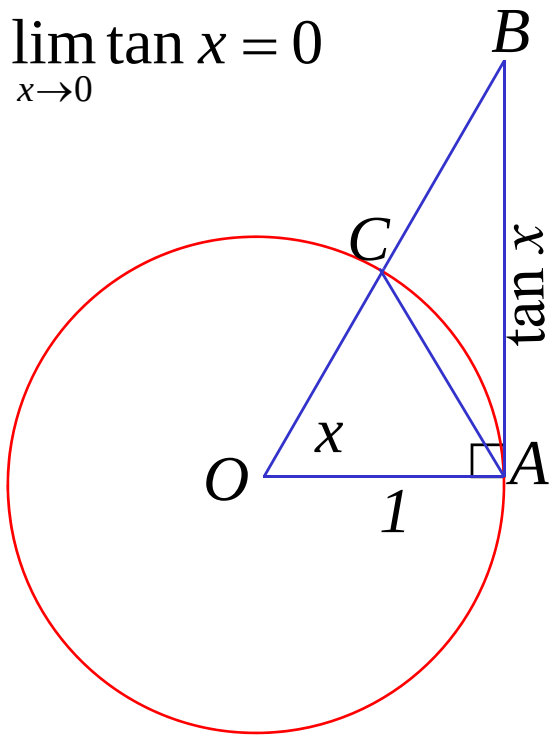
Differentiating Trig

$\lim_{x\to 0} \sin x = 0$

$\lim_{x\to 0} \cos x = 1$

$\lim_{x\to 0} \tan x = 0$



Area $\Delta AOC <$ Area Sector $OAC <$ Area ΔAOB

$$\frac{1}{2}(1)(1)\sin x < \frac{1}{2}(1)^2 x < \frac{1}{2}(1)\tan x$$

$$\sin x < x < \tan x$$

$$\frac{\sin x}{\sin x} < \frac{x}{\sin x} < \frac{\tan x}{\sin x}$$

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$$

as $x \to 0$

$$1 < \frac{x}{\sin x} < 1$$

$$\boxed{\lim_{x\to 0} \frac{x}{\sin x} = 1}$$

e.g. $(i)\lim_{x\to 0}\dfrac{5x}{\sin 5x} = 1$

$(ii)\lim_{x\to 0}\dfrac{x}{\sin 3x} = \lim_{x\to 0}\dfrac{1}{3}\times\dfrac{3x}{\sin 3x}$

$= \dfrac{1}{3}$

$y = \sin x$

$$\frac{dy}{dx} = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

$$= \lim_{h\to 0}\frac{\sin x\cosh + \cos x\sinh - \sin x}{h}$$

$$= \lim_{h\to 0}\cos x\left(\frac{\sinh}{h}\right) + \sin x\left(\frac{\cosh - 1}{h}\right)$$

$$= \lim_{h\to 0}\cos x\left(\frac{\sinh}{h}\right) + \sin x\left(\frac{2\cos^2\frac{h}{2} - 2}{h}\right) \quad \left[\cos 2\theta = 2\cos^2\theta - 1\right]$$

$$= \lim_{h\to 0}\cos x\left(\frac{\sinh}{h}\right) + \sin x\left(\frac{2\sin^2\frac{h}{2}}{h}\right)$$

$$= \lim_{h\to 0} \cos x \left(\frac{\sinh}{h} \right) + \sin x \left(\frac{\sin^2 \frac{h}{2}}{\frac{h}{2}} \right)$$

$$= \lim_{h\to 0} \cos x \left(\frac{\sinh}{h} \right) + \sin x \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \times \sin \frac{h}{2} \right)$$

$$= (\cos x)(1) + (\sin x)(0)$$

$$= \cos x$$

$$y = \cos x$$

$$y = \sin\left(\frac{\pi}{2} - x \right)$$

$$\frac{dy}{dx} = -\cos\left(\frac{\pi}{2} - x \right)$$

$$= -\sin x$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x)\cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x)\sin f(x)$$

$$y = \tan x$$

$$y = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x)\sec^2 f(x)$$

e.g. $(i)\ y = \sin x^3$

$$\frac{dy}{dx} = 3x^2 \cos x^3$$

$(ii)\ y = \tan \frac{1}{x}$

$$\frac{dy}{dx} = -\frac{1}{x^2}\sec^2 \frac{1}{x}$$

$(iii)\ y = \log \cos x$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x}$$

$$= -\tan x$$

$(iv)\ y = \tan^5 x$

$$\frac{dy}{dx} = 5\tan^4 x \sec^2 x$$

$(v)\ y = \cos e^x$

$$\frac{dy}{dx} = -e^x \sin e^x$$

Exercise 14F; 2, 4, 5

Exercise 14G; 2ace etc, 3ace etc, 5ace etc, 6, 7ab(i), 8, 12, 13c, 16a*

Exercise 14H; 2 ac, 4bd, 6, 8, 14, 17, 20