

$$4f) \quad \frac{2 \tan \frac{11\pi}{12}}{1 - \tan^2 \frac{11\pi}{12}}$$

$$A = \tan \frac{11\pi}{12} = \tan \frac{\theta}{2}$$

$$\begin{aligned} \frac{2t}{1-t^2} &= \tan \theta \\ &= \tan \frac{11\pi}{6} \\ &= -\tan \frac{\pi}{6} \\ &= -\frac{1}{\sqrt{3}} \end{aligned}$$

5g)

$$\frac{\tan 2\alpha + \cot \alpha}{\tan 2\alpha - \tan \alpha} = \cot^2 \alpha$$

Let  $t = \tan \alpha$

$$\begin{aligned} \frac{\tan 2\alpha + \cot \alpha}{\tan 2\alpha - \tan \alpha} &= \frac{\frac{2t}{1-t^2} + \frac{1}{t}}{\frac{2t}{1-t^2} - t} \\ &= \frac{2t^2 + (1-t^2)}{2t^2 - t(1-t^2)} \\ &= \frac{t^2 + 1}{t^2 + t} \\ &= \frac{t^2 + 1}{t(1+t)} \\ &= \frac{1}{t} = \cot^2 \alpha \end{aligned}$$

$$6a) \quad t = \tan 112\frac{1}{2}^\circ = \tan \frac{\theta}{2}$$

$$\frac{2t}{1-t^2} = \tan \theta = \tan 225^\circ$$
$$= \tan 45^\circ$$
$$\therefore t = 1$$

$$b) \text{ (i) } \quad \frac{2t}{1-t^2}$$

$$1-t^2 = 1$$

$$2t = 1-t^2$$

$$t^2 + 2t - 1 = 0$$

$$t = \frac{-2 \pm \sqrt{8}}{2}$$

$$t = -1 \pm \sqrt{2}$$

$112\frac{1}{2}^\circ$  is obtuse

$$\therefore \tan 112\frac{1}{2}^\circ = -1 - \sqrt{2}$$

$$\text{(ii) } \quad \frac{2t}{1-t^2} = 1$$

$$\tan 2\theta = 1$$

$$2\theta = 45^\circ \text{ or } 2\theta = 135^\circ$$

$$\therefore \tan 22\frac{1}{2}^\circ = \frac{1 - \sqrt{2}}{1 + \sqrt{2}}$$

$$8/ \quad \tan \alpha = -\frac{1}{3}$$

$$\frac{\pi}{2} < \alpha < \pi$$

≡ 2nd quad

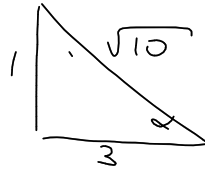
$$\text{let } t = \tan \alpha$$

$$\begin{aligned} \tan 2\alpha &= \frac{2t}{1-t^2} \\ &= \frac{2(-\frac{1}{3})}{1-\frac{1}{9}} \\ &= \frac{-6}{8} \\ &= \underline{\underline{-\frac{3}{4}}} \end{aligned}$$

$$8b) \tan \alpha = -\frac{1}{3}$$

$$\frac{\pi}{2} < \alpha < \pi$$

$$\begin{aligned} \sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ &= 2 \left( \frac{1}{\sqrt{10}} \right) \left( -\frac{3}{\sqrt{10}} \right) \\ &= -\frac{6}{10} = -\frac{3}{5} \end{aligned}$$



let  $t = \tan \alpha$

$$b) \sin 2\alpha = \frac{2t}{1+t^2}$$

$$\begin{aligned} &= \frac{2\left(-\frac{1}{3}\right)}{1+\left(-\frac{1}{3}\right)^2} \\ &= \frac{-\frac{2}{3}}{\frac{10}{9}} \\ &= -\frac{3}{5} \end{aligned}$$

d)  $\tan \frac{\alpha}{2}$

Let  $t = \tan \frac{\alpha}{2}$

$$\frac{2t}{1-t^2} = -\frac{1}{3}$$

$$6t = t^2 - 1$$

$$t^2 - 6t - 1 = 0$$

$$t = \frac{6 \pm \sqrt{40}}{2}$$

$$= 3 \pm \sqrt{10}$$

$$\tan \frac{\alpha}{2} > 0$$

$$\therefore \tan \frac{\alpha}{2} = \underline{3 + \sqrt{10}}$$

10a)

$$\begin{aligned}
 x &= \tan \theta + \sec \theta \\
 &= \frac{2t}{1-t^2} + \frac{1+t^2}{1-t^2} \\
 &= \frac{t^2 + 2t + 1}{1-t^2} \\
 &= \frac{(t+1)^2}{(1+t)(1-t)} \\
 &= \frac{t+1}{1-t}
 \end{aligned}$$

$$\frac{x^2 - 1}{x^2 + 1} = \sin \theta$$

$$\begin{aligned}
 \frac{x^2 - 1}{x^2 + 1} &= \frac{(t+1)^2}{(1-t)^2} - 1 \\
 &= \frac{(t+1)^2}{(1-t)^2} + 1 \\
 &= \frac{(t+1)^2 - (1-t)^2}{(t+1)^2 + (1-t)^2} \\
 &= \frac{2t^2 + 2}{4t} \\
 &= \frac{2t}{t^2 + 1} \\
 &= \sin \theta
 \end{aligned}$$