6a) $x - 3 + \sqrt{y+2} = -1 + \sqrt{5}$

$x - 3 = -1 \qquad y + 2 = 5$

$x = 2 \text{ , } y = 3$.

7a)

$$(a+b\sqrt{2})^2 = 3+2\sqrt{2}$$

$$a^2 + 2ab\sqrt{2} + 2b^2 = 3+2\sqrt{2}$$

$$a^2 + 2b^2 = 3$$

$$2ab = 2 \Rightarrow b = \frac{1}{a}$$

$$a^2 + \frac{2}{a^2} = 3$$

$$a^4 - 3a^2 + 2 = 0$$

$$(a^2-2)(a^2-1) = 0$$

$a^2 = 2$ or $a^2 = 1$

$a = \pm\sqrt{2}$ not a solution

$a = \pm 1$

$\therefore a=1, b=1$ or $a=-1, b=-1$

7c) $(a+b\sqrt{3})^2 = 13-4\sqrt{3}$

$a^2 + 2ab\sqrt{3} + 3b^2 = 13-4\sqrt{3}$

$a^2 + 3b^2 = 13$ $\qquad 2ab = -4$

$a^2 + \frac{12}{a^2} = 13$ $\qquad b = -\frac{2}{a}$

$a^4 - 13a^2 + 12 = 0$

$(a^2-12)(a^2-1) = 0$

$a^2 = 12$ or $a^2 = 1$

no solutions $\qquad a = \pm 1$

$\therefore a = 1, b = -2$ or $a = -1, b = 2$

8a) $\sqrt{15-6\sqrt{6}} = x-\sqrt{y}$

$\sqrt{216}$ $\qquad$ $\sqrt{4x^2y}$

$15-6\sqrt{6} = x^2-2x\sqrt{y}+y$

$x^2+y=15$

$4x^2y=216$

$x^2y=54$

$y=\frac{54}{x^2}$

$x^2+\frac{54}{x^2}=15$

$x^4-15x^2+54=0$

$(x^2-9)(x^2-6)=0$

$x=\pm 3$ or $x=\pm\sqrt{6}$ not a solution

$\therefore x=3, y=6$ or $x=-3, y=6$

$\sqrt{15-6\sqrt{6}} = 3-\sqrt{6}$