5, r)
$$\frac{2x}{\sqrt{x}\left(\sqrt{x+2}+\sqrt{x}\right)}$$

$$= \frac{2\sqrt{x}}{\sqrt{x+2}+\sqrt{x}} \times \frac{\sqrt{x+2}-\sqrt{x}}{\sqrt{x+2}-\sqrt{x}}$$

$$= \frac{2\sqrt{x^2+2x}-2x}{x+2-x}$$

$$= \sqrt{x^2+2x}-x$$

6a) $$\frac{3}{2+\sqrt{2}} + \frac{3}{\sqrt{2}}$$

$$= \frac{3\sqrt{2} + 6 + 3\sqrt{2}}{2\sqrt{2} + 2}$$

$$= \frac{6\sqrt{2} + 6}{2\sqrt{2} + 2}$$

$$\frac{6(\sqrt{2}+1)}{2(\sqrt{2}+1)}$$

$$= \underline{\underline{3}}$$

$$= \frac{3\sqrt{2} + 3}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1}$$

$$= \frac{6 - 3\sqrt{2} + 3\sqrt{2} - 3}{2 - 1}$$

$$= \underline{\underline{3}}$$

9b) $a = 2 - \sqrt{3}$

$$a + \frac{1}{a} = 2 - \sqrt{3} + \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= 2 - \sqrt{3} + \frac{2 + \sqrt{3}}{4 - 3}$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3}$$

$$= \underline{\underline{4}}$$

10c)

a) $\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

b) $y = 2+\sqrt{5}$

$y + \frac{1}{y} = 2\sqrt{5}$

$y^2 + \frac{1}{y^2} = \left(y+\frac{1}{y}\right)^2 - 2$

$= \left(2\sqrt{5}\right)^2 - 2$

$= \underline{\underline{18}}$

12a)

$$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$$

$$= \frac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{2-\sqrt{6}-\sqrt{10}+\sqrt{6}-3-\sqrt{15}+\sqrt{10}-\sqrt{15}-5}$$

$$= \frac{\sqrt{2}-\sqrt{3}-\sqrt{5}}{-6-2\sqrt{15}} \times \frac{-6+2\sqrt{15}}{-6+2\sqrt{15}}$$

$$= \frac{-6\sqrt{2}+6\sqrt{3}+6\sqrt{5}+2\sqrt{30}-\underbrace{2\sqrt{45}}_{6\sqrt{5}}-\underbrace{2\sqrt{75}}_{10\sqrt{3}}}{36-60}$$

$$= \frac{-6\sqrt{2}-4\sqrt{3}+2\sqrt{30}}{-24}$$

$$= \frac{3\sqrt{2}+2\sqrt{3}-\sqrt{30}}{12}$$

12b $\dfrac{1}{\sqrt[3]{2}} \times \dfrac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$

$= \dfrac{\sqrt[3]{4}}{2}$

c) $\dfrac{1}{\sqrt[3]{2}-1} \times \dfrac{\sqrt[3]{2^2}+\sqrt[3]{2}+1}{\sqrt[3]{2^2}+\sqrt[3]{2}+1}$

$a^3-b^3 = a-b \times a^2+ab+b^2$

$= \dfrac{\sqrt[3]{4}+\sqrt[3]{2}+1}{1}$

$= \sqrt[3]{4}+\sqrt[3]{2}+1$

14/ $\sqrt{17} \doteq 4.12$ (to 2 dp)

a) $\dfrac{1}{\sqrt{17}-4} = \dfrac{1}{4.12-4}$

$= \dfrac{1}{0.12}$

$= 8.\dot{3} = \dfrac{25}{3}$

b) $\dfrac{1}{\sqrt{17}-4} \times \dfrac{\sqrt{17}+4}{\sqrt{17}+4}$

$= \dfrac{\sqrt{17}+4}{17-16}$

$= \dfrac{\sqrt{17}+4}{}$

$= 4+4.12$

$= 8.12$

$\dfrac{1}{\sqrt{17}-4}$

$= 8.123\ldots$

15

$$\frac{a}{b+\sqrt{c}} + \frac{d}{\sqrt{c}}$$

$$= \frac{a\sqrt{c} + d(b+\sqrt{c})}{\sqrt{c}(b+\sqrt{c})}.$$

$$= \frac{(a+d)\sqrt{c} + bd}{b\sqrt{c}+c} \times \frac{b\sqrt{c}-c}{b\sqrt{c}-c}$$

$$= \frac{(a+d)bc - c(a+d)\sqrt{c} + b^2 d\sqrt{c} - bcd}{b^2c - c^2}$$

$$= \frac{(a+d)bc - bcd + (b^2d - c(a+d))\sqrt{c}}{b^2c - c^2}$$

to be rational

$$b^2 d - c(a+d) = 0$$

$$b^2 d = c(a+d)$$

If $\frac{a}{b+\sqrt{c}} + \frac{d}{\sqrt{c}}$ is rational

then $db^2 = c(a+d)$

If $\frac{a}{1+\sqrt{c}} + \frac{d}{\sqrt{c}}$ rational

then $d = c(a+d)$

$$d = c(a+d)$$

$$= ac + cd$$

$$d - cd = ac$$

$$d = \frac{ac}{1-c}$$

as c is a positive integer

$$1 - c < 0$$

$$\therefore d < 0$$

however $d > 0$

$\therefore \frac{a}{1+\sqrt{c}} + \frac{d}{\sqrt{c}}$ is not rational

16 a) $x = \sqrt{2} = \dfrac{x^2+2}{2x}$

b) $\dfrac{p}{q} = \sqrt{2} + \varepsilon$, ε is very small, positive number

$$\frac{p^2+2q^2}{2pq} = \frac{p}{2q} + \frac{q}{p}$$

$$= \frac{\sqrt{2}+\varepsilon}{2} + \frac{1}{\sqrt{2}+\varepsilon}$$

$$= \frac{\sqrt{2}+\varepsilon}{2} + \frac{\sqrt{2}-\varepsilon}{2-\varepsilon^2}$$

$$= \frac{\sqrt{2}}{2} + \frac{\varepsilon}{2} + \frac{\sqrt{2}}{2-\varepsilon^2} - \frac{1}{\varepsilon}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2-\varepsilon^2} + \frac{\varepsilon}{2} - \frac{1}{\varepsilon}$$

$$> \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\varepsilon}{2} - \frac{1}{\varepsilon}$$

$$= \frac{2\sqrt{2}}{2} + \frac{\varepsilon}{2} - \frac{1}{\varepsilon}$$

$$= \sqrt{2} + \frac{\varepsilon}{2} - \frac{1}{\varepsilon}$$

$$> \sqrt{2} \qquad \left(\frac{\varepsilon}{2} - \frac{1}{\varepsilon} > 0\right)$$