Expanding Binomials

$$(1+x)^n = (1+x)(1+x)(1+x)(1+x)\cdots(1+x)$$

when expanding parentheses we choose a term from each set of parentheses and multiply them together.

$$x \times 1 \times x \times 1 \times 1 \times 1 \times \cdots \times x = x^3$$

Question: How many different ways could you end up with x^3 ?

OR How many different ways can you choose three x's from n sets of parentheses?

Answer: ${}^nC_3$

General Expansion of Binomials

${}^nC_k$ is the coefficient of x^k in $(1+x)^k$

$${}^nC_k = \binom{n}{k}$$

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$$

which extends to;

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots + {}^nC_{n-1} a b^{n-1} + {}^nC_n b^n$$

$$e.g.\ (2+3x)^4 = {}^4C_0 2^4 + {}^4C_1 2^3 (3x) + {}^4C_2 2^2 (3x)^2 + {}^4C_3 2 (3x)^3 + {}^4C_4 (3x)^4$$

$$\underline{= 16 + 96x + 216x^2 + 216x^3 + 81x^4}$$

Pascal's Triangle Relationships

$$(1)\quad {}^{n}C_k = {}^{n-1}C_{k-1} + {}^{n-1}C_k \quad \text{where } 1 \leq k \leq n-1$$

$$(1+x)^n = (1+x)(1+x)^{n-1}$$

$$= (1+x)\left({}^{n-1}C_0 + {}^{n-1}C_1 x + \ldots + {}^{n-1}C_{k-1}x^{k-1} + {}^{n-1}C_k x^k + \ldots + {}^{n-1}C_{n-1}x^{n-1}\right)$$

looking at coefficients of x^k

$LHS = {}^{n}C_k$

$$RHS = (1)\left({}^{n-1}C_{k-1}\right) + (1)\left({}^{n-1}C_k\right)$$

$$= {}^{n-1}C_{k-1} + {}^{n-1}C_k \qquad \therefore {}^{n}C_k = {}^{n-1}C_{k-1} + {}^{n-1}C_k$$

$$(2)\quad {}^{n}C_k = {}^{n}C_{n-k} \quad \text{where } 1 \leq k \leq n-1$$

"Pascal's triangle is symmetrical"

$$(3)\quad {}^{n}C_0 = {}^{n}C_n = 1$$

Exercise 5B; 2ace, 5, 6ac, 10ac, 11, 14