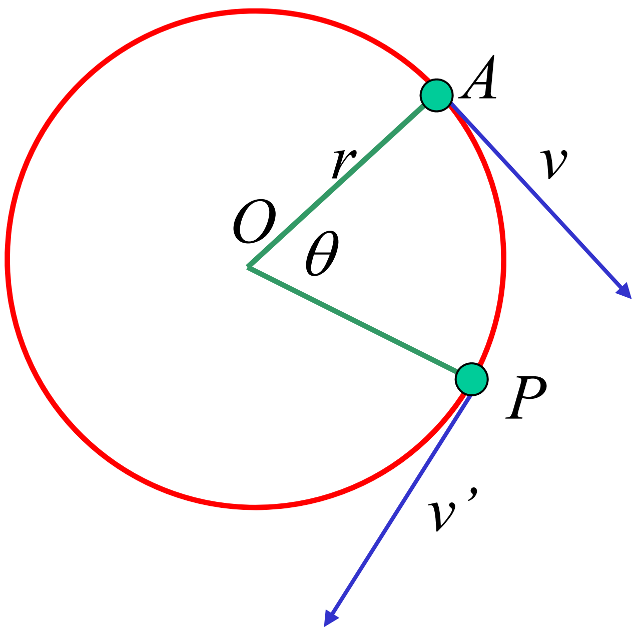
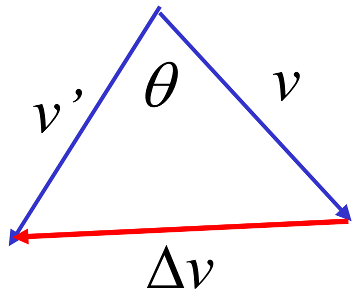
Acceleration with Uniform Circular Motion

Uniform circular motion is when a particle moves with constant angular velocity. $(\therefore$ the magnitude of the linear velocity will also be constant$)$

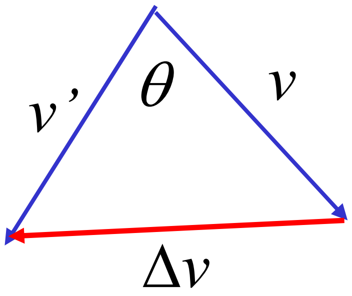


A particle moves from *A* to *P* with constant angular velocity.

The acceleration of the particle is the change in velocity with respect to time.



This triangle of vectors is similar to ΔOAP



$$\frac{\Delta v}{AP} = \frac{v}{r}$$

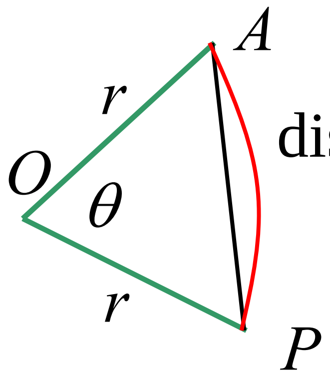
$$\Delta v = \frac{v \cdot AP}{r}$$

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v \cdot AP}{r \cdot \Delta t}$$

But, as $\Delta t \to 0$, $AP = arcAP$

$$\therefore a = \lim_{\Delta t \to 0} \frac{v \cdot arcAP}{r \cdot \Delta t}$$



$$\text{distance} = \text{speed} \times \text{time}$$

$$arcAP = v \cdot \Delta t$$

$$\therefore a = \lim_{\Delta t \to 0} \frac{v \cdot v \cdot \Delta t}{r \cdot \Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{v^2}{r}$$

$$= \frac{v^2}{r}$$

$\therefore$ the acceleration in uniform circular motion has magnitude $\frac{v^2}{r}$ and is directed towards the centre

Acceleration Involved in Uniform Circular Motion	Forces Involved in Uniform Circular Motion
$a = \frac{v^2}{r}$	$F = \frac{mv^2}{r}$
OR	*OR*
$a = r\omega^2$	$F = mr\omega^2$

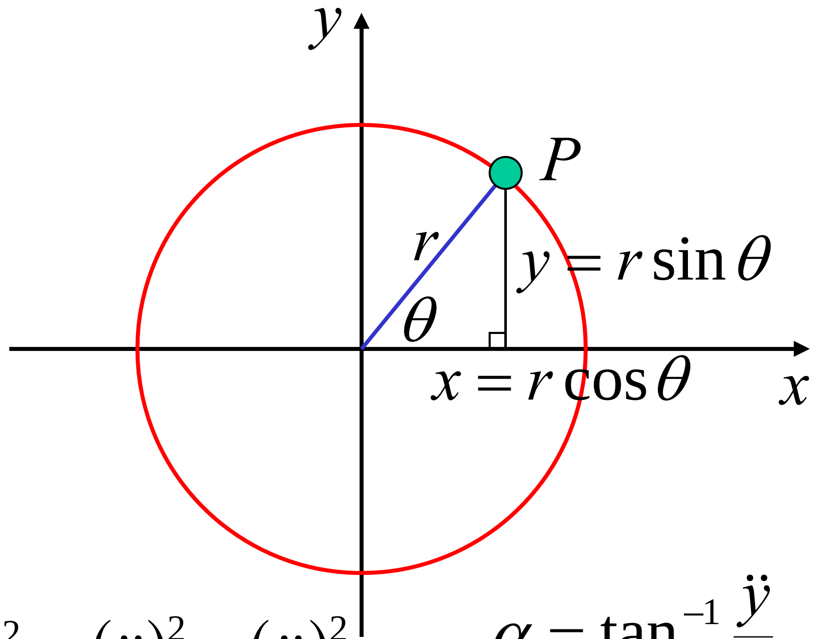
e.g. (*i*) (2003)

A particle P of mass m moves with constant angular velocity ω on a circle of radius r. Its position at time t is given by;

$$x = r\cos\theta$$

$$y = r\sin\theta, \quad \text{where } \theta = \omega t$$

a) Show that there is an inward radial force of magnitude $mr\omega^2$ acting on P.



$$x = r\cos\theta$$

$$\dot{x} = -r\sin\theta \cdot \frac{d\theta}{dt}$$

$$= -r\omega\sin\theta$$

$$\ddot{x} = -r\omega\cos\theta \cdot \frac{d\theta}{dt}$$

$$= -r\omega^2\cos\theta$$

$$= -\omega^2 x$$

$$y = r\sin\theta$$

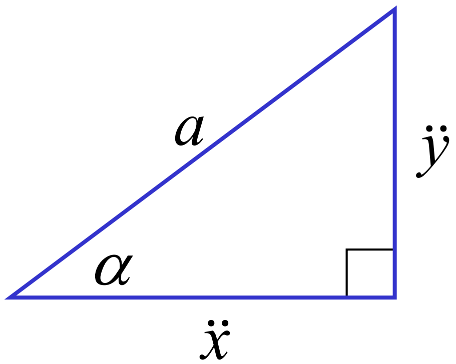
$$\dot{y} = r\cos\theta \cdot \frac{d\theta}{dt}$$

$$= r\omega\cos\theta$$

$$\ddot{y} = -r\omega\sin\theta \cdot \frac{d\theta}{dt}$$

$$= -r\omega^2\sin\theta$$

$$= -\omega^2 y$$



$$a^2 = (\ddot{x})^2 + (\ddot{y})^2$$

$$= \omega^4 x^2 + \omega^4 y^2$$

$$= \omega^4 (x^2 + y^2)$$

$$= \omega^4 r^2$$

$$a = r\omega^2$$

$$F = ma$$

$$\therefore F = mr\omega^2$$

$$\alpha = \tan^{-1}\frac{\ddot{y}}{\ddot{x}}$$

$$= \tan^{-1}\left(\frac{-\omega^2 y}{-\omega^2 x}\right)$$

$$= \tan^{-1}\frac{y}{x}$$

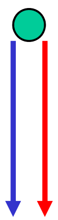
$$= \theta$$

$\therefore$ There is a force, $F = mr\omega^2$, acting towards the centre

b) A telecommunications satellite, of mass m, orbits Earth with constant angular velocity ω at a distance r from the centre of the Earth. The gravitational force exerted by Earth on the satellite is $\frac{Am}{r^2}$ where A is a constant. By considering all other forces on the satellite to be negligible, show that;

$$r = \sqrt[3]{\frac{A}{\omega^2}}$$

$$m\ddot{x} = mr\omega^2$$



$$\frac{Am}{r^2}$$

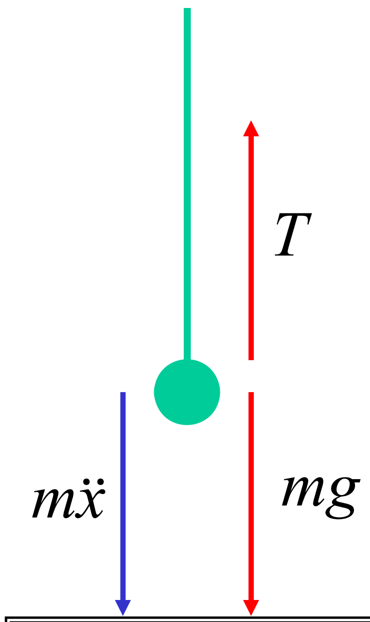
$$mr\omega^2 = \frac{Am}{r^2}$$

$$r^3 = \frac{Am}{m\omega^2}$$

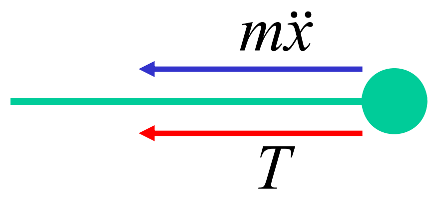
$$= \frac{A}{\omega^2}$$

$$r = \sqrt[3]{\frac{A}{\omega^2}}$$

(ii) A string is 50cm long and it will break if a ,mass exceeding 40kg is hung from it.
A mass of 2kg is attached to one end of the string and it is revolved in a circle.
Find the greatest angular velocity which may be imparted without breaking the string.



$$m\ddot{x} = mg - T$$
$$0 = mg - T$$
$$T = (40)(9.8)$$
$$= 392N$$



$$T = mr\omega^2$$
$$392 = (2)(0.5)\omega^2$$
$$\omega^2 = 392$$
$$\omega = \sqrt{392}$$
$$\omega = 2\sqrt{98}\text{rad/s}$$

“*Cambridge*” Exercise 9A; all

***Patel* Exercise 9B; all**