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# *Pymble Ladies' College*

## HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION 2014

# Mathematics Extension 1

### General Instructions

- Reading time – 5 minutes.
- Working time – 2 hours.
- Write using pencil for Questions 1-10.
- Write using black or blue pen for Questions 11-14. Black pen is preferred.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- In Questions 11-14, show relevant mathematical reasoning and/or calculations.

Total Marks – 70

**Section I** Pages 1-4

10 marks

- Attempt all Questions 1-10
- Allow about 15 mins for this section

**Section II** Pages 5-11

60 marks

- Attempt Questions 11-14
- Allow about 1 hour 45 minutes for this section

<b>Mark</b>	<b>/70</b>
<b>Highest Mark</b>	<b>/70</b>
<b>Rank</b>	

## Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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- 1 The roots of the equation  $x^3 - 5x^2 + 4 = 0$  are  $\alpha, \beta$  and  $\gamma$ .

The value of  $\alpha + \beta + \gamma$  and the value of  $\alpha\beta\gamma$  are respectively.

- (A) 5 and 4
- (B) 5 and  $-4$
- (C)  $-5$  and 4
- (D)  $-5$  and  $-4$

- 2 Evaluate  $\sin^{-1}\left(\sin\frac{4\pi}{3}\right)$ .

- (A)  $\frac{4\pi}{3}$
- (B)  $\frac{\pi}{3}$
- (C)  $\frac{-2\pi}{3}$
- (D)  $\frac{-\pi}{3}$

- 3 When the polynomial  $P(x) = x^4 + ax + 2$  is divided by  $x^2 + 1$  the remainder is  $2x + 3$ .

The value of  $a$  is

- (A) 1
- (B) 2
- (C) 0
- (D) 3

- 4 Given the points  $A(7, 14)$  and  $B(1, 2)$ ,  $C$  is a point on  $AB$  produced such that  $AB : BC = 2 : 1$ .

Find the coordinates of  $C$ .

- (A)  $(-5, -10)$
- (B)  $(-2, -4)$
- (C)  $(3, 6)$
- (D)  $(5, 10)$

- 5 Find  $\int \frac{1}{\sqrt{1-3x^2}} dx$ .

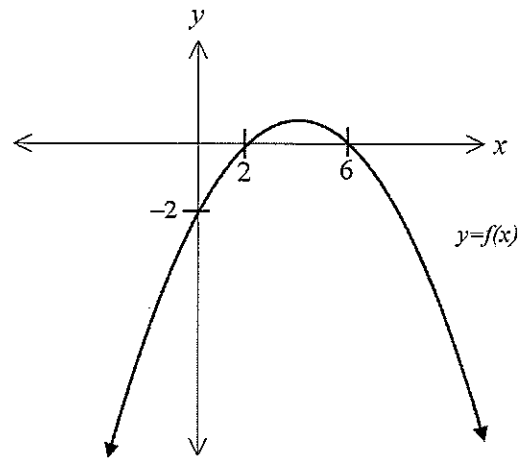
- (A)  $3\sin^{-1}(3x) + C$
- (B)  $\frac{1}{3}\sin^{-1}(3x) + C$
- (C)  $\sqrt{3}\sin^{-1}(\sqrt{3}x) + C$
- (D)  $\frac{1}{\sqrt{3}}\sin^{-1}(\sqrt{3}x) + C$

- 6 Evaluate  $\int_0^{\frac{\pi}{6}} \sin^2\theta d\theta$ .

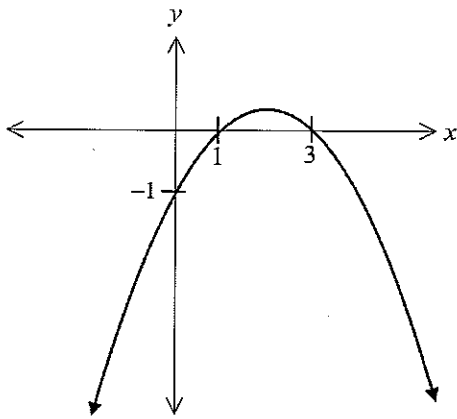
- (A)  $\frac{\pi}{12} - \frac{\sqrt{3}}{8}$
- (B)  $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$
- (C)  $\frac{1}{24}$
- (D) 1

7 The figure on the right shows the graph of  $y = f(x)$ .

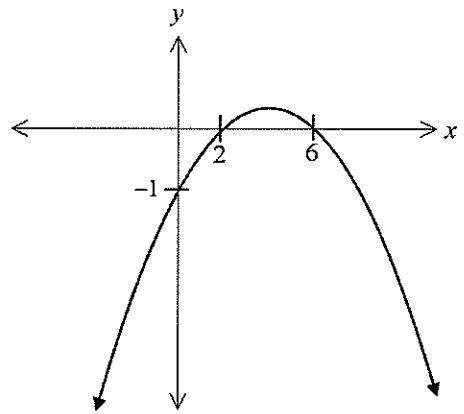
If  $2f(x) = g(x)$ , which of the following may represent the graph of  $y = g(x)$ ?



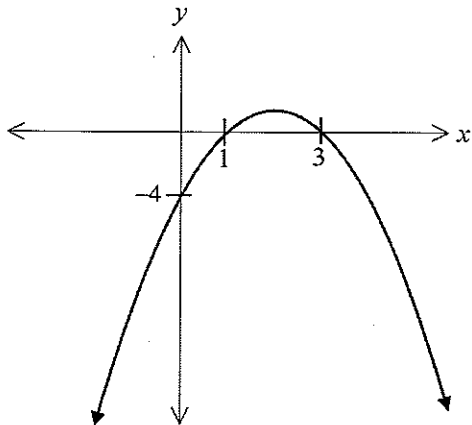
(A)



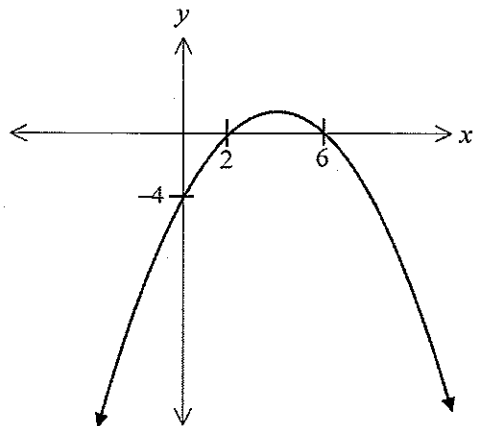
(B)



(C)



(D)



- 8 If  $\int_{-a}^a f(x) dx = 0$ , then which one of the following statements is false?
- (A)  $f(x)$  is an odd function
- (B)  $\int_0^a f(x) dx = \int_{-a}^0 f(-x) dx$
- (C)  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$
- (D) The area bounded by the curve  $y = f(x)$ , the  $x$  axis and the lines  $x = a$  and  $x = -a$  is twice the area bounded by the curve  $y = f(x)$ , the  $x$  axis and the lines  $x = 0$  and  $x = a$ .

- 9 For  $0^\circ \leq \theta \leq 90^\circ$ , the least value of  $\frac{30}{3\sin^2 \theta + 2\sin^2(90^\circ - \theta)}$  is

- (A) 5  
 (B) 6  
 (C) 10  
 (D) 15

- 10 Given  $n$  is an integer, the general solution of  $\tan\left(2x + \frac{\pi}{4}\right) = \sqrt{3}$  is

- (A)  $x = \frac{(12n+1)\pi}{24}$
- (B)  $x = \frac{(3n+1)\pi}{6}$
- (C)  $x = \frac{(12n-1)\pi}{24}$
- (D)  $x = \frac{(6n+1)\pi}{6}$

## Section II

60 marks

Attempt Questions 11-14

Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11.** (15 marks). Use a **Separate Booklet**.

**Marks**

(a) Given  $f(x) = x^4 + x^2 - 80$ .

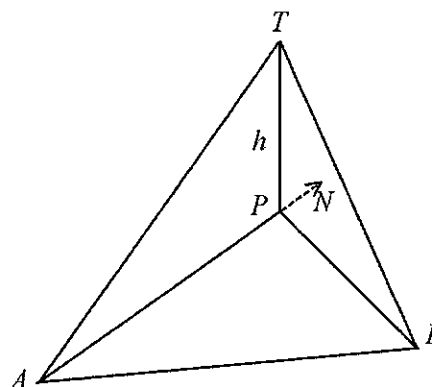
2

Assume there is a zero near  $x = 3$ . Use Newton's method once to find a better approximation to the root correct to 2 significant figures.

- (b) From a point  $A$  due south of a tower,  $TP$ , the angle of elevation of the top of the tower,  $T$  is  $25^\circ$  and from a point  $B$  due east of the tower is  $32^\circ$ .

The distance from  $A$  to  $B$  is 50 metres.

Let the height of tower  $TP$  be  $h$  metres.



- (i) Copy the diagram in your answer booklet and complete with all given information.

1

- (ii) Find an expression for  $PA$  in terms of  $h$ .

1

- (iii) Find the height of the tower,  $h$ , correct to 1 decimal place.

3

Question 11 continues on page 6.

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(c) The function  $f(x)$  is defined as  $f(x) = \frac{3x-4}{x+2}$ , where  $x \neq -2$ .

(i) Find an expression for  $f^{-1}(x)$ . 2

(ii) Write down the domain of  $f^{-1}(x)$ . 1

(d) Solve  $\frac{4}{(x-1)^2} > 1$ . 3

(e) Find  $\int \frac{\ln x}{2x} dx$  using the substitution  $u = \ln x$ . 2

**End of Question 11**

(a) Find the term independent of  $x$  in the expansion of  $\left(2x + \frac{1}{x^2}\right)^6$ . **2**

(b) (i) Show that  $\tan x = \frac{\sin 2x}{1 + \cos 2x}$ . **2**

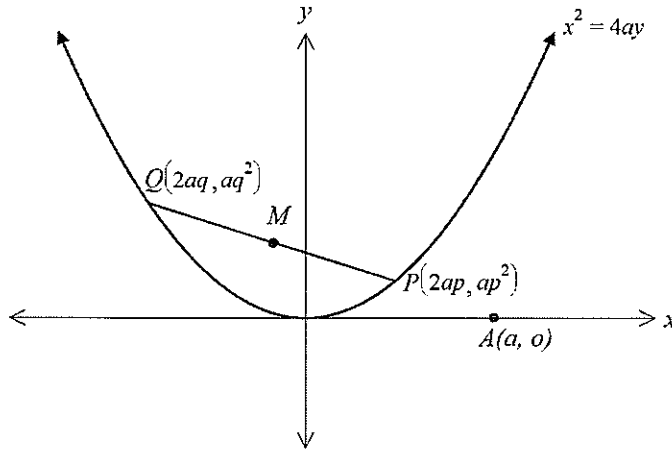
(ii) Hence evaluate  $\tan \frac{\pi}{12}$  in simplest form. **2**

(c) Prove by mathematical induction that  $8^n - 3^n$  is divisible by 5, where  $n$  is a positive integer. **3**

**Question 12 continues on page 8.**



(d)



In the diagram above, the points  $P(2ap, ap^2)$  and  $Q(2aq, aq^2)$  lie on the parabola with equation  $x^2 = 4ay$ .

- (i) Write down the coordinates of the midpoint  $M$  of the chord  $PQ$ . 1
  
- (ii) Show that the equation of the chord  $PQ$  is  $y = \frac{(p+q)x}{2} - apq$ . 2
  
- (iii) Show that the condition for the chord  $PQ$  produced to pass through the point  $A(a, 0)$  is  $p+q = 2pq$ . 1
  
- (iv) Find the cartesian equation of the locus of  $M$ , as the points  $P$  and  $Q$  move on the parabola subject to the constraint that  $PQ$  pass through  $A(a, 0)$ . 2

**End of Question 12**

- (a) Find the acute angle between the tangents on the curve  $y = \tan^{-1} x$  at the points where  $x = 0$  and  $x = 1$ . Answer correct to the nearest degree. 2

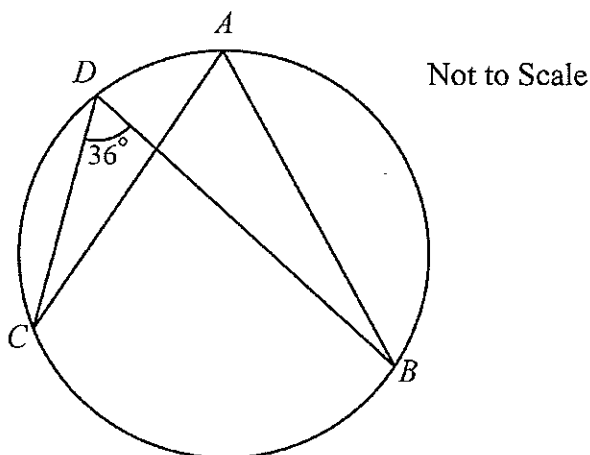
- (b) During a chemical reaction, the amount,  $R$  kg, of chemical formed at time  $t$  hours is modelled by the differential equation

$$\frac{dR}{dt} = 4 - \frac{R}{15}$$

- (i) Show that  $R = 60 - 50e^{-\frac{t}{15}}$  is a solution to  $\frac{dR}{dt} = 4 - \frac{R}{15}$ . 2

- (ii) How long will it take for 20 kg of the chemical to form? Give your answer correct to 2 significant figures. 2

- (c) In the figure below,  $BD$  is a diameter of the circle  $ABCD$ . If  $AB=AC$  and  $\angle BDC = 36^\circ$ , find  $\angle ABD$ . 3



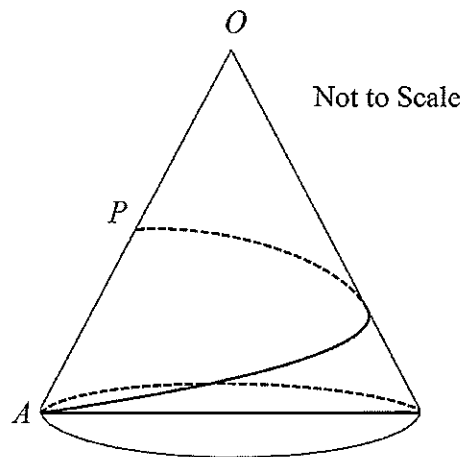
Question 13 continues on page 10.

(d) A thin sheet of smooth metal is in the shape of a sector of a circle with  $OA$ ,  $OB$  as bounding radii each of length 10 cm, and the angle  $AOB$  is  $60^\circ$ .

(i) Find the length of the arc  $AB$ .

1

(ii) The sheet is now bent to form a right circular cone by welding the radii  $OA$  and  $OB$  together (and inserting a circular disc to close in the cone at the base).



(a) Find the volume of the cone in terms of  $\pi$ .

3

(Note: The volume of a right circular cone is,  $\frac{1}{3}\pi r^2 h$ .)

(b) On the surface of this cone a thin string is pulled tight starting with one end fixed at the point  $A$  and passing once round the cone to the other end  $P$  which is at the midpoint of  $OA$  (as shown in diagram).

2

Find the exact length of this string.

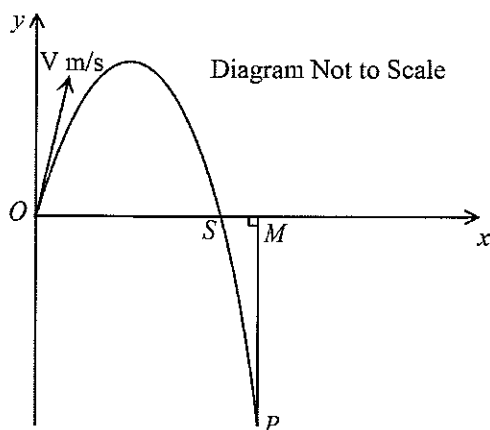
End of Question 13

(a) Solve  $\sin x - 3\cos x = 3$  for  $0^\circ \leq x \leq 360^\circ$ . 3

(b) A projectile is fired from a point  $O$  with initial speed of  $V$  m/s at an angle of elevation  $\theta$ . If  $x$  and  $y$  are the horizontal and vertical displacements of the projectile in metres from  $O$  at time  $t$  seconds later then

$$x = Vt \cos \theta \text{ and } y = Vt \sin \theta - \frac{1}{2}gt^2 \text{ where } g \text{ m/s}^2 \text{ is the acceleration due to gravity.}$$

The projectile falls to a point  $P$  below the level of  $O$  such that  $PM = OM$ .



(i) Prove that the time taken to reach  $P$  is  $2V \frac{(\sin \theta + \cos \theta)}{g}$  seconds. 1

(ii) Show that the distance  $OM$  is  $\frac{V^2}{g}(\sin 2\theta + \cos 2\theta + 1)$  metres. 2

(iii) If  $OS = r$ ,  $OM = \frac{4r}{3}$  and  $r > 0$ , prove that  $\sin 2\theta - 3\cos 2\theta = 3$ . 3

(iv) Hence, by using Question 14 part (a), find the value of  $\theta$ . 2

(v) Find an expression for the horizontal and vertical components of the velocity. 1

(vi) If the magnitude of the velocity of the projectile at  $P$  is  $kV$  m/s, find the exact value of  $k$ . 3

End of Paper

1. B      2. D      3. B      4. B      5. D  
 6. A      7. D      8. C      9. C      10. A

Q 11

a)  $f(x) = x^4 + x^2 - 80$

$f'(x) = 4x^3 + 2x$

$x_2 = 3 - \frac{f(3)}{f'(3)}$

$= 3 - \frac{3^4 + 3^2 - 80}{4 \times 3^3 + 2 \times 3}$

$= 2.912 \dots$

(2)

b) iii) In  $\Delta P T_3$

$\tan 25^\circ = \frac{h}{PA}$

$PA = h \cot 25^\circ$  (1)

iii) In  $\Delta B P T_3$

$\tan 32^\circ = \frac{h}{PB}$

$PB = h \cot 32^\circ$  A

In  $\Delta A P P B_3$

$50^2 = (h \cot 25^\circ)^2 + (h \cot 32^\circ)^2$

$h^2 = \frac{50^2}{\cot^2 25^\circ + \cot^2 32^\circ}$

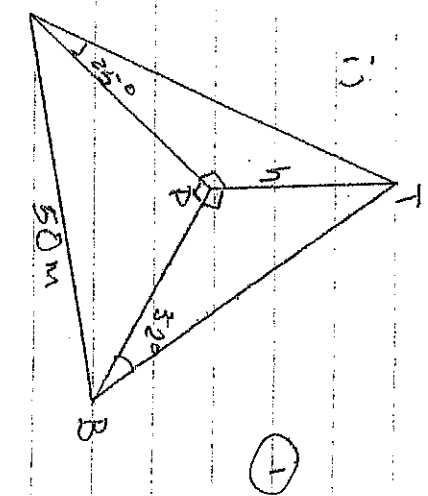
$= 349.16 \dots$

$h = 18.68 \dots$

$= 18.7 \text{ m}$

(h is a height of tower)

(3)



$$c) f(x) = \frac{3x-4}{x+2}$$

$$ii) x = \frac{3y-4}{y+2}$$

$$xy + 2x = 3y - 4$$

$$xy - 3y = -4 - 2x$$

$$3y - xy = 2x + 4$$

$$y = \frac{2x+4}{3-x}$$

$$f^{-1}(x) = \frac{2x+4}{3-x} \quad (2)$$

iii) Domain: all real  $x$ ;  $x \neq 3$ . (1)

$$d) \frac{4}{(x-1)^2} > 1, \quad x \neq 1$$

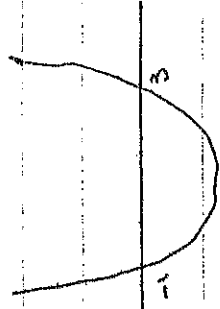
$$4 > x^2 - 2x + 1$$

$$x^2 - 2x - 3 < 0$$

$$(x-3)(x+1) < 0$$

$$-1 < x < 3$$

$\therefore -1 < x < 3$  except  $x=1$ . (3)



$$e) \int \frac{\ln x}{2x} dx, \quad u = \ln x$$

$$= \frac{1}{2} \int \ln x \cdot \frac{1}{x} dx, \quad du = \frac{1}{x} dx$$

$$= \frac{1}{2} \int u du$$

$$= \frac{1}{2} \left( \frac{1}{2} u^2 \right) + C$$

$$= \frac{1}{4} u^2 + C$$

$$= \frac{1}{4} (\ln x)^2 + C \quad (2)$$

Q 12

$$a) \left( 2x + \frac{1}{x^2} \right)^6$$

$$= \sum_{r=0}^6 {}^6C_r (2x)^r (x^{-2})^{6-r}$$

Term independent of  $x$

$$= {}^6C_4 (2x)^4 (x^{-2})^2$$

$$= {}^6C_4 \cdot 2^4$$

$$= 240$$

(2)

$$b) \text{ in RHS } = \frac{\sin 2x}{1 + \cos 2x}$$

$$= \frac{2 \sin x \cos x}{1 + 2 \cos^2 x - 1}$$

$$= \frac{2 \sin x \cos x}{2 \cos^2 x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$= \text{LHS}$$

(2)

$$\text{ii) } \tan \frac{\pi}{12}$$

$$= \frac{\sin \frac{\pi}{6}}{1 + \cos \frac{\pi}{6}}$$

$$= \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{1}{2}}{\frac{\sqrt{3} + 2}{2}}$$

$$= \frac{1}{\sqrt{3} + 2}$$

$$= 2 - \sqrt{3}$$

(2)

c) Let the statement be  $8^n - 3^n = 5P$  where  $P$  is an integer.

When  $n = 1$   $8^1 - 3^1 = 8 - 3 = 5$  which is divisible by 5.

Assume the statement is true for  $n = k$ ,

i.e.  $8^k - 3^k = 5P \Rightarrow 8^k = 5P + 3^k$

Prove that the statement is true for  $n = k + 1$ ;

i.e. Prove that  $8^{k+1} - 3^{k+1} = 5Q$  where  $Q$

is an integer.

$$\text{LHS} = 8^{k+1} - 3^{k+1}$$

$$= 8(8^k) - 3(3^k)$$

$$= 8(5P + 3^k) - 3(3^k); \text{ from *}$$

$$= 5 \cdot 8P + 8 \cdot 3^k - 3 \cdot 3^k$$

$$= 5 \cdot 8P + 5 \cdot 3^k$$

$$= 5(8P + 3^k)$$

$$= 5Q; \quad Q = 8P + 3^k \text{ is also an integer.}$$

Since the statement is true for  $n = 1$ ,

assume true for  $n = k$  and proved true for  $n = k + 1$ ,

So the statement is true for  $n = 1 + 1 = 2$ ,

$n = 2 + 1 = 3, \dots$ ,  $\therefore$  the statement is true for all positive integers of  $n$ .

3

1 for intro, \* and conclusion

1 for case  $n = 1$

1 for correct steps showing

$$8^{k+1} - 3^{k+1} = 5Q, \quad Q \text{ an integer}$$



As is Midpoint M of PQ

$$= \left( \frac{2ap + 2aq}{2}, \frac{ap^2 + aq^2}{2} \right)$$

$$= (ap + aq, \frac{ap^2 + aq^2}{2}) \quad \text{--- (1)}$$

$$\text{ii) } M_{PQ} = \frac{ap^2 - aq^2}{2ap - 2aq}$$

$$= \frac{a(p+q)(p-q)}{2a(p-q)}$$

$$= \frac{1}{2}(p+q)$$

Equation of chord PQ is

$$\frac{p+q}{2}x - \frac{y - ap^2}{x - 2ap}$$

$$\left( \frac{p+q}{2} \right) x - 2ap \left( \frac{p+q}{2} \right) = y - ap^2$$

$$\frac{(p+q)x}{2} - (ap^2 + a pq) + ap^2 = y \quad \text{--- (2)}$$

$$y = \frac{(p+q)x}{2} - apq$$

iii) IF chord PQ passes through A(a, 0),

$$\text{then } 0 = \frac{1}{2}a(p+q) - apq \quad \text{--- (1)}$$

$$\frac{1}{2}a(p+q) = apq$$

$$p+q = 2pq$$

$$\text{iv) } X_M = ap + aq$$

$$X_M = 2apq \quad \therefore p+q = 2pq$$

$$Y_M = \frac{1}{2}a(p^2 + q^2) - 2pq$$

$$= \frac{1}{2}a[(p+q)^2 - 2pq] - 2pq$$

$$2ay = x^2 - ax$$

$$x^2 - ax - 2ay = 0 \quad \text{--- (2)}$$

Q13

$$a) y = \tan^{-1} x$$
$$y' = \frac{1}{1+x^2}$$

$$m_1 = \frac{1+0^2}{1+1^2} = 1 \quad ; \quad m \text{ of tangent at } x=0 = 1$$
$$m_2 = \frac{1+1^2}{1+1^2} = \frac{1}{2} \quad ; \quad m \text{ of tangent at } x=1 = \frac{1}{2}$$

$$\tan \theta = \frac{1 - \frac{1}{2}}{1 + (1)(\frac{1}{2})}$$

$$\tan \theta = \frac{1}{3}$$

$$\text{The acute angle } \theta = 18.43^\circ \dots$$
$$= 18^\circ \quad \text{---}$$

$$b) \text{ i) } R = 60 - 50e^{-\frac{t}{15}}$$
$$\Rightarrow 50e^{-\frac{t}{15}} = 60 - R$$

$$\frac{dR}{dt} = 0 = 50 \left( -\frac{1}{15} e^{-\frac{t}{15}} \right)$$

$$= \frac{1}{15} (50e^{-\frac{t}{15}})$$

$$= \frac{1}{15} (60 - R)$$

$$= 4 - \frac{R}{15}$$

$$\text{ii) } 20 = 60 - 50e^{-\frac{t}{15}}$$

$$-40 = -50e^{-\frac{t}{15}}$$

$$\frac{4}{5} = \ln \frac{4}{5}$$

$$t = -15 \ln \frac{4}{5}$$

$$= 3.347 \dots$$

$$\approx 3.3 \text{ hours}$$

(2)

(2)

c)  $\angle BAC = \angle BDC$  (Angles at the circumference standing on the same chord BC)

$= 36^\circ$

$\angle ACB = \angle ABC$  (Angles opposite equal sides AB and AC in isosceles  $\triangle ABC$ )

$\angle ACB + \angle ABC + \angle BAC = 180^\circ$  (Angles sum of  $\triangle ABC$ )

$2\angle ACB + 36^\circ = 180^\circ$

$\angle ACB = 72^\circ$

$\angle DCA = \angle DBA$  (Angles at the circumference standing on the same arc AD)

(Angles at the circumference in a semicircle, BD is the diameter)

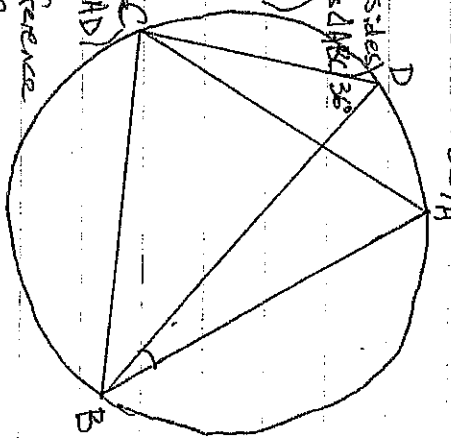
$\angle DCB = 90^\circ$

$\angle DCA + \angle ACB = \angle DCB$  (adjacent angles)

$\angle DBA + 72^\circ = 90^\circ$

$\angle DBA = 90^\circ - 72^\circ$

$= 18^\circ$



$$\text{In } \angle AOB = 60^\circ = \frac{\pi}{3}$$

$$\text{Length of arc AB} = 10 \times \frac{\pi}{3}$$

$$= \frac{10\pi}{3} \text{ cm} \quad \textcircled{1}$$

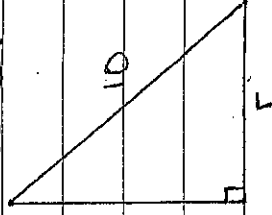
ii) Let  $r$  be the radius of the cone and  $h$  be the height of the cone.

$$2\pi r = \frac{10\pi}{3}$$

$$r = \frac{5}{3} \text{ cm}$$

$$h = \sqrt{10^2 - \left(\frac{5}{3}\right)^2}$$

$$= \frac{5\sqrt{35}}{3} \text{ cm}$$



$$\text{Volume of cone} = \frac{1}{3} \times \pi \times \left(\frac{5}{3}\right)^2 \times \frac{5\sqrt{35}}{3}$$

$$= \frac{125\sqrt{35}}{81} \pi \text{ cm}^3$$

iii) In  $\triangle BPO$ :

$$PB^2 = 5^2 + 10^2 - 2(5)(10)(\cos 60^\circ)$$

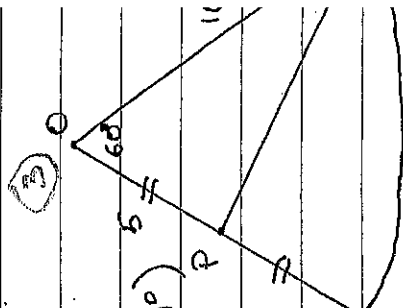
$$= 125 - 100 \times \frac{1}{2}$$

$$= 75$$

$$PB = \sqrt{75}$$

$$= 5\sqrt{3}$$

$\therefore$  Length of string,  $PB = 5\sqrt{3} \text{ cm}$



- 1 - correct set up
- 2 - correct set up and acute soln

Q 14

3 - All correct

a)  $\sin X - 3 \cos X = 3$

$\sqrt{10} \sin (X - \theta) = 3$  ;  $\tan \theta = 3 \Rightarrow \theta \approx 71.56^\circ$

$\sin (X - \theta) = \frac{3}{\sqrt{10}}$  ;  $-\theta \leq X - \theta \leq 360^\circ - \theta$

$X - \theta = 71.56^\circ \dots$  OR  $X - \theta = 180^\circ - 71.56^\circ \dots$

$X = 143.13^\circ \dots$  OR  $X = 180^\circ$

$\therefore X = 143^\circ$  OR  $180^\circ$  (nearest degree)

b)  $X = Vt \cos \theta$

$y = Vt \sin \theta - \frac{1}{2}gt^2$

i) Since PM = DM, at P  $X = -y$  ;

$Vt \cos \theta = \frac{1}{2}gt^2 - Vt \sin \theta$

$\frac{1}{2}gt^2 = Vt(\cos \theta + \sin \theta)$

$t = 0$  OR  $\frac{1}{2}gt = V(\sin \theta + \cos \theta)$

However at P,  $t \neq 0$  ;

$\frac{1}{2}gt = V(\sin \theta + \cos \theta)$  (1) r/w

$t = \frac{2V(\sin \theta + \cos \theta)}{g}$

ii) When  $t = \frac{2V(\sin \theta + \cos \theta)}{g}$  ;

$X = \frac{V \cos \theta \cdot 2V(\sin \theta + \cos \theta)}{g}$  (1)

$= \frac{V^2}{g} (2 \sin \theta \cos \theta + 2 \cos^2 \theta)$

$= -\frac{V^2}{g} (\sin 2\theta + \cos 2\theta + 1)$  ;  $\sin 2\theta = 2 \sin \theta \cos \theta$   
and  $\cos 2\theta = 2 \cos^2 \theta - 1$

$\Rightarrow 2 \cos^2 \theta = \cos$

(2)

iii) OS = r

$\Rightarrow$  When  $y = 0$  ;  $X = r$

$Vt \sin \theta - \frac{1}{2}gt^2 = 0$

$t(V \sin \theta - \frac{1}{2}gt) = 0$

$\frac{1}{2}gt = \frac{V \sin \theta}{g}$  ;  $t \neq 0$

$t = \frac{2V \sin \theta}{g}$  (1)

When  $t = \frac{2V \sin \theta}{g}$

$$X = V \cos \theta \cdot \frac{2V \sin \theta}{g} = R$$

$$\frac{2V^2 \sin \theta \cos \theta}{g} = R \quad \textcircled{1}$$

$$\frac{V^2 \sin 2\theta}{g} = R \quad *$$

$$OM = \frac{4R}{3} = \frac{V^2}{g} (\sin 2\theta + \cos 2\theta + 1), \text{ from (i)}$$

$$\frac{4V^2 \sin 2\theta}{3g} = \frac{V^2 \sin 2\theta}{g} + \frac{V^2}{g} (\cos 2\theta + 1), \text{ from (ii)}$$

$$\frac{V^2 \sin 2\theta}{3g} = \frac{V^2}{g} (\cos 2\theta + 1) \quad \textcircled{1}$$

$$\frac{\sin 2\theta}{3} = \cos 2\theta + 1 \quad \text{Tying together.}$$

$$\sin 2\theta = 3 \cos 2\theta + 3 \quad \textcircled{2}$$

$$\sin 2\theta - 3 \cos 2\theta = 3 \quad \textcircled{3}$$

$$\text{inv } 2\theta = 143.13^\circ \text{ OR } 180^\circ$$

$$\theta = 71.56^\circ \text{ OR } 90^\circ \quad \textcircled{1}$$

However  $\theta$  is the angle of elevation

and the projectile is not fired vertically,  $0^\circ < \theta < 90^\circ$

$\therefore \theta = 71.56^\circ \dots$   $\textcircled{1}$  Explanation

and exclusion

i)  $X = Vt \cos \theta \Rightarrow \dot{X} = V \cos \theta$

$y = Vt \sin \theta - \frac{1}{2}gt^2 \Rightarrow \dot{y} = V \sin \theta - gt$   $\textcircled{1}$

ii) From (a),  $\tan \theta = 3$

$$\Rightarrow \sin \theta = \frac{3}{\sqrt{10}}$$

$$\& \cos \theta = \frac{1}{\sqrt{10}} \quad \textcircled{1}$$

$$\dot{X} = \frac{V}{\sqrt{10}} \text{ and } \dot{y} = \frac{3V}{\sqrt{10}} - g \left( \frac{2V}{g} (\sin \theta + \cos \theta) \right)$$

from

$$\dot{y} = \left( \frac{3}{\sqrt{10}} - \frac{6}{\sqrt{10}} - \frac{2}{\sqrt{10}} \right) V \quad \textcircled{1}$$

$$= \frac{-5V}{\sqrt{10}}$$

$$(\text{Velocity})^2 = \dot{X}^2 + \dot{y}^2 = (kV)^2$$

$$\left( \frac{V}{\sqrt{10}} \right)^2 + \left( \frac{-5V}{\sqrt{10}} \right)^2 = k^2 V^2 \quad \textcircled{1}$$

$$\frac{26V^2}{10} = k^2 V^2$$

$$k = \sqrt{\frac{13}{5}} \quad \text{since the magnitude of } k > 0$$