Applications of Calculus To The Physical World

Displacement (x)

Distance from a point, with direction.

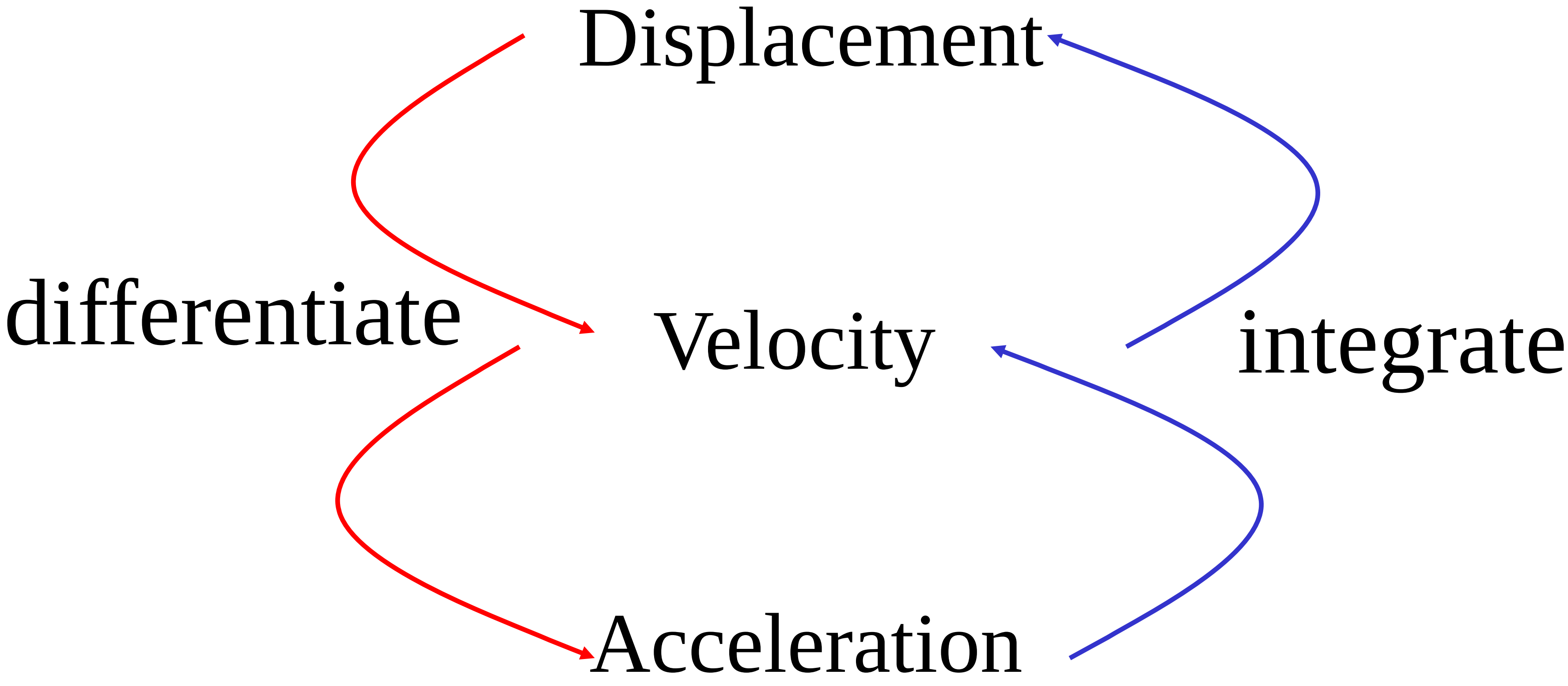
Velocity $\left(v, \frac{dx}{dt}, \dot{x} \right)$

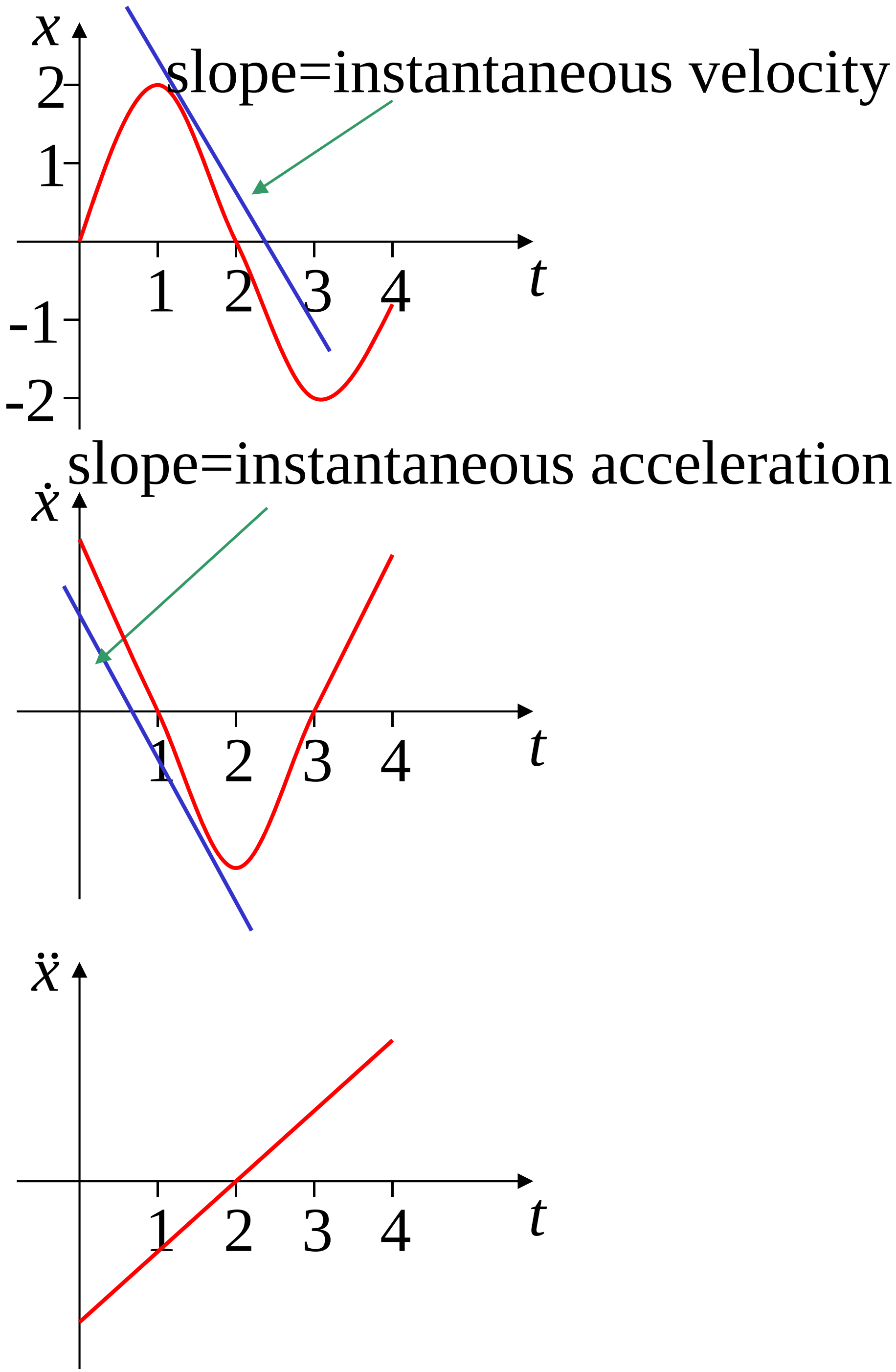
The rate of change of displacement with respect to time i.e. speed with direction.

Acceleration $\left(a, \frac{dv}{dt}, \frac{d^2x}{dt^2}, \ddot{x}, \dot{v} \right)$

The rate of change of velocity with respect to time

NOTE: "deceleration" or slowing down is when acceleration is in the opposite direction to velocity.


Displacement
differentiate
Velocity
integrate
Acceleration



e.g. (i) distance traveled $= 7\text{m}$

(ii) total displacement $= -1\text{m}$

(iii) average speed $= \dfrac{7}{4}\ \text{m/s}$

(iv) average velocity $= \dfrac{-1}{4}\ \text{m/s}$

e.g. (*i*) The displacement x from the origin at time t seconds, of a particle traveling in a straight line is given by the formula $x = t^3 - 21t^2$

a) Find the acceleration of the particle at time t.

$$x = t^3 - 21t^2$$

$$v = 3t^2 - 42t$$

$$a = 6t - 42$$

b) Find the times when the particle is stationary.

Particle is stationary when $v = 0$

$$\text{i.e. } 3t^2 - 42t = 0$$

$$3t(t - 14) = 0$$

$$t = 0 \quad \text{or} \quad t = 14$$

Particle is stationary initially and again after 14 seconds

(ii) A particle is moving on the x axis. It started from rest at $t = 0$ from the point $x = 7$.
If its acceleration at time t is $2 + 6t$ find the position of the particle when $t = 3$.

$$a = 2 + 6t$$

$$v = 2t + 3t^2 + c$$

$$\text{when } t = 0, v = 0$$

$$\text{i.e. } 0 = 0 + 0 + c$$

$$c = 0$$

$$\therefore v = 2t + 3t^2$$

$$x = t^2 + t^3 + c$$

$$\text{when } t = 0, x = 7$$

$$\text{i.e. } 7 = 0 + 0 + c$$

$$c = 7$$

$$\therefore x = t^2 + t^3 + 7$$

$$\text{when } t = 3, x = 3^2 + 3^3 + 7$$

$$= 43$$

after 3 seconds the particle is 43 units to the right of O.

OR

$$\frac{dv}{dt} = 2 + 6t$$

$$\int_0^v dv = \int_0^t (2 + 6t)\,dt$$

$$v = \left[2t + 3t^2\right]_0^t$$

$$v = 2t + 3t^2$$

$$\frac{dx}{dt} = 2t + 3t^2$$

$$\int_7^x dx = \int_0^3 (2t + 3t^2)\,dt$$

$$[x]_7^x = \left[t^2 + t^3\right]_0^3$$

$$x - 7 = 3^2 + 3^3 - 0$$

$$x = 43$$

e.g. **2001 HSC Question 7c)**

A particle moves in a straight line so that its displacement, in metres, is given by $x = \frac{t-2}{t+2}$

where t is measured in seconds.

(i) What is the displacement when $t = 0$?

$$\text{when } t = 0, x = \frac{0-2}{0+2}$$
$$= -1$$

$\therefore$ the particle is 1 metre to the left of the origin

(ii) Show that $x = 1 - \frac{4}{t+2}$

Hence find expressions for the velocity and the acceleration in terms of t.

$$1 - \frac{4}{t+2} = \frac{t+2-4}{t+2}$$
$$= \frac{t-2}{t+2}$$
$$\therefore x = 1 - \frac{4}{t+2}$$

$$v = -\frac{4(-1)}{(t+2)^2}$$
$$v = \frac{4}{(t+2)^2}$$

$$a = \frac{4 \times -2(t+2)^1(1)}{(t+2)^4}$$
$$a = \frac{-8}{(t+2)^3}$$

(iii) Is the particle ever at rest? Give reasons for your answer.

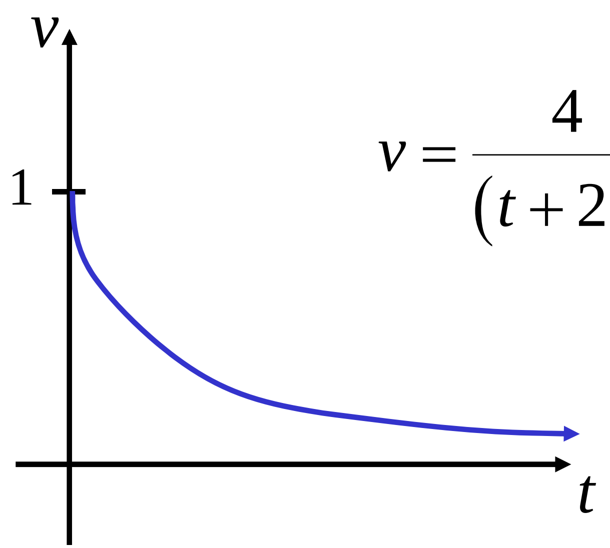
$$v = \frac{4}{(t+2)^2} \neq 0$$

$\therefore$ the particle is never at rest

(iv) What is the limiting velocity of the particle as *t* increases indefinitely?

$$\lim_{t\to\infty} v = \lim_{t\to\infty} \frac{4}{(t+2)^2}$$
$$= 0$$

OR



$$v = \frac{4}{(t+2)^2}$$

$\therefore$ the limiting velocity of the particle is 0 m/s

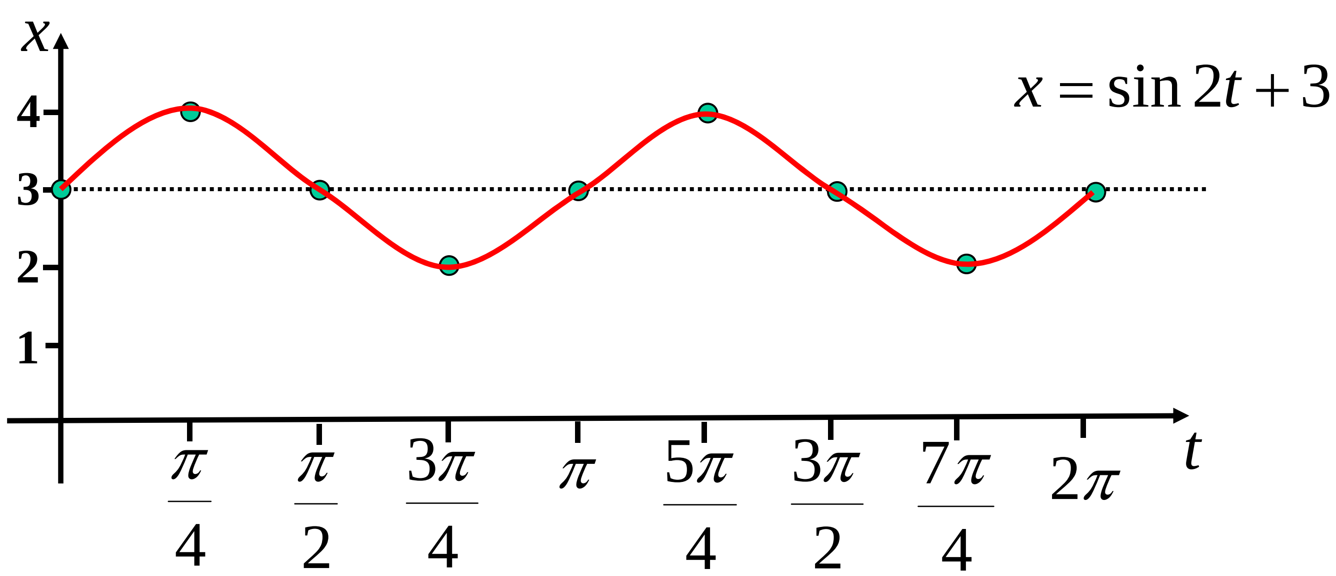
(ii) **2002 HSC Question 8b)**

A particle moves in a straight line. At time t seconds, its distance x metres from a fixed point O in the line is given by $x = \sin 2t + 3$

(i) Sketch the graph of x as a function of t for $0 \leq t \leq 2\pi$

amplitude $= 1$ unit period $= \frac{2\pi}{2}$ divisions $= \frac{\pi}{4}$

shift $= \uparrow 3$ units $= \pi$



(ii) Using your graph, or otherwise, find the times when the particle is at rest, and the position of the particle at those times.

Particle is at rest when velocity = 0

$$\frac{dx}{dt} = 0 \quad \text{i.e. the stationary points}$$

when $t = \dfrac{\pi}{4}$ seconds, $x = 4$ metres

$t = \dfrac{3\pi}{4}$ seconds, $x = 2$ metres

$t = \dfrac{5\pi}{4}$ seconds, $x = 4$ metres

$t = \dfrac{7\pi}{4}$ seconds, $x = 2$ metres

(iii) Describe the motion completely.

The particle oscillates between x=2 and x=4 with a period of π seconds

Exercise 3B; 2, 4, 6, 8, 10, 12