

# (C) Transformations

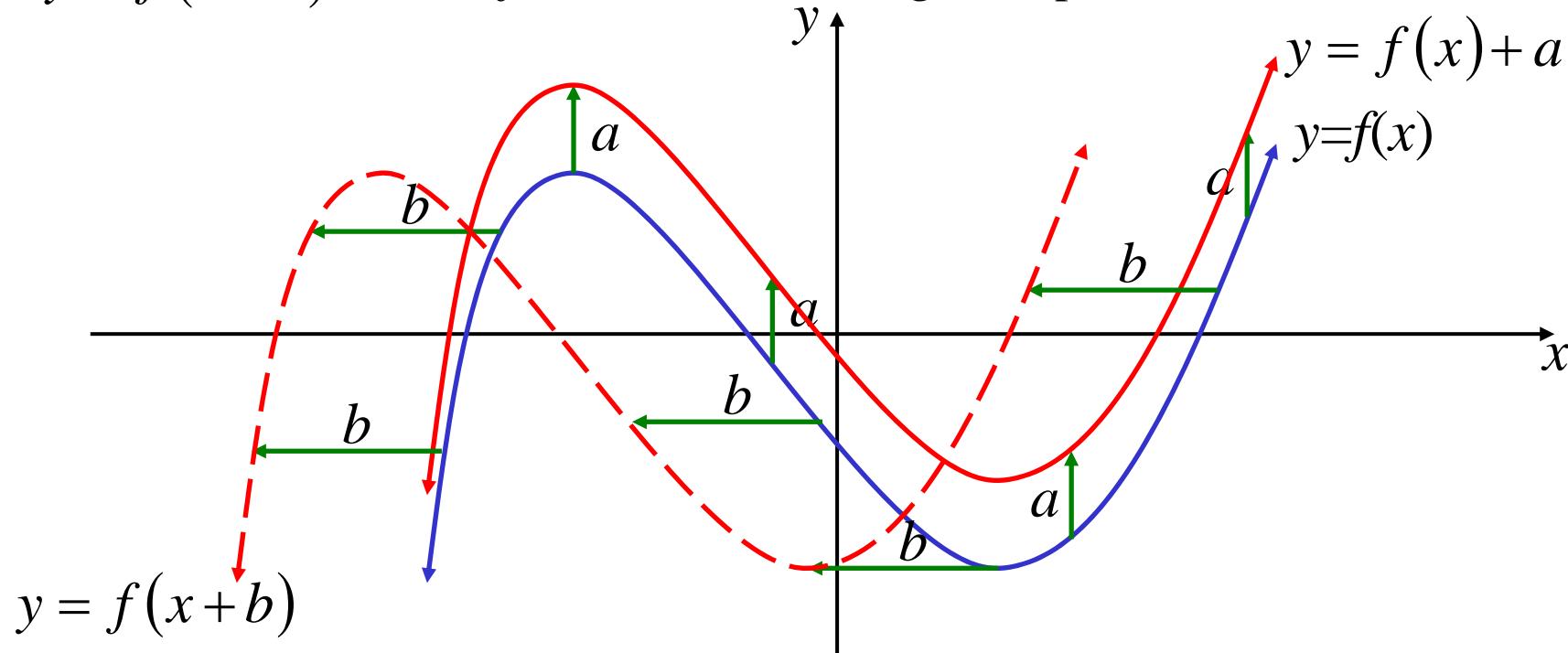
Given that the graph  $y = f(x)$  can be sketched, then it is possible to build other sketches through appropriate transformations.

## 1. Shifting Curves

- $y = f(x) \pm a$  OR  $y \mp a = f(x)$

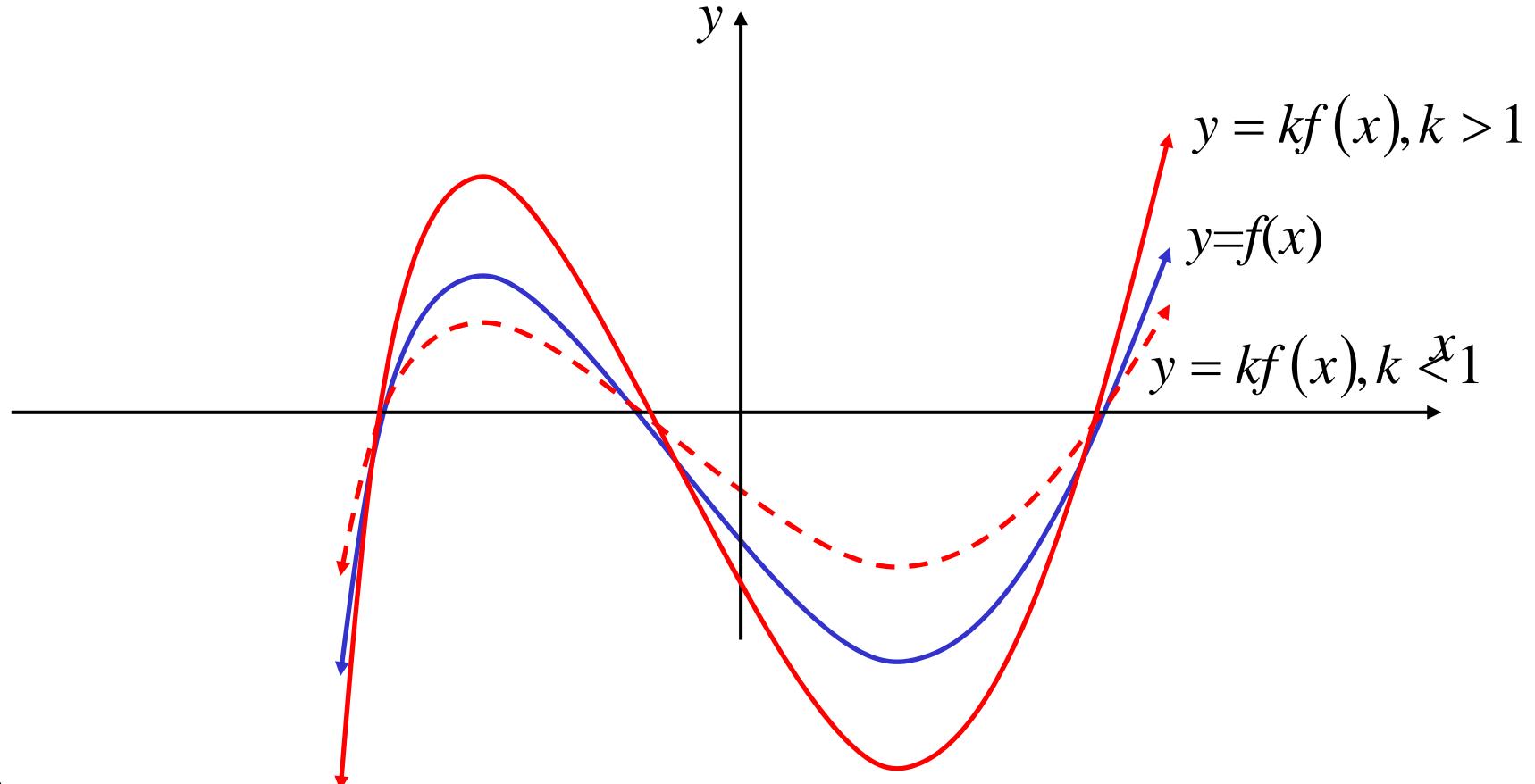
(to shift  $a$  units up, replace  $y$  with  $y - a$ )

- $y = f(x \pm a)$  (to shift  $a$  units to the right, replace  $x$  with  $x - a$ )



## 2. Stretching Curves

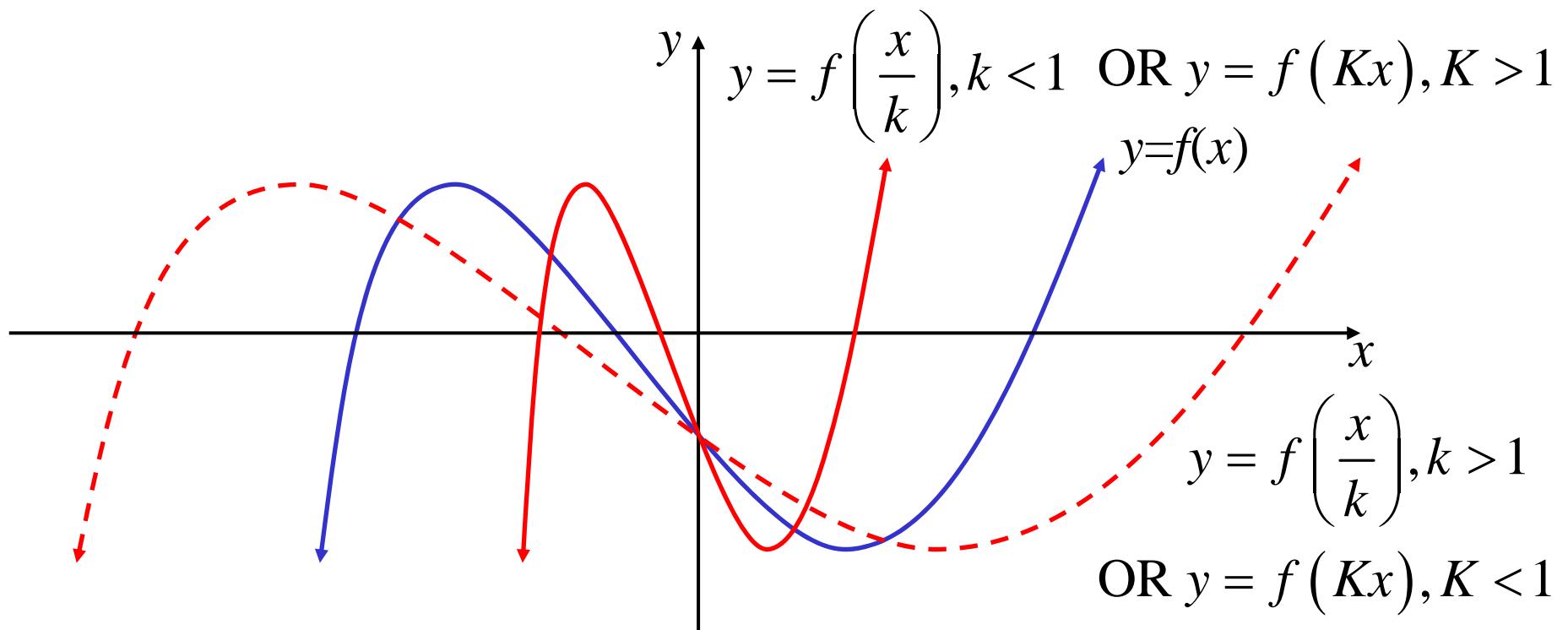
- $y = kf(x)$  (to stretch vertically by factor  $k$ , replace  $y$  with  $\frac{y}{k}$ )  
 $(k < 1 \text{ shallower}; k > 1 \text{ steeper})$



Note:

- domain remains same
- $x$  values of key points remain the same
- $x$  intercepts remain same

- $y = f\left(\frac{x}{k}\right)$  (to stretch horizontally by factor  $k$ , replace  $x$  with  $\frac{x}{k}$ )  
( $k < 1$  steeper,  $k > 1$  shallower)

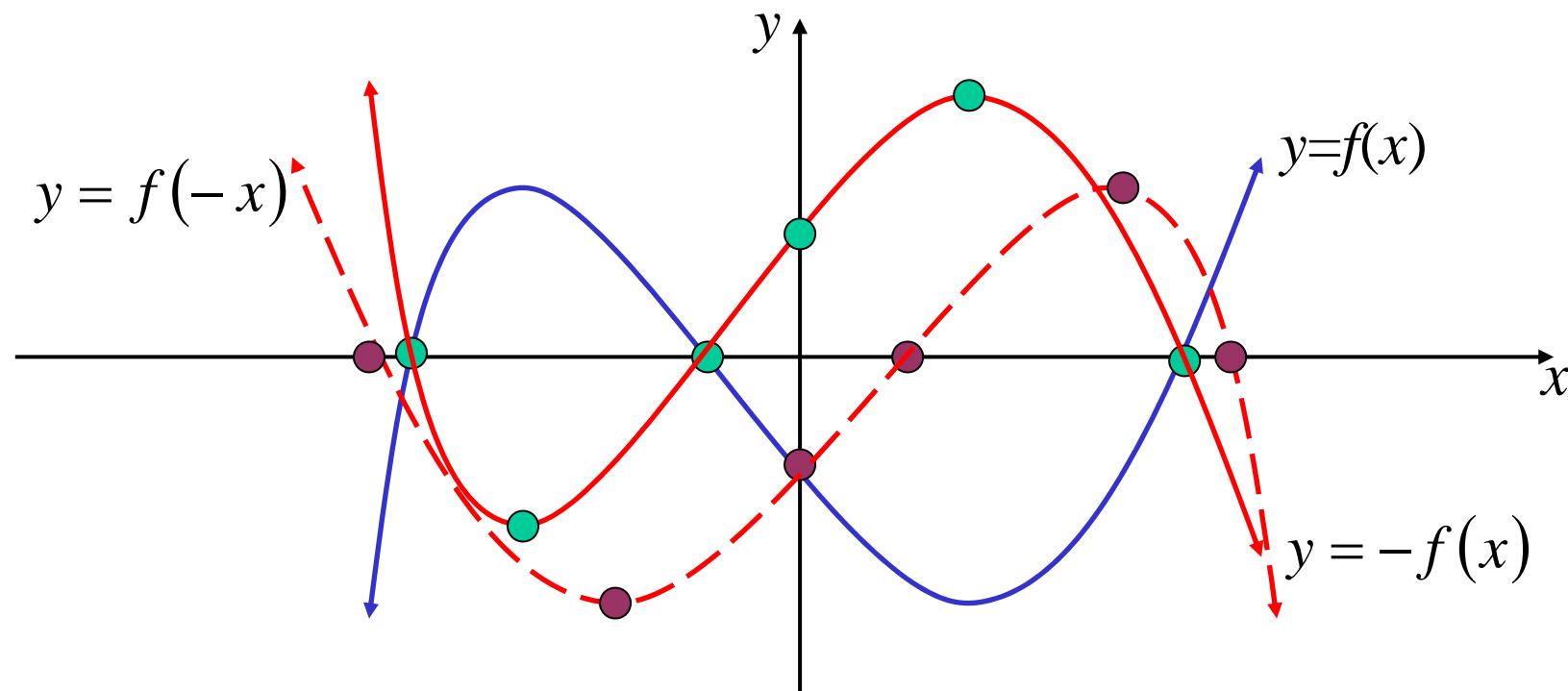


Note:

- range remains same
- y values of key points remain the same
- y intercepts remain same

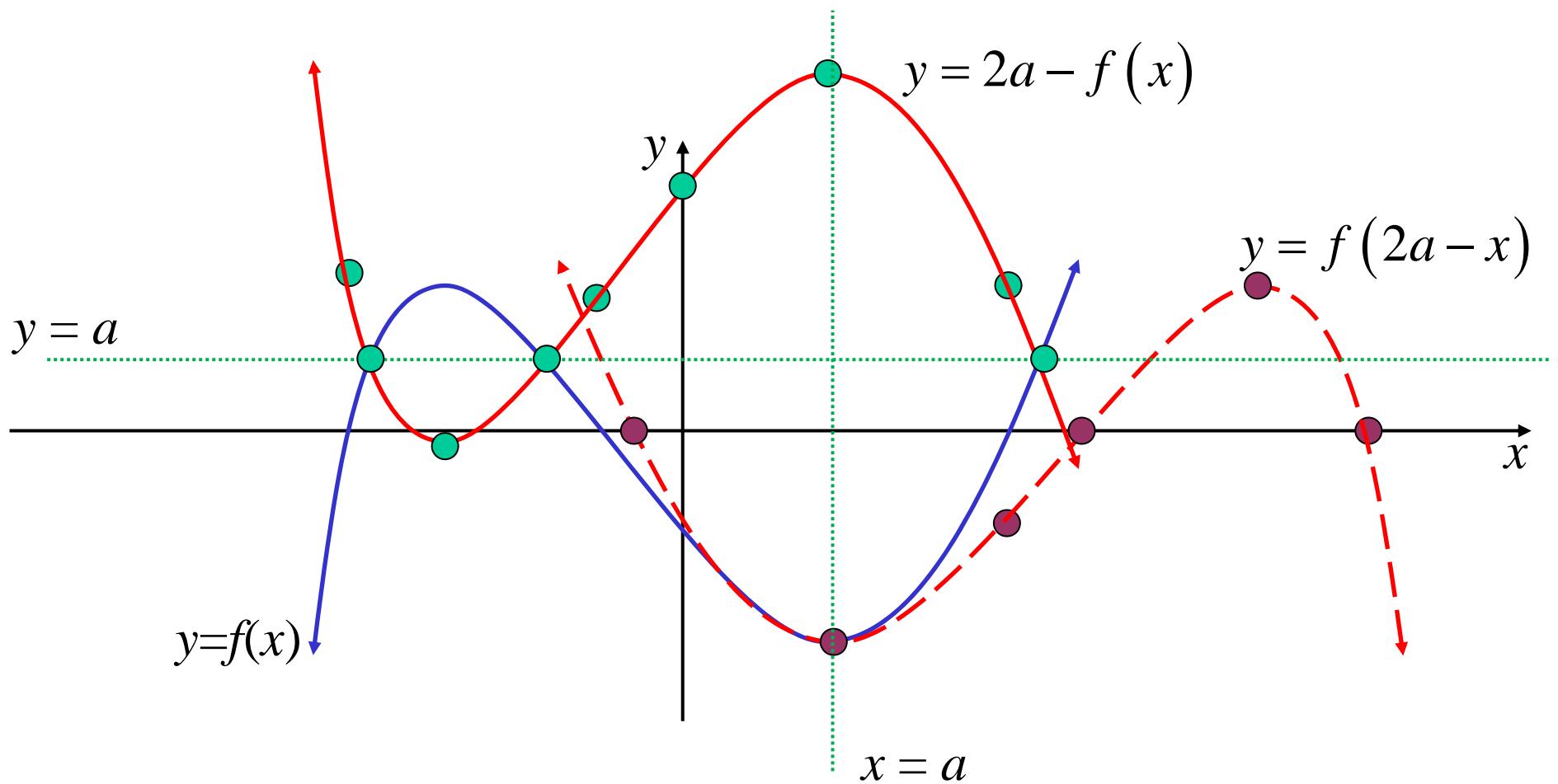
### 3. Reflecting Curves

- $y = -f(x)$  (reflect  $f(x)$  in the  $x$  axis)
- $y = f(-x)$  (reflect  $f(x)$  in the  $y$  axis)



- $y = f(2a - x)$  (reflect  $f(x)$  in the line  $x = a$ )  
(replace  $x$  with  $2a - x$ )

- $y = 2a - f(x)$  (reflect  $f(x)$  in the line  $y = a$ )  
(replace  $y$  with  $2a - y$ )



*“Cambridge”*: Exercise 8A; 1, 2a, 4, 5, 8, 9