Geometrical Theorems about Parabola

(1) Focal Chords

e.g. Prove that the tangents drawn from the extremities of a focal chord intersect at right angles on the directrix.

1 Prove $pq = -1$

2 Show that the slope of the tangent at P is p, and the slope of the tangent at Q is q.

$$pq = -1$$

Tangents are perpendicular to each other

3 Show that the point of intersection, T, of the tangents is

$\{a(p+q), apq\}$

$$y = apq$$

$$\therefore y = -a \qquad (\because pq = -1)$$

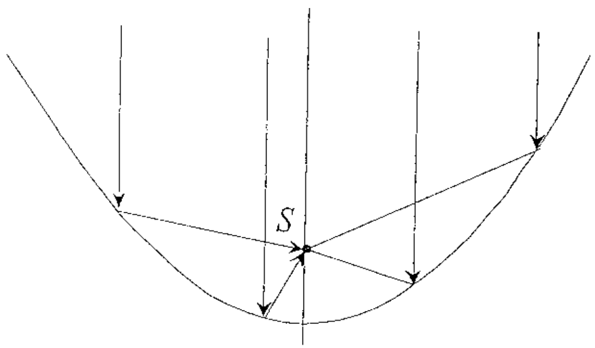
Tangents meet on the directrix

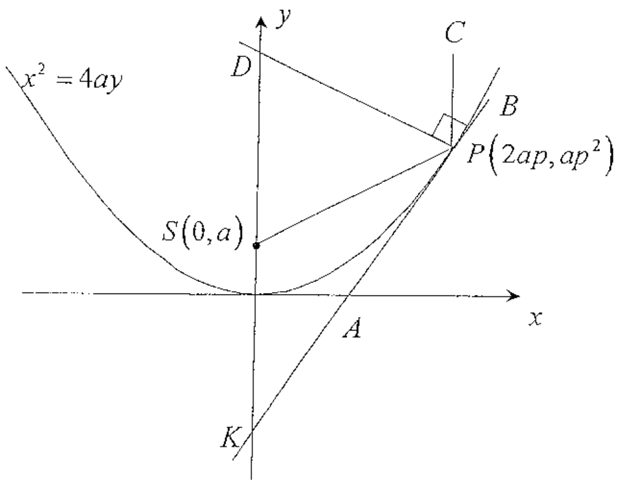
(2) Reflection Property

Any line parallel to the axis of the parabola is reflected towards the focus.

Any line from the focus of the parabola is reflected parallel to the axis.

Thus a line and its reflection are equally inclined to the normal, as well as to the tangent.





Prove: $\angle SPK = \angle CPB$

(angle of incidence = angle of reflection)

Data: $CP \parallel y$ axis

1 Show tangent at P is $y = px - ap^2$

2 tangent meets y axis when $x = 0$

$\therefore K$ is $(0, -ap^2)$

$d_{SK} = a + ap^2$

$$d_{PS} = \sqrt{(2ap-0)^2 + \left(ap^2 - a\right)^2}$$
$$= \sqrt{4a^2p^2 + a^2p^4 - 2a^2p^2 + a^2}$$
$$= a\sqrt{p^4 + 2p^2 + 1}$$
$$= a\sqrt{\left(p^2+1\right)^2}$$
$$= a\left(p^2+1\right) = d_{SK}$$
$$\therefore \Delta SPK \text{ is isosceles} \quad (\text{two} = \text{sides})$$

$$\angle SPK = \angle SKP \quad (\text{base } \angle\text{'s isosceles}\Delta \; =)$$

$$\angle SKP = \angle CPB \quad (\text{corresponding } \angle\text{'s } =, SK \,||\, CP)$$
$$\therefore \angle SPK = \angle CPB$$

Exercise 9I; 1, 2, 4, 7, 11, 12, 17, 18, 21