

$$3 \text{ a) (i) } \sin 3x + \sin x = 2 \sin 2x \cos x.$$

$$\text{(ii) } \sin 3x + \sin x = 0 \quad 0 \leq x \leq \pi$$

$$2 \sin 2x \cos x = 0$$

$$\sin 2x = 0 \quad \text{or} \quad \cos x = 0$$

$$2x = 0, \pi, 2\pi \quad x = \frac{\pi}{2}$$

$$x = 0, \frac{\pi}{2}, \pi$$

$$\therefore \underline{x = 0, \frac{\pi}{2}, \pi}$$

$$9b) \quad \sin 3x + \sin x = 2 \sin 2x \cos x$$

$$\sin 3x + \sin 2x + \sin x = 0$$

$$\sin 2x + 2 \sin 2x \cos x = 0$$

$$\sin 2x (1 + 2 \cos x) = 0$$

$$\sin 2x = 0 \quad \text{or} \quad \cos x = -\frac{1}{2}$$

$$2x \in \{0, \pi, 2\pi, 3\pi, 4\pi\} \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

$$x = 0, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, 2\pi$$

$$10a) \cos 5x + \cos x = 0$$

$$2 \cos 3x \cos 2x = 0$$

$$\cos 3x = 0 \quad \text{or} \quad \cos 2x = 0$$

$$3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$10b) \sin 4x - \sin x = 0$$

$$\sin 4x + \sin(-x) = 0$$

$$2 \sin \frac{3x}{2} \cos \frac{5x}{2} = 0$$

$$\sin \frac{3x}{2} = 0 \quad \text{or} \quad \cos \frac{5x}{2} = 0$$

$$\frac{3x}{2} = 0, \pi, 2\pi, 3\pi$$

$$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$$

$$\frac{5x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$$

$$x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$$

10e)

$$\sin x + \sin 2x + \sin 3x + \sin 4x = 0$$

$$2 \sin \frac{3x}{2} \cos \frac{x}{2} + 2 \sin \frac{7x}{2} \cos \frac{x}{2} = 0$$

$$2 \cos \frac{x}{2} \left(\sin \frac{3x}{2} + \sin \frac{7x}{2} \right) = 0$$

$$2 \cos \frac{x}{2} \left(2 \sin \frac{5x}{2} \cos x \right) = 0$$

$$4 \cos \frac{x}{2} \sin \frac{5x}{2} \cos x = 0$$

$$\cos \frac{x}{2} = 0$$

$$\frac{x}{2} = 90^\circ$$

$$x = 180^\circ$$

$$\sin \frac{5x}{2} = 0$$

$$\frac{5x}{2} = 0, 180, 360, 540, 720, 900$$

$$x = 0, 72^\circ, 144^\circ, 216^\circ, 288^\circ, 360^\circ$$

$$\cos x = 0$$

$$x = 90^\circ, 270^\circ$$

$$\underline{x = 0, 72^\circ, 90^\circ, 144^\circ, 180^\circ, 216^\circ, 270^\circ, 288^\circ, 360^\circ}$$