10/ d) $f(x) = x^3 + 1$

f^{-1}: $x = y^3 + 1$

$y^3 = x - 1$

$y = \sqrt[3]{x-1}$

inverse function

$f^{-1}(f(x))$

$= \sqrt[3]{x^3 + 1 - 1}$

$= \sqrt[3]{x^3}$

$= \underline{\underline{x}}$

$f(f^{-1}(x))$

$= \left(\sqrt[3]{x-1}\right)^3 + 1$

$= x - 1 + 1$

$= \underline{\underline{x}}$

10f, $f(x) = 9 - x^2$, $x \geq 0$

has inverse function

f^{-1}: $x = 9 - y^2$, $y \geq 0$

$y^2 = 9 - x$

$y = \sqrt{9 - x}$

$f^{-1}(f(x)) = \sqrt{9 - (9 - x^2)}$

$= \sqrt{x^2}$

$= \underline{x}$

$f(f^{-1}(x)) = 9 - (\sqrt{9 - x})^2$

$= 9 - 9 + x$

$= \underline{\underline{x}}$

10g) $f(x) = 3^{-x^2}$ NO INVERSE

OR

$x = 3^{-y^2}$

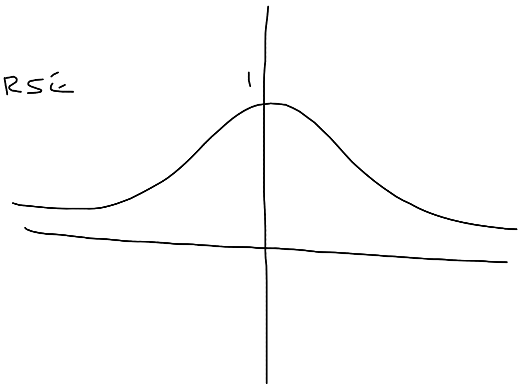
$-y^2 = \log_3 x$

$y^2 = -\log_3 x$

$y = \pm\sqrt{-\log_3 x}$

NOT UNIQUE $\therefore$

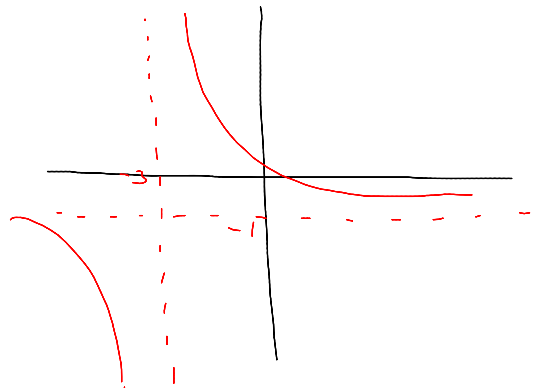
no inverse function



10h) $f(x) = \dfrac{1-x}{3+x}$

$= -1 + \dfrac{4}{3+x}$

has inverse function



OR

$x = \dfrac{1-y}{3+y}$

$3x + xy = 1-y$

$xy + y = 1-3x$

$y(x+1) = 1-3x$

$y = \dfrac{1-3x}{x+1}$

UNIQUE

$\therefore$ has inverse function.

$$f^{-1}(f(x)) = \frac{1 - 3\left(\frac{1-x}{3+x}\right)}{\left(\frac{1-x}{3+x}\right) + 1}$$

$$= \frac{3+x-3+3x}{1-x+3+x}$$

$$= \frac{4x}{4}$$

$$= x \checkmark$$

$$f(f^{-1}(x)) = \frac{1 - \left(\frac{1-3x}{x+1}\right)}{3 + \left(\frac{1-3x}{x+1}\right)}$$

$$= \frac{x+1-1+3x}{3x+3+1-3x}$$

$$= \frac{4x}{4}$$

$$= x \checkmark$$

10j) $f(x) = x^2 - 2x$, $x \geq 1$

has an inverse function

f^{-1}: $x = y^2 - 2y$, $y \geq 1$

$y^2 - 2y - x = 0$

$y = \dfrac{2 + \sqrt{4 + 4x}}{2}$

$= \dfrac{2 + 2\sqrt{1+x}}{2}$

$\underline{y = 1 + \sqrt{1+x}}$

$f^{-1}(f(x)) = 1 + \sqrt{1 + x^2 - 2x}$

$= 1 + \sqrt{x^2 - 2x + 1}$

$= 1 + \sqrt{(x-1)^2}$

$= 1 + x - 1$

$= \underline{x}$

$f(f^{-1}(x)) = (1 + \sqrt{1+x})^2 - 2(1 + \sqrt{1+x})$

$= 1 + 2\sqrt{1+x} + 1 + x - 2 - 2\sqrt{1+x}$

$= \underline{x}$

11) a) $y = \frac{ax+b}{x+c}$ is inverse of $y = \frac{b-cx}{x-a}$

b) show it's its own inverse iff $a+c=0$

$$\frac{ax+b}{x+c} = \frac{b-cx}{x-a}$$

$$ax^2 - a^2x + \cancel{bx} - ab = \cancel{bx} - cx^2 + bc - c^2x$$

$$(a+c)x^2 - (a^2-c^2)x - ab - bc = 0$$

$$(a+c)x^2 - (a+c)(a-c)x - b(a+c) = 0 \qquad a+c \neq 0$$

$$x^2 - (a+c)x - b = 0$$

$$x = \frac{(a+c) \pm \sqrt{(a+c)^2 + 4b}}{2}$$

$\therefore$ equation not always true if $a+c \neq 0$

ie equation is always true if $a+c=0$