4f)

$$\frac{2\tan\frac{11\pi}{12}}{1-\tan^2\frac{11\pi}{12}}$$

$t = \tan\frac{11\pi}{12} = \tan\frac{\theta}{2}$

$$\frac{2t}{1-t^2} = \tan\theta$$

$= \tan\frac{11\pi}{6}$

$= -\tan\frac{\pi}{6}$

$= \underline{\underline{-\frac{1}{\sqrt{3}}}}$

5g) $$\boxed{\frac{\tan 2\alpha + \cot\alpha}{\tan 2\alpha - \tan\alpha} = \cot^2\alpha}$$

Let $t = \tan\alpha$

$$\frac{\tan 2\alpha + \cot\alpha}{\tan 2\alpha - \tan\alpha} = \frac{\frac{2t}{1-t^2} + \frac{1}{t}}{\frac{2t}{1-t^2} - t}$$

$$= \frac{2t^2 + (1-t^2)}{2t^2 - t^2(1-t^2)}$$

$$= \frac{t^2+1}{t^2+t^4}$$

$$= \frac{t^2+1}{t^2(1+t^2)}$$

$$= \frac{1}{t^2} = \underline{\underline{\cot^2\alpha}}$$

6a) $t = \tan 112\frac{1}{2}^\circ = \tan \frac{\theta}{2}$

$$\frac{2t}{1-t^2} = \tan\theta = \tan 225^\circ$$
$$= \tan 45^\circ$$
$$\therefore = 1$$

b) (i) $\dfrac{2t}{1-t^2} = 1$

$2t = 1 - t^2$

$t^2 + 2t - 1 = 0$

$t = \dfrac{-2 \pm \sqrt{8}}{2}$

$t = -1 \pm \sqrt{2}$

$112\frac{1}{2}^\circ$ is obtuse

$\therefore \tan 112\frac{1}{2}^\circ = -1 - \sqrt{2}$

(ii) $\dfrac{2t}{1-t^2} = 1$

$\tan 2\theta = 1$

$2\theta = 225^\circ$ or $2\theta = 45^\circ$

$\therefore \tan 22\frac{1}{2}^\circ = -1 + \sqrt{2}$

8/ $\tan\alpha = -\frac{1}{3}$ $\frac{\pi}{2} < \alpha < \pi$

$\therefore$ 2nd quad

let $t = \tan\alpha$

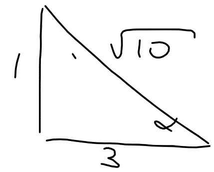
$$\tan 2\alpha = \frac{2t}{1-t^2}$$

$$= \frac{2\left(-\frac{1}{3}\right)}{1-\frac{1}{9}}$$

$$= \frac{-6}{8}$$

$$= \frac{-3}{4}$$

8b) $\tan\alpha = -\frac{1}{3}$ $\qquad \frac{\pi}{2} < \alpha < \pi$



$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$= 2\left(\frac{1}{\sqrt{10}}\right)\left(-\frac{3}{\sqrt{10}}\right)$$
$$= -\frac{6}{10} = -\frac{3}{5}$$

let $t = \tan\alpha$

b) $\sin 2\alpha = \dfrac{2t}{1+t^2}$

$$= \frac{2\left(-\frac{1}{3}\right)}{1+\left(-\frac{1}{3}\right)^2}$$
$$= \frac{-\frac{2}{3}}{\frac{10}{9}}$$
$$= -\frac{3}{5}$$

d) $\tan\frac{\alpha}{2}$

Let $t = \tan\frac{\alpha}{2}$

$$\frac{2t}{1-t^2} = -\frac{1}{3}$$
$$6t = t^2 - 1$$
$$t^2 - 6t - 1 = 0$$
$$t = \frac{6 \pm \sqrt{40}}{2}$$
$$= 3 \pm \sqrt{10}$$
$\tan\frac{\alpha}{2} > 0$ $\quad 3 + \sqrt{10}$

$\therefore \tan\frac{\alpha}{2} = 3 + \sqrt{10}$

10a)

$$x = \tan\theta + \sec\theta$$

$$= \frac{2t}{1-t^2} + \frac{1+t^2}{1-t^2}$$

$$= \frac{t^2+2t+1}{1-t^2}$$

$$= \frac{(t+1)^2}{(1+t)(1-t)}$$

$$= \frac{t+1}{1-t}$$

$$\frac{x^2-1}{x^2+1} = \sin\theta$$

$$\frac{x^2-1}{x^2+1}$$

$$= \frac{\frac{(t+1)^2}{(1-t)^2} - 1}{\frac{(t+1)^2}{(1-t)^2} + 1}$$

$$= \frac{(t+1)^2-(1-t)^2}{(t+1)^2+(1-t)^2}$$

$$= \frac{4t}{2t^2+2}$$

$$= \frac{2t}{t^2+1}$$

$$= \sin\theta$$