Equations of the form asinx + bcosx = c

Method 1: Using the *t* results

eg (i) $3\cos\theta + 4\sin\theta = 2$ $\qquad 0 \le \theta \le 360^\circ$

let $t = \tan\frac{\theta}{2}$ $\qquad 3\left(\frac{1-t^2}{1+t^2}\right) + 4\left(\frac{2t}{1+t^2}\right) = 2$ $\qquad 0 \le \frac{\theta}{2} \le 180^\circ$

$$3 - 3t^2 + 8t = 2 + 2t^2$$

$$5t^2 - 8t - 1 = 0$$

$$t = \frac{8 \pm \sqrt{84}}{10}$$

$\tan\frac{\theta}{2} = \frac{4-\sqrt{21}}{5}$ or $\tan\frac{\theta}{2} = \frac{4+\sqrt{21}}{5}$

Q2 $\tan\alpha = \frac{\sqrt{21}-4}{5}$ $\qquad$ Q1 $\tan\alpha = \frac{4+\sqrt{21}}{5}$

$\alpha = 6^\circ 39'$ $\qquad \alpha = 59^\circ 47'$

$\frac{\theta}{2} = 173^\circ 21'$ $\qquad \frac{\theta}{2} = 59^\circ 47'$

$\theta = 346^\circ 42'$ $\qquad \theta = 119^\circ 33'$

$Test: \theta = 180^\circ$

$3\cos 180^\circ + 4\sin 180^\circ$

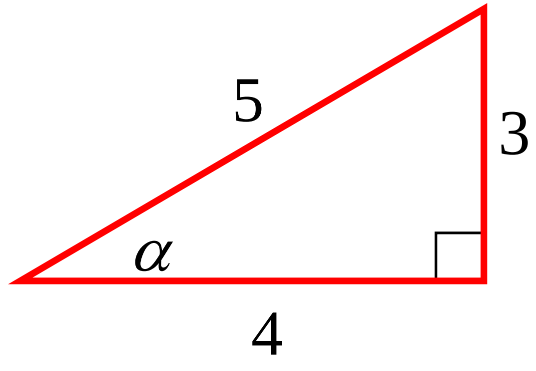
$= -4 \ne 2$

$\therefore \theta = 119^\circ 33', 346^\circ 42'$

Method 2: Auxiliary Angle Method

(i) Change into a sine function

eg (i) $3\cos\theta + 4\sin\theta = 2$ $\qquad 0 \le \theta \le 360^\circ$



$$3\cos\theta + 4\sin\theta = 2$$

$$5\times\left(\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right) = 2$$

$$5\sin(\alpha+\theta) = 2$$

$$\sin(\alpha+\theta) = \frac{2}{5}$$

Q1, Q2

$$\sin\beta = \frac{2}{5}$$

$$\beta = 23^\circ 35'$$

$$36^\circ 52' + \theta = 23^\circ 35', 156^\circ 25'$$

$$\theta = -13^\circ 17', 119^\circ 33'$$

$$\tan\alpha = \frac{3}{4}$$

$$\alpha = 36^\circ 52'$$

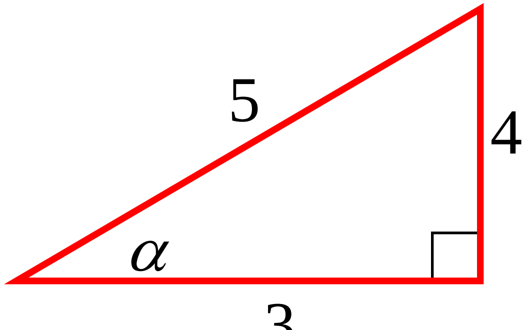
$$\therefore \theta = 119^\circ 33', 346^\circ 43'$$

Method 2: Auxiliary Angle Method

(ii) Change into a cosine function

eg (i) $3\cos\theta + 4\sin\theta = 2$ $\qquad 0 \le \theta \le 360^\circ$

$$\cos\alpha\cos\theta + \sin\alpha\sin\theta$$



$$3\cos\theta + 4\sin\theta = 2$$

$$5\times\left(\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right) = 2$$

$$5\cos(\theta-\alpha) = 2$$

$$\cos(\theta-\alpha) = \frac{2}{5}$$

Q1, Q4

$$\cos\beta = \frac{2}{5}$$

$$\beta = 66^\circ 25'$$

$$\tan\alpha = \frac{4}{3}$$

$$\alpha = 53^\circ 8'$$

$$\theta - 53^\circ 8' = 66^\circ 25', 293^\circ 35'$$

$$\therefore \theta = 119^\circ 33', 346^\circ 43'$$

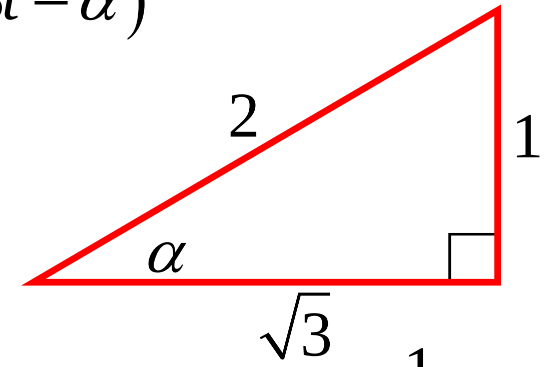
2005 Extension 1 HSC Q5c) (i)

(ii) Express $\sqrt{3}\sin 3t - \cos 3t$ in the form $R\sin(3t - \alpha)$

$$(\sin 3t\cos\alpha - \cos 3t\sin\alpha)$$

$$\sqrt{3}\sin 3t - \cos 3t = 2\sin(3t - \alpha)$$

$$= 2\sin(3t - 30^\circ)$$



$$\tan\alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = 30^\circ$$

Exercise 2E;

6, 7, 10bd, 11, 13, 14, 16ac, 20a, 23

2003 Extension 1 HSC Q2e) (i)

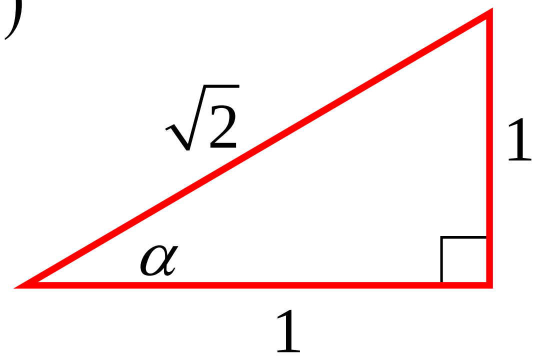
(iii) Express $\sin x - \cos x$ in the form $R\cos(x + \alpha)$

$$(\cos x\cos\alpha - \sin x\sin\alpha)$$

$$\sin x - \cos x = -(\cos x - \sin x)$$

$$= -\sqrt{2}\cos(x + \alpha)$$

$$= -\sqrt{2}\cos(x + 45^\circ)$$



$$\tan\alpha = 1$$

$$\alpha = 45^\circ$$