4a) $\sin x - \sqrt{3}\cos x = 0$

$\sin x = \sqrt{3}\cos x$

$\tan x = \sqrt{3}$

Q1, 3

$\tan \alpha = \sqrt{3}$

$\alpha = 60°$

$x = 60°, 240°$

c) $4\cos 2x = 3\sec 2x$

$\cos^2 2x = \frac{3}{4}$

$\cos 2x = \pm \frac{\sqrt{3}}{2}$

Q1, 2, 3, 4

$\cos \alpha = \frac{\sqrt{3}}{2}$

$\alpha = 30°$

$2x = 30°, 150°, 210°, 330°, 390°, 510°, 570°, 690°$

$x = 15°, 75°, 105°, 165°, 195°, 255°, 285°, 345°$

5a) $\sin^2\alpha = \sin\alpha$

$\sin^2\alpha - \sin\alpha = 0$

$\sin\alpha(\sin\alpha - 1) = 0$

$\sin\alpha = 0$ or $\sin\alpha = 1$

$\alpha = 0°, 180°, 360°$ $\quad \alpha = 90°$

$\alpha = 0°, 90°, 180°, 360°$

c) $\cos^2\alpha = \sin\alpha\cos\alpha$

$\cos^2\alpha - \sin\alpha\cos\alpha = 0$

$\cos\alpha(\cos\alpha - \sin\alpha) = 0$

$\cos\alpha = 0$ or $\cos\alpha = \sin\alpha$

$\alpha = 90°, 270°$ $\quad \tan\alpha = 1$

$\alpha = 45°, 225°$

$\alpha = 45°, 90°, 225°, 270°$

5 i) $\operatorname{cosec}^3 2\alpha = 4\operatorname{cosec} 2\alpha$

$\operatorname{cosec} 2\alpha\left(\operatorname{cosec}^2 2\alpha - 4\right) = 0$

$\operatorname{cosec} 2\alpha = 0$ or $\operatorname{cosec} 2\alpha = \pm 2$

no solutions

$\sin 2\alpha = \pm \frac{1}{2}$

$2\alpha = 30^\circ, 150^\circ, 210^\circ, 330^\circ,$
$390^\circ, 510^\circ, 570^\circ, 690^\circ$

$\alpha = 15^\circ, 75^\circ, 105^\circ, 165^\circ, 195^\circ, 205^\circ,$
$285^\circ, 345^\circ$

9d) $\sqrt{3}\,\text{cosec}^2 A = 4\cot A$

$\sqrt{3} + \sqrt{3}\cot^2 A = 4\cot A$

$\sqrt{3}\cot^2 A - 4\cot A + \sqrt{3} = 0$

$(\sqrt{3}\cot A - 1)(\cot A - \sqrt{3}) = 0$

$\cot A = \frac{1}{\sqrt{3}}$ or $\cot A = \sqrt{3}$

$\tan A = \sqrt{3}$ $\tan A = \frac{1}{\sqrt{3}}$

9g)

$$2(\cos A - \sec A) = \tan A$$

$$2\left(\cos A - \frac{1}{\cos A}\right) = \frac{\sin A}{\cos A}$$

$$2\cos^2 A - 2 = \sin A$$

$$2 - 2\sin^2 A - 2 = \sin A$$

$$2\sin^2 A + \sin A = 0$$

$$\sin A(2\sin A + 1) = 0$$

$$\sin A = 0 \quad \text{or} \quad \sin A = -\frac{1}{2}$$

9 i) $\sin^2 A - 2\sin A\cos A - 3\cos^2 A = 0$

$(\sin A - 3\cos A)(\sin A + \cos A) = 0$

$\sin A = 3\cos A$ or $\sin A = -\cos A$

$\tan A = 3$ $\tan A = -1$

9) $\tan^2 A + 8\cos^2 A = 5$

$\sec^2 A - 1 + 8\cos^2 A = 5$

$\frac{1}{\cos^2 A} + 8\cos^2 A = 6$

$1 + 8\cos^4 A = 6\cos^2 A$

$8\cos^4 A - 6\cos^2 A + 1 = 0$

$(4\cos^2 A - 1)(2\cos^2 A - 1) = 0$

$\cos^2 A = \frac{1}{4}$ or $\cos^2 A = \frac{1}{2}$

$\cos A = \pm\frac{1}{2}$ $\cos A = \pm\frac{1}{\sqrt{2}}$

10d) $8\sin^2\theta\cos^2\theta = 1$

$2 \times (2\sin\theta\cos\theta)^2 = 1$

$2\sin^2 2\theta = 1$

$\sin^2 2\theta = \frac{1}{2}$

$\sin 2\theta = \pm\frac{1}{\sqrt{2}}$

$2\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ, 405^\circ, 495^\circ, 585^\circ, 675^\circ$

$\theta = 22\frac{1}{2}^\circ, 67\frac{1}{2}^\circ, 112\frac{1}{2}^\circ, 157\frac{1}{2}^\circ, 202\frac{1}{2}^\circ, 247\frac{1}{2}^\circ, 292\frac{1}{2}^\circ, 337\frac{1}{2}^\circ$

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g) $10\cos\theta + 13\cos\frac{1}{2}\theta = 5$

$20\cos^2\frac{\theta}{2} - 10 + 13\cos\frac{\theta}{2} = 5$

$20\cos^2\frac{\theta}{2} + 13\cos\frac{\theta}{2} - 15 = 0$

$(5\cos\frac{\theta}{2} - 3)(4\cos\frac{\theta}{2} + 5) = 0$

$\cos\frac{\theta}{2} = \frac{3}{5}$ or $\cos\frac{\theta}{2} = -\frac{5}{4}$ no solutions

$\frac{\theta}{2} = 53^\circ$

$\theta = 106^\circ$

10j) $\cos 2\theta + 3 = 3\sin 2\theta$

$\cos^2\theta - \sin^2\theta + 3\cos^2\theta + 3\sin^2\theta = 6\sin\theta\cos\theta$

$4\cos^2\theta - 6\sin\theta\cos\theta + 2\sin^2\theta = 0$

$2\cos^2\theta - 3\sin\theta\cos\theta + \sin^2\theta = 0$

$(2\cos\theta - \sin\theta)(\cos\theta - \sin\theta) = 0$

$2\cos\theta - \sin\theta = 0$ or $\cos\theta - \sin\theta = 0$

$\tan\theta = 2$ $\quad$ $\tan\theta = 1$

Q1, 3 $\quad$ Q1, 3

$\theta = 63°26', 243°26'$ $\quad$ $\theta = 45°, 225°$

$\theta = 45°, 63°26', 225°, 243°26'$

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$$2\cos x - 1 = 2\cos 2x$$

$$= 4\cos^2 x - 2$$

$$4\cos^2 x - 2\cos x - 1 = 0$$

$$\cos x = \frac{2 \pm \sqrt{20}}{8}$$

$$= \frac{1 \pm \sqrt{5}}{4}$$

$\cos x = \frac{1+\sqrt{5}}{4}$ or $\cos x = \frac{1-\sqrt{5}}{4}$

Q1, 4 Q2, 3

$x = 36^\circ, 354^\circ$

$\cos\alpha = \frac{\sqrt{5}-1}{4}$

$\alpha = 72^\circ$

$x = 108^\circ, 252^\circ$

$x = 36^\circ, 108^\circ, 252^\circ, 354^\circ$

24/ $\sin\theta + \cos\theta = \sin 2\theta$

a) $\sin^2 2\theta - \sin 2\theta - 1 = 0$

b) $\sin 2\theta = \frac{1 \pm \sqrt{5}}{2}$

$\sin 2\theta = \frac{1-\sqrt{5}}{2}$ $\left(\frac{1+\sqrt{5}}{2} > 1\right)$

Q3, 4

$\sin\alpha = \frac{\sqrt{5}-1}{2}$

$\alpha = 38^\circ 10'$

$2\theta = 218^\circ 10', 321^\circ 50', 578^\circ 10', 681^\circ 50'$

$\theta = 109^\circ 5', 160^\circ 55', 289^\circ 5', 340^\circ 55'$

$\theta = 160^\circ 55', 289^\circ 5'$ (4 per checking)