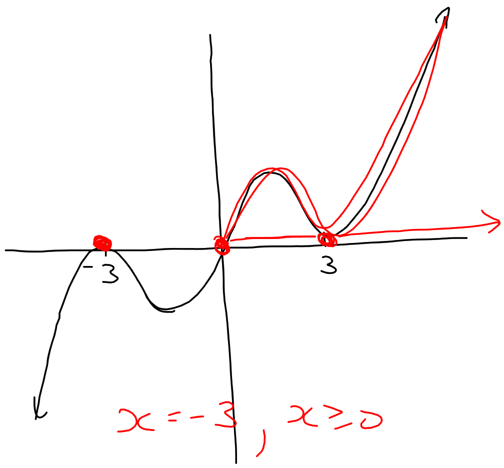
3f)

$x(x-3)^2(x+3)^2 \geq 0$



$x = -3, \; x \geq 0$

$x(x-3)^2(x+3)^2 > 0$

$0 < x < 3, \; x > 3$ *

$x > 0, \; x \neq 3$

9

a) $f(x) = \dfrac{x}{x-3}$

$= \boxed{1 + \dfrac{3}{x-3}}$

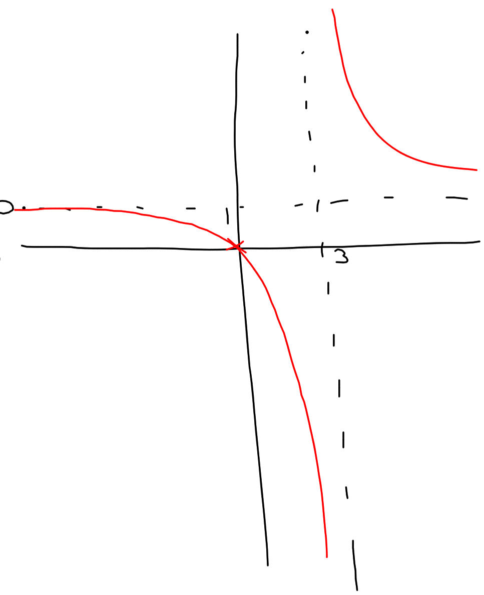
<u>x intercept</u> $f(x) = 0$

$\dfrac{x}{x-3} = 0$

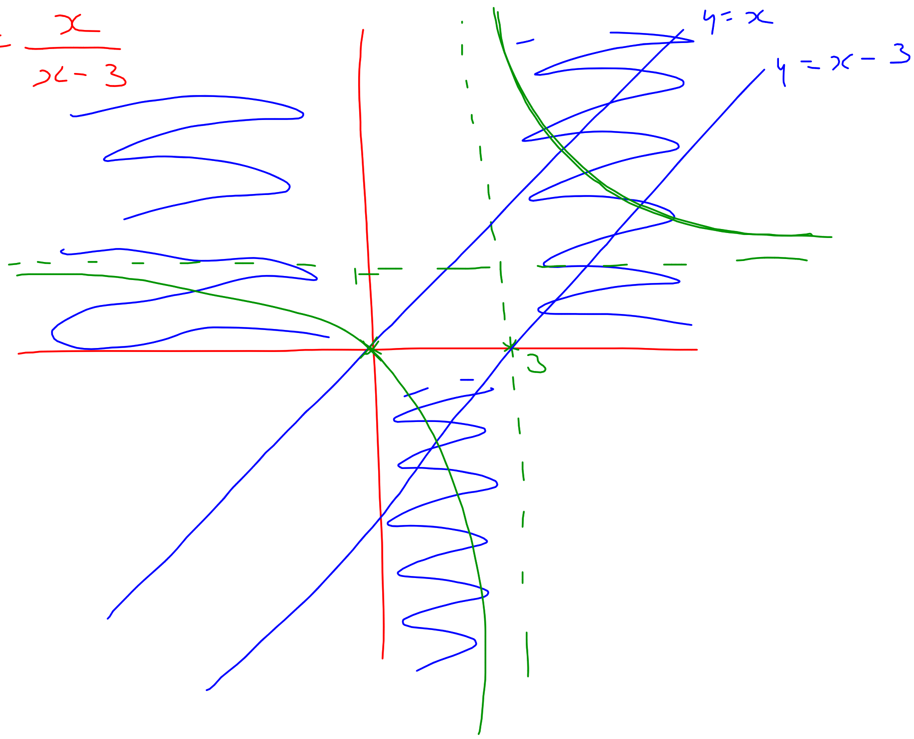
$x = 0$

$x - 3 \neq 0$

$x \neq 3$



$y = \dfrac{x}{x-3}$



10/a) $\frac{4}{x+3} \geq x$

$\frac{4}{(x+3)} \times (x+3)^2 \geq x(x+3)^2$

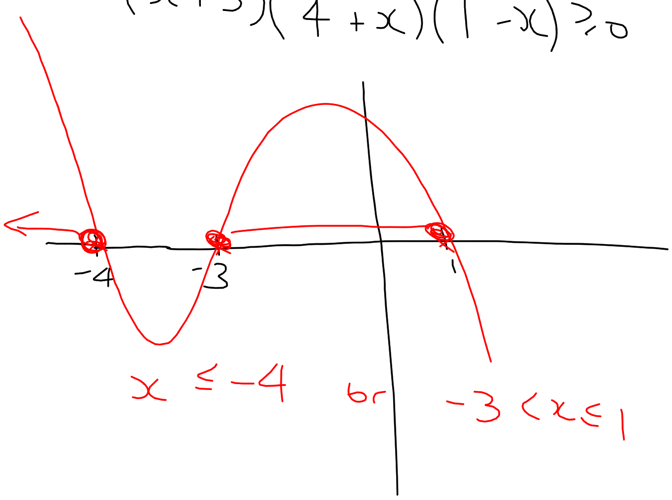
$4(x+3) \geq x(x+3)^2$

$4(x+3) - x(x+3)^2 \geq 0$

$(x+3)(4 - x(x+3)) \geq 0$

$(x+3)(4-3x-x^2) \geq 0$

$(x+3)(4+x)(1-x) \geq 0$



$x \leq -4$ or $-3 < x \leq 1$

$\frac{4}{x+3} \geq x$

$x+3 \neq 0$

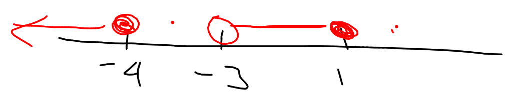
$x \neq -3$

$4 = x^2+3x$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=-4$ or $x=1$



$x \leq -4$ or $-3 < x \leq 1$

12/

a) $f(x) = 1 + x + x^2$

$= (x + \frac{1}{2})^2 + \frac{3}{4} > 0$

b) $f(x) = 1 + x + x^2 + x^3 + x^4$

If $x > 0$	$x = 0$	$x < 0$
$f(x) > 0$	$f(x) = 1 > 0$	$1 - x > 0$

$1 - x^5 > 0$

$(1 - x)(1 + x + x^2 + x^3 + x^4) > 0$

$\therefore (1 + x + x^2 + x^3 + x^4) > 0$

$(\because 1 - x > 0)$

c) $f(x) = 1 + x + x^2 + \dots + x^{2n-1} + x^{2n} > 0$.

If $x > 0$

$f(x) > 0$

If $x = 0$

$f(x) = 1 > 0$

If $x < 0$

$\underline{1 - x > 0}$

$1 - x^{2n+1} > 0$

$(1-x)(1 + x + x^2 + \dots + x^{2n}) > 0$

$\therefore 1 + x + x^2 + \dots + x^{2n} > 0 \quad (\because 1 - x > 0)$

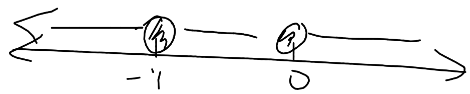
d) Prove $x=-1$, is the only zero of $f(x)=1+x+x^2+\dots+x^{2n-1}$

$f(0)=1$

a) $1+x+x^2>0$

b) $x^3+x^4 = x^3(x+1) \geq 0$

critical pts are $0, -1$



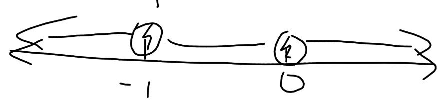
all real x

$\therefore \ 1+x+x^2+x^3+x^4>0$

c) $x^{2n-1} + x^{2n}$

$= x^{2n-1}(x+1) \geqslant 0$

critical pts are $0, -1$



$\therefore 1 + x + x^2 \dots + x^{2n} > 0$

d) $1 + x + x^2 + \dots + x^{2n-2} > 0$

solve $\underbrace{1 + x + x^2 + \dots + x^{2n-2}} + x^{2n-1} = 0$

$x^{2n-1} = -(1 + x + x^2 + \dots + x^{2n-2})$

$$1+x+x^2+\dots+x^{2n-1}=0$$

$$1(1+x)+x^2(1+x)+\dots+x^{2n-2}(1+x)=0$$

$$(1+x)(1+x^2+\dots+x^{2n-2})=0$$

but $1+x^2+\dots+x^{2n-2}>0$

$$\therefore (1+x)=0$$

$$\underline{\underline{x=-1}}$$