STUDENT NUMBER: ____________________

TEACHER: ____________________

Founded 1982

THE HILLS GRAMMAR SCHOOL

Trial Higher School Certificate Examination 2014

MATHEMATICS EXTENSION 1

Time Allowed: Two hours (plus five minutes reading time)

Weighting: %

Outcomes: H6, H7, H8, H9, HE1, HE2, HE4, HE7, HE9

General Instructions:	Total Marks – 70
• Board-approved calculators may be used • Attempt all questions • Start all questions on a new sheet of paper • The marks for each question are indicated on the examination • Show all necessary working for Questions 11-14 • The diagrams are not drawn to scale • A table of standard integrals is provided	**Section I** Questions 1-10 **10 Marks** Allow about 15 minutes for this section **Section II** Questions 11-14 **60 Marks** Allow about 1 hour and 45 minutes for this section

MCQ	Question 11	Question 12	Question 13	Question 14	TOTAL
/10	/15	/15	/15	/15	/70

Section 1 Multiple Choice (10 Marks)

1 When $2x^3 - 3x^2 + 2a - 4$ is divided by $x - 1$ the remainder is -5. The value of a is:

(A) 2 (C) -2

(B) 0 (D) -3

2 The domain and range of $f(x) = 3\sin^{-1}\left(\dfrac{x}{2}\right)$ is given by:

(A) x is real
$-3 \le y \le 3$

(B) $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$
$-3 \le y \le 3$

(C) $-\dfrac{1}{2} \le x \le \dfrac{1}{2}$
$-\dfrac{3\pi}{2} \le y \le \dfrac{3\pi}{2}$

(D) $-2 \le x \le 2$
$-\dfrac{3\pi}{2} \le y \le \dfrac{3\pi}{2}$

3 The angle between $y = 2x + 3$ and $y = x^2$ when $x = 3$ is given by:

(A) 0^o (C) 90^o

(B) $\tan^{-1}\left(\dfrac{4}{13}\right)$ (D) $\tan^{-1}\left(-\dfrac{8}{11}\right)$

4 If the interval AB is divided externally in the ratio 3:1 by the point P, the coordinates of P given $A(-2,3)$ and $B(3,-4)$ are:

(A) $\left(\dfrac{11}{2}, -\dfrac{15}{2}\right)$

(B) $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$

(C) $\left(-\dfrac{11}{2}, \dfrac{15}{2}\right)$

(D) $\left(-\dfrac{1}{2}, -\dfrac{1}{2}\right)$

5 The equation of the tangent to the parabola $y^2 = 4ax$ at the point $\left(ap^2, 2ap\right)$ is given by:

(A) $px - y - ap^2 = 0$

(B) $x - py + ap^2 = 0$

(C) $px + y - ap^2 = 0$

(D) $x - py - ap^2 = 0$

6 The coefficient of x^5 in $\left(x^2 - \dfrac{2}{x}\right)^7$ is:

(A) ${}^7C_3(-2)^3$

(B) ${}^7C_4(-2)^4$

(C) ${}^7C_5(-2)^5$

(D) ${}^7C_4(-2)^3$

7 Evaluate $\lim\limits_{x \to 0} \dfrac{x}{\tan 2x}$:

(A) 0

(B) ∞

(C) 2

(D) 0.5

8 The derivative of $\tan^{-1}\left(\dfrac{x^3}{3}\right)$ is:

(A) $\dfrac{3x^2}{9 + x^6}$

(B) $\dfrac{x^2}{9 + x^6}$

(C) $\dfrac{3x^2}{1 + x^6}$

(D) $\dfrac{9x^2}{9 + x^6}$

9 If $t = \tan\left(\dfrac{\theta}{2}\right)$ the correct expression for $\dfrac{\sec^2\theta}{\operatorname{cosec}^2\theta}$ is:

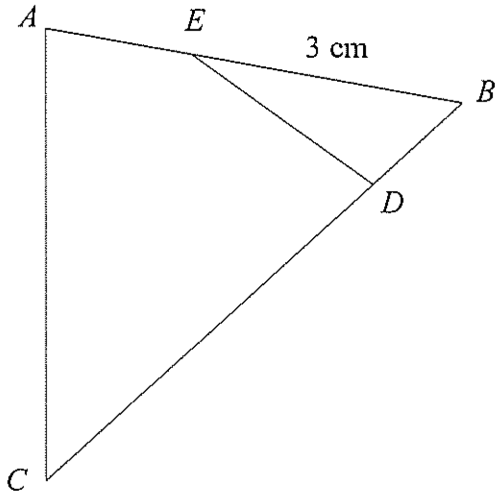
(A) $\dfrac{4t^2}{(1-t^2)^2}$

(B) $\dfrac{(1+t^2)^2}{(1-t^2)^2}$

(C) $\dfrac{(1+t^2)}{(1-t^2)^2}$

(D) $\dfrac{(1-t^2)^2}{4t^2}$

10 In the diagram below $BE = 3$ cm, $AE = BD = x$, $DC = 11x$ and $\angle BDE = \angle BAC$.



What is the value of x?

(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) 1

(D) $1\frac{1}{2}$

Section 2 **Marks**

Question 11 (15 marks)

(a) Use the substitution $u = 1 + x$ to evaluate $15\int_{-1}^{0} x\sqrt{1+x}\ dx$ **3**

(b) Let $f(x) = 3x^2 + x$. Use the definition $f'(\mathrm{x}) = \lim_{h \to 0} \frac{f(\mathrm{x}+h) - f(\mathrm{x})}{h}$ to find the derivative of $f(x)$ at the point $x = a$. **2**

(c) Find

(i) $\int \frac{e^x}{1+e^x}\ dx$ **1**

(ii) $\int_0^{\pi} \cos^2 3x\ dx$ **3**

(d) Find the term independent of x in the binomial expansion of $\left(x^2 - \frac{1}{x}\right)^9$ **3**

(e) By using the binomial expansion,

(i) show that $(q+p)^n - (q-p)^n = 2\binom{n}{1}q^{n-1}p + 2\binom{n}{3}q^{n-3}p^3 + \ldots..$ **1**

(ii) What is the last term in the expansion if n is odd? **1**

(iii) What is the last term in the expansion if n is even? **1**

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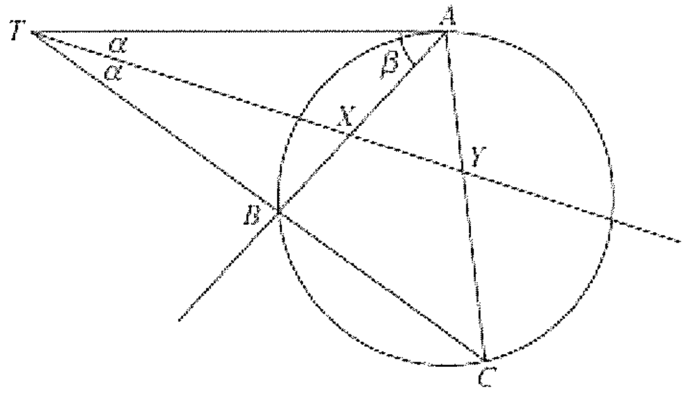
Question 12 (15 marks)

(a) In the diagram the points A, B and C lie on the circle and CB produced meets the tangent from A at the point T. The bisector of the angle ATC intersects AB and AC at X and Y respectively. Let $\angle TAB = \beta$.



NOT TO SCALE

Copy or trace the diagram into your writing booklet.

(i) Explain why $\angle ACB = \beta$ **1**

(ii) Hence prove that triangle AXY is isosceles. **2**

(b) A household iron is cooling in a room of constant temperature 22°C. At time t minutes its temperature T decreases according to the equation

$$\frac{dT}{dt} = -k(T - 22) \text{ where } k \text{ is a positive constant.}$$

The initial temperature of the iron is 80°C and it cools to 60°C after 10 minutes.

(i) Verify that $T = 22 + Ae^{-kt}$ is a solution of this equation, where A is a constant. **1**

(ii) Find the values of A and k. (give answers to 2 significant figures) **2**

(iii) How long will it take for the temperature of the iron to cool to 30°C? (Give your answer to the nearest minute.) **2**

(c) The polynomial $P(x) = x^3 - 2x^2 + kx + 24$ has roots $\alpha,\ \beta,\ \gamma$.

(i) Find the value of $\alpha + \beta + \gamma$. **1**

(ii) Find the value of $\alpha\beta\gamma$. **1**

(iii) It is known that two of the roots are equal in magnitude but opposite in sign. Find the third root and hence find the value of k. **2**

(d) Use the principle of mathematical induction to show that

$2 \times 1! + 5 \times 2! + 10 \times 3! + \ldots\ldots.. + (n^2 + 1)n! = n(n+1)!$ for all positive integers n. **3**

START A NEW PAGE

Question 13 (15 marks)

(a) If $f(x) = \ln(x+3)$

(i) find $f^{-1}(x)$. **1**

(ii) Sketch $y = x$, $f(x)$ and $f^{-1}(x)$ on the same axes. **2**

(b) A particle moves in a straight line and its position at time t is given by

$$x = 4\sin\left(2t + \frac{\pi}{3}\right)$$

(i) Show that the particle is undergoing simple harmonic motion. **2**

(ii) Find the amplitude of the motion. **1**

(iii) When does the particle first reach maximum speed after time $t = 0$? **1**

(c) The acceleration of a particle P is given by the equation

$$\frac{d^2x}{dt^2} = 8x\left(x^2 + 4\right)$$

where x metres is the displacement of P from a fixed point O after t seconds. Initially the particle is at O and has velocity $8\ \text{ms}^{-1}$ in the positive direction.

(i) Show that the speed at any position x is given by $2(x^2 + 4)\ \text{ms}^{-1}$. **2**

(ii) Hence find the time taken for the particle to travel 2 metres from O. **2**

(d) A particle is projected from the origin with velocity v ms^{-1} at an angle α to the horizontal. The position of the particle at time t seconds is given by the parametric equations

$$x = vt\cos\alpha$$

$$y = vt\sin\alpha - \frac{1}{2}gt^2 \qquad \text{(Do not prove these equations.)}$$

(i) Show that the maximum height reached, h metres, is given by

$$h = \frac{v^2 \sin^{2}\cancel{2}\alpha}{2g}$$ **2**

(ii) Show that it returns to the initial height at $x = \frac{v^2}{g}\sin 2\alpha$ **2**

START A NEW PAGE

Question 14 (15 marks)

(a) (i) Write $8\cos x + 6\sin x$ in the form $A\cos(x - \alpha)$ where $A > 0$ and $0 \le \alpha \le \frac{\pi}{2}$, **2**

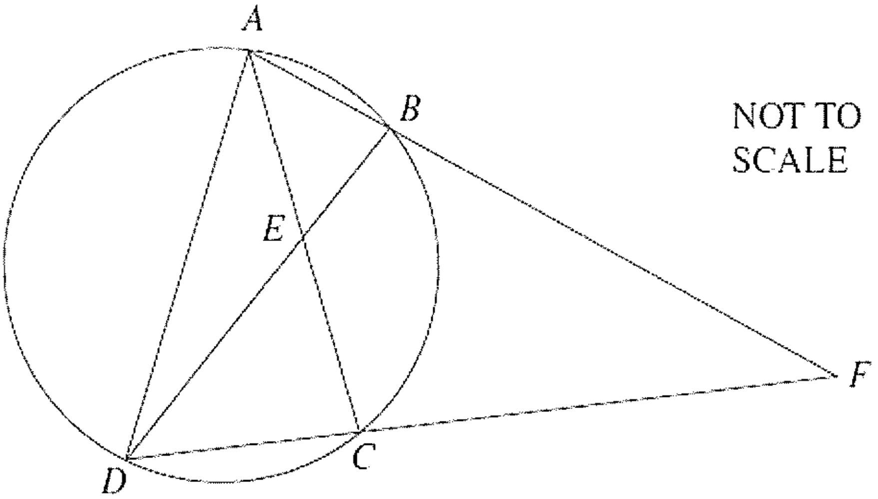
(ii) Hence, or otherwise, solve the equation $8\cos x + 6\sin x = 5$ for $0 \le \alpha \le 2\pi$. **2**
Give your answers correct to three decimal places.

(b) The two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are on the parabola $x^2 = 4ay$.

(i) The equation of the tangent to $x^2 = 4ay$ $(2at, at^2)$ at P is $y = px - ap^2$. (Do not prove this.)
Show that the tangents at the points P and Q meet at R, where R is the point $\left[a(p+q), apq\right]$. **2**

(ii) As P varies, the point Q is always chosen so that $\angle POQ$ is a right angle, where O is the origin. Using this condition and the result of part (i) find the locus of R. **2**

(c)



The points A, B, C and D are placed on a circle of radius r such that AC and BD meet at E. The lines AB and DC are produced to meet at F, and $BECF$ is a cyclic quadrilateral.
Copy or trace this diagram into your writing booklet.

(i) Find the size of $\angle DBF$, giving reasons for your answer. **2**

(ii) Explain why AD equals $2r$. **1**

(d)

(i) Show that for all positive integers n,

$x[(1+x)^{n-1} + (1+x)^{n-2} + \dots\dots + (1+x)^2 + (1+x) + 1] = (1+x)^n - 1$ **2**

(ii) Hence explain why

$$\binom{n-1}{k-1}+\binom{n-2}{k-1}+\binom{n-3}{k-1}+\dots\dots+\binom{k-1}{k-1}=\binom{n}{k} \qquad \text{for } 1 \le k \le n$$ **1**

(iii) Show that $n\binom{n-1}{k}=(k+1)\binom{n}{k+1}$ **1**

END OF ASSESSMENT

Trial Exam 2014 Draft 1 Ext 1

Q 11, 13 MCQ

1, $P(x) = 2x^3 - 3x^2 + 2a - 4$

$P(1) = 2 - 3 + 2a - 4$

$2a - 5 = -5$

B B

2, $f(x) = 3\sin^{-1}\frac{x}{2}$

domain $-1 \le \frac{x}{2} \le 1$

$-2 \le x \le 2$

range $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$

D

3, $y = 2x + 3 \Rightarrow m_1 = 2$

$y = x^2$

$\frac{dy}{dx} = 2x$ at $x = 3$ $m_2 = 6$.

$\tan\theta = \frac{6-2}{1+6\times 2} = \frac{4}{13}$

B

4, $A(-2, 3) \longleftrightarrow B(3, -4)$

$3 : -1$

$P = \left(\frac{2+9}{2}, \frac{-3-12}{2}\right) = \left(\frac{11}{2}, -\frac{15}{2}\right)$

A

5, $y^2 = 4ax$

$2y\frac{dy}{dx} = 4a$

$\frac{dy}{dx} = \frac{2a}{y}$ when $y = 2ap$.

$\frac{dy}{dx} = \frac{1}{p}$

equa of tangent $\frac{y - 2ap}{x - ap^2} = \frac{1}{p}$

$py - 2ap^2 = x - ap^2$

$x - py + ap^2 = 0$

B

6/ $\left(x^2 - \frac{2}{x}\right)^7$ $T_{r+1} = \binom{7}{r} (x^2)^{7-r} \left(\frac{-2}{x}\right)^r$

$= \binom{7}{r} x^{14-2r} \frac{(-2)^r}{x^r} = \binom{7}{r}(-2)^r x^{14-3r}$

for x^5 term $14 - 3r = 5$

$-3r = -9 \Rightarrow r = 3$

$T_4 = \binom{7}{3}(-2)^3$

(A)

7/ $\lim_{x \to 0} \frac{x}{\tan 2x} = \frac{1}{2} \lim_{2x \to 0} \frac{2x}{\tan 2x} = \frac{1}{2}$

(D)

8/ $y = \tan^{-1} \frac{x^3}{3}$

$\frac{dy}{dx} = \frac{x^2}{1 + \frac{x^6}{9}} = \frac{9x^2}{9 + x^6}$

(D)

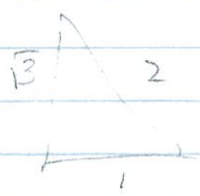
9/ $t = \tan\left(\frac{\theta}{2}\right)$ $\frac{\sec^2\theta}{\csc^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta = \left(\frac{2t}{1-t^2}\right)^2$

(A)

10/ $2\sin^2 2x - \sin 2x = 0$

$\sin 2x\,(2\sin 2x - 1) = 0$

$\sin 2x = 0$ or $\sin 2x = \frac{1}{2}$



$\downarrow$ $\downarrow$

$2x = \pm k\pi$ $2x = \frac{\pi}{6} + 2k\pi$ or $\frac{-\pi}{6} + (2k+1)\pi$

$x = \pm \frac{k\pi}{2}$ $x = \frac{\pi}{12} + k\pi$ $\frac{-\pi}{12} + \frac{2k+1}{2}\pi$

(C)

Quest 11 | Comments

(a) $15\int_{-1}^{0} x\sqrt{1+x}\, dx$

let $u = 1 + x$ when $x=-1, u=0$

$\frac{du}{dx} = 1$ $\quad x=0, u=1$ ① mark

$I = 15\int_0^1 (u-1)\, u^{\frac{1}{2}}\, du$ ① mark

$= 15\int_0^1 u^{\frac{3}{2}} - u^{\frac{1}{2}} du$

$= 15\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_0^1$

$= 15\left(\frac{2}{5} - \frac{2}{3}\right)$

$= 15\left(\frac{6-10}{15}\right) = -4$ ① mark.

• Some students failed to convert surd form to indice form.

(b) $f(x) = 3x^2 + x$

$f(a) = 3a^2 + a$

$f(a+h) = 3(a^2 + 2ah + h^2) + a + h.$ ① mark

$\frac{f(a+h) - f(a)}{h} = \frac{6ah + 3h^2 + h}{h}$

$= 6a + 3h + 1$ ① mark.

$f'(a) = \lim_{h \to 0}(6a + 3h + 1) = 6a + 1$

• Many from 2nd class omitted $\lim_{h\to 0}$, docked 1 mark.

(c)(i) $\int \frac{e^x}{1+e^x} dx = \ln(1 + e^x) + c$ ① mark

(ii) $\int_0^{\pi} \cos^2 3x\, dx = \frac{1}{2}\int_0^{\pi} \cos 6x + 1\, dx$ ① mark

$= \frac{1}{2}\left[\frac{1}{6}\sin 6x + x\right]_0^{\pi}$ ① mark

$= \frac{\pi}{2}$ ① mark.

• Many students mixed terminals
• could not rewrite $\cos^2 3x$ in terms of $\cos 6x$.

Comments

(d) $\left(x^2 - \frac{1}{x}\right)^9$ ① mark

$$T_{r+1} = \binom{9}{r}(x^2)^{9-r}\left(\frac{-1}{x}\right)^r$$

$$= \binom{9}{r}(-1)^r x^{18-2r-r}$$

for const. term $18 - 3r = 0$

$r = 6.$ — ① mark

$$T_7 = \binom{9}{6}(-1)^6 x^{18-18} = \binom{9}{6}$$

$= 84$ ① mark.

· If $(-1)^6$ not shown, 1 mark docked.

(e) show $(q+p)^n - (q-p)^n = 2\binom{n}{1}q^{n-1}p + 2\binom{n}{3}q^{n-3}p^3$

answered poorly by 2nd class.

$$\text{LHS} = q^n + \binom{n}{1}q^{n-1}p + \binom{n}{2}q^{n-2}p^2 + \binom{n}{3}q^{n-3}p^3$$

$$- \left(q^n - \binom{n}{1}q^{n-1} + \binom{n}{2}q^{n-2}p^2 - \binom{n}{3}q^{n-3}p^3\right)$$ ① mark

$$= 2\binom{n}{1}q^{n-1}p + 2\binom{n}{3}q^{n-3}p^3 + 2\binom{n}{5}q^{n-5}p^5 + \dots$$

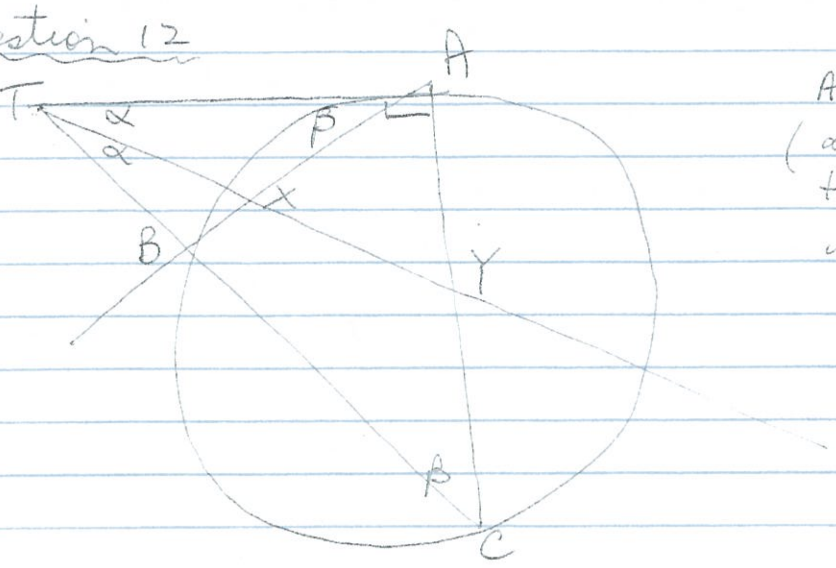
If n is odd last term $2p^n$ ① mark

If n is even last term $2\binom{n}{n-1}q\,p^{n-1}$ ① mark

many swapped ans for odd + even.

Question 12

(a)



$A\hat{C}B = \beta$
(angle between chord & tangent equals angle in alternate segment)
① mark.

Comments
Learn words!

Students made part (ii) too complicated.

Prove $\triangle AXY$ is isosceles

using $\triangle TAX$ $\quad \angle AXY = \alpha + \beta$ (ext. ∠ of △) ① mark

~~$\angle BAY = 90 - \beta$ ...~~

using $\triangle TCY$ $\quad \angle AYX = \alpha + \beta$ (ext ∠ of △). ① mark

$\therefore \triangle AXY$ is isosceles

(b) $\dfrac{dT}{dt} = -k(T-22)$ $\quad$ when $t=0$, $T=80$
when $t=10$, $T=60$

(i) $T = 22 + Ae^{-kt}$

$\dfrac{dT}{dt} = -kAe^{-kt}$

$= -k(T-22)$ ① mark.

(ii) when $t=0$

$80 = 22 + Ae^{0} \Rightarrow A = 58$ ① mark

when $t = 10$, $T = 60$

$60 = 22 + 58e^{-k\times 10}$

$38 = 58e^{-10k}$

$-10k = \ln\dfrac{38}{58}$

$k = \ln\dfrac{38}{58} \div -10$ ① mark

$\doteqdot 0.0423 \Rightarrow$ memory

Some students got the answer wrong because they did not use the memory key.

(iii) for $T = 30°$

$30 = 22 + 58e^{-kt}$

$-kt = \ln\left(\dfrac{8}{58}\right)$ ① mark

$t = \ln\left(\dfrac{8}{58}\right) \div -k$

$= 46.85$

$\doteqdot 47$ mins ① mark

Comments

(c) $P(x) = x^3 - 2x^2 + kx + 24$
has roots α, β, γ

(i) $\alpha + \beta + \gamma = 2$ ① mark

(ii) $\alpha\beta\gamma = -24$ ① mark

(iii) Let roots be $\alpha, -\alpha, \beta$
$\beta = 2$
$-\alpha^2\beta = -24$
$\alpha^2 = 12$
$\alpha = \pm 2\sqrt{3}$ ① mark

$k = \alpha\beta - \alpha\beta + \alpha \times -\alpha.$
$= -12$ ① mark.

(d) Show $2\times1! + 5\times2! + 10\times3! + \cdots + (n^2+1)n! = n(n+1)!$

Test $n=1$ $LHS = 2\times1! = 2$
$RHS = 1(1+1) = 2$
$\therefore$ True for $n=1$ ① mark
Assume true for $n=k$
$2\times1! + 5\times2! + \cdots + (k^2+1)k! = k(k+1)!$ $*S_k$
① mark
Test $n=k+1$
Is $2\times1! + 5\times2! + \cdots + (k^2+1)k! + [(k+1)^2+1](k+1)! = (k+1)(k+2)!$
$*S_{k+1}$

$LHS = k(k+1)! + [(k+1)^2+1](k+1)!$

Need to work with LHS only.

$= (k+1)!\,[k + k^2 + 2k + 1 + 1]$

$= (k+1)!\,[k^2 + 3k + 2]$

$= (k+1)!\,(k+2)(k+1)$ ① mark
$= (k+1)(k+2)! = RHS$
$\therefore$ If $n=k$ true then $n=k+1$ is true
It is true for $n=1$
$\therefore$ True for all $n \in \mathbb{Z}^+$

Comments

Question 13

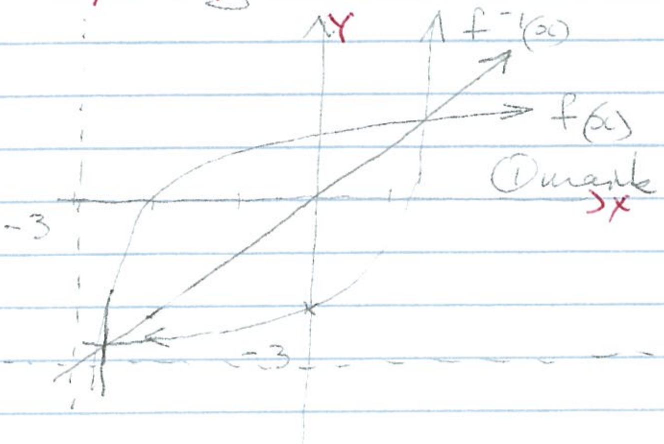
(a) $f(x) = \ln(x+3)$

$f^{-1}(x)$ is $x = \ln(y+3)$

$e^{x} = y + 3$

$f^{-1}(x)$ ~~y~~ $= e^{x} - 3$ ① mark

• if not written as $f^{-1}(x)$, no mark awarded.



① mark

• many did not sketch properly - labels of graphs axes omitted

(b)(i) $x = 4\sin\left(2t + \frac{\pi}{3}\right)$

$\dot{x} = 8\cos\left(2t + \frac{\pi}{3}\right)$

$\ddot{x} = -16\sin\left(2t + \frac{\pi}{3}\right)$ ① mark

① mark

$\ddot{x} = -4x$ This is SHM

• students failed to write -16 as $-2^2 x$

(ii) amplitude $= 4$ ① mark ✓

(iii) for max speed

$\cos\left(2t + \frac{\pi}{3}\right) = 1$ or $\sin\left(2t + \frac{\pi}{3}\right) = 0$

• ans poorly.

$2t + \frac{\pi}{3} = 0, \pi$ etc

$2t = \frac{2\pi}{3}$

$t = \frac{\pi}{3}$ secs ① mark.

Comments

(c) $\frac{d^2x}{dt^2} = 8x(x^2+4)$ when $t=0$, $x=0$, $v=8$

$= 8x^3+32x$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 8x^3+32x$

$\frac{1}{2}v^2 = 2x^4+16x^2+c$ ① mark

$v^2 = 4x^4+32x^2+2c$

when $x=0$, $v=8$ $\Rightarrow$ $2c=64$

$v^2 = 4x^4+32x^2+64$

$v^2 = 4(x^4+8x^2+16)$

$v = \pm 2(x^2+4)\ ms^{-1}$

when particle commences it has $v>0$ and $\dot{x}>0$

$\therefore v = 2(x^2+4)\ ms^{-1}$ ① mark.

- poorly ans.
- students failed to calc c
- lots of fudging in c + d.

(ii) $\frac{dx}{dt} = 2(x^2+4)$

$\frac{dt}{dx} = \frac{1}{2(x^2+4)}$ ① mark

$t = \frac{1}{2}\times\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^2$

$t = \frac{1}{4}\tan^{-1}1$

$t = \frac{\pi}{16}$ secs. ① mark

(d) $x = vt\cos\alpha$

$y = vt\sin\alpha - \frac{1}{2}gt^2$

- poorly answered

(i) $\dot{y} = v\sin\alpha - gt$

for max height $\dot{y}=0$

$gt = v\sin\alpha$

$t = \frac{v\sin\alpha}{g}$ ① mark

$h = v\sin\alpha\frac{v\sin\alpha}{g} - \frac{1}{2}g\frac{v^2\sin^2\alpha}{g^2}$

$= \frac{v^2\sin^2\alpha}{g} - \frac{1}{2}\frac{v^2\sin^2\alpha}{g}$

$= \frac{v^2\sin^2\alpha}{2g}$

- students did not get to $\dot{y}=0$ for max h.

- 1 Marks awarded for $t=\frac{v\sin\alpha}{g}$
- 2 Marks given for attempt to subst in y.

Comments

(ii) $x = vt\cos\alpha$ ①, $y = vt\sin\alpha - \frac{gt^2}{2}$ ②

$t = \frac{x}{v\cos\alpha}$ ③

sub ③ in ②

$y = v\sin\alpha \frac{x}{v\cos\alpha} - \frac{g}{2}\frac{x^2}{v^2\cos^2\alpha}$

$y = x\tan\alpha - \frac{g}{2}\frac{x^2}{v^2\cos^2\alpha}$

for $y = 0$

$x\tan\alpha = \frac{g}{2}\frac{x^2}{v^2\cos^2\alpha}$ ① mark

$x = 0$ at start for $x \neq 0$

$\frac{gx}{2v^2\cos^2\alpha} = \tan\alpha$

$x = \frac{2v^2\cos^2\alpha}{g} \cdot \frac{\sin\alpha}{\cos\alpha}$

$x = \frac{v^2\sin 2\alpha}{g}$ ① mark.

. poorly answered
. lots of fudging.

Question 14

a) i) let $8\cos x + 6\sin x = A\cos(x-\alpha)$

$= A\cos x\cos\alpha + A\sin x\sin\alpha$

$A\cos\alpha = 8$
$A\sin\alpha = 6$ } α in 1st quad

$\tan\alpha = \frac{6}{8} = \frac{3}{4}$

$\alpha = \tan^{-1}\left(\frac{3}{4}\right)$ ① mark.

$A^2 = 6^2 + 8^2$

$A^2 = 100$

$A = \pm 10$ ① mark.

$8\cos x + 6\sin x = 10\cos(x-\alpha)$

ii) $8\cos x + 6\sin x = 5$

$10\cos(x-\alpha) = 5$

$\cos(x-\alpha) = \frac{1}{2}$ ① mark

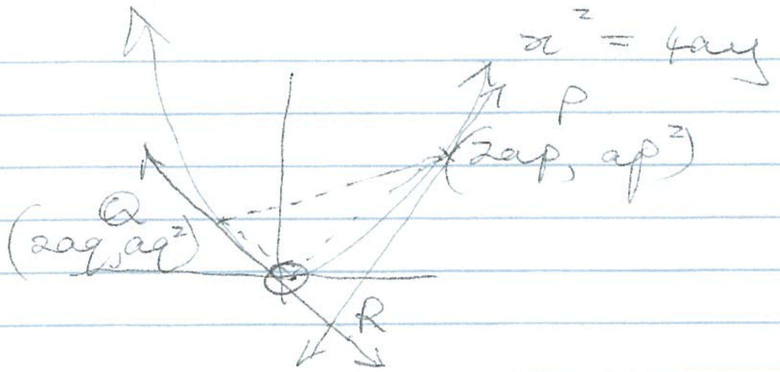
$x - \alpha = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

① mark

$x = \frac{\pi}{3} + \tan^{-1}\left(\frac{3}{4}\right), \frac{5\pi}{3} - \tan^{-1}\left(\frac{3}{4}\right)$

Comments

(b)



tangent thru P is $y = px - ap^2$ ①
" thru Q is $y = qx - aq^2$ ②

equate ① & ②

$$px - ap^2 = qx - aq^2$$
$$(p-q)x = ap^2 - aq^2$$
$$x = a(p+q) \quad \text{① mark}$$

sub in ① $y = ap(p+q) - ap^2$
$= apq$ ① mark

$\therefore R = (a(p+q), apq)$

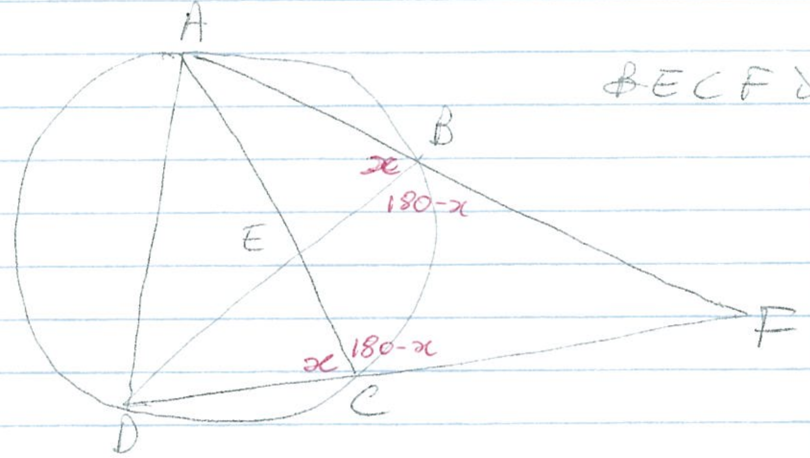
(ii) ~~for~~ $\angle POQ = 90°$ $\cancel{pq = -1}$ grad of $OP = \frac{ap^2}{2ap} = \frac{p}{2}$

$\therefore \frac{pq}{4} = -1$ gradients ① mark $pq = -4$

$\therefore x = a(p+q)$
$y = -4a$ ← locus
$\cancel{x = y(p+q)}$

$\therefore$ locus is line $y = -4a$ ① mark
ie directrix

4)



BECF is cyclic quad

(i) let $\angle ABD = x$ ① mark
then $\angle ACD = x$ (angles on same arc AD)
$\therefore \angle EBF = \angle ECF$ (supp. ∠'s on line)
$\angle EBF + \angle ECF = 180°$ (opp ∠'s in cyclic quad)
$180 - x + 180 - x = 180 \therefore x = 90°$ ① mark

Comments

ii) If $\angle ABD = 90^\circ$
then AD is a diameter (ΔADB is right $\angle$ angled)
hence $AD = 2r$ (1) mark.

(d) Show $x\left[(1+x)^{n-1} + (1+x)^{n-2} + \dots + (1+x)^2 + (1+x) + 1\right] = (1+x)^n - 1$

(i) LHS = $x \times$ GP with $a = 1$, $r = (1+x)$, n terms.
(1) mark

$= x\left(\dfrac{a\left((1+x)^n - 1\right)}{(1+x) - 1}\right)$

$= (1+x)^n - 1$

= RHS (1) mark

Some students used Induction.

(ii) x^k term on RHS $= \binom{n}{k}$

x^k term on LHS $= \binom{n-1}{k-1} + \binom{n-2}{k-1} + \dots + \binom{k-1}{k-1}$

(iii) Show that $n\binom{n-1}{k} = (k+1)\binom{n}{k+1}$.

LHS $= \dfrac{n\,(n-1)!}{k!\,(n-1-k)!}$

$= \dfrac{n!}{k!\,(n-1-k)!}$

$= \dfrac{(k+1)\,n!}{(k+1)\,k!\,(n-(k+1))!}$

$= \dfrac{(k+1)\,n!}{(k+1)!\,(n-(k+1))!}$

$= (k+1)\binom{n}{k+1}$