3b) $x = ct$ $\quad y = \frac{c}{t} = ct^{-1}$

$\frac{dx}{dt} = c$ $\quad \frac{dy}{dt} = -ct^{-2}$

$= -\frac{c}{t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= -\frac{c}{t^2} \times \frac{1}{c}$

$= -\frac{1}{t^2}$

when $t = -1$, $\frac{dy}{dx} = -1$

4(i) $y = \dfrac{1}{1+x^2}$

$= (1+x^2)^{-1}$

$\dfrac{dy}{dx} = -(1+x^2)^{-2}(2x)$

$= \dfrac{-2x}{(1+x^2)^2}$

tangent is horizontal when $\frac{dy}{dx} = 0$

ie $-2x = 0$

$x = 0$

$\therefore$ pt is $(0, 1)$

5a) $y = (5x-4)^4$

$\frac{dy}{dx} = 4(5x-4)^3(5)$

$= 20(5x-4)^3$

$x=1, \frac{dy}{dx} = 20(1)^3$

$= 20$

T/ $y - 1 = 20(x-1)$

$y - 1 = 20x - 20$

$y = 20x - 19$

5d)

$$y = \sqrt{x-2}$$

$$= (x-2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x-2)^{-\frac{1}{2}}(1)$$

$$= \frac{1}{2\sqrt{x-2}}$$

When $x = 1$,

$$\frac{dy}{dx} =$$

$\therefore$ none as $x = 1$ not in domain.

(6b) $y = (4x-7)^3$

$\frac{dy}{dx} = 3(4x-7)^2(4)$

$= 12(4x-7)^2$

tangent is perpendicular $x + 12y + 6 = 0$

$12y = -x - 6$

$y = -\frac{1}{12}x - \frac{1}{2}$

$\therefore$ slope required is 12

$\frac{dy}{dx} = 12$

$12(4x-7)^2 = 12$

$4x - 7 = \pm 1$

$4x = 7 \pm 1$

$4x = 8$ or $4x = 6$

$x = 2$ $\quad x = \frac{3}{2}$

7b)

$x = (t-1)^2$ $\quad y = (t-1)^3$

when $t = 3$, $x = 4$ $\quad y = 8$

$$\frac{dx}{dt} = 2(t-1)^1(1) = 2(t-1)$$

$$\frac{dy}{dt} = 3(t-1)^2(1) = 3(t-1)^2$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= 3(t-1)^2 \times \frac{1}{2(t-1)}$$

$$= \frac{3(t-1)}{2}$$

when $t = 3$, $\frac{dy}{dx} = 3$

$\therefore$ slope of tangent is 3

$y - 8 = 3(x-4)$

$y - 8 = 3x - 12$

$y = 3x - 4$

11

$y = a(x+b)^2 - 8$

has tangent $y = 2x$ at $P(4,8)$

$\frac{dy}{dx} = 2a(x+b)^1(1)$

$= 2a(x+b)$

when $x = 4$, $\frac{dy}{dx} = 2$

$2 = 2a(4+b)$

$1 = a(4+b)$

$a = \frac{1}{4+b}$

when $x = 4, y = 8$

$8 = a(4+b)^2 - 8$

$16 = a(4+b)^2$

$16 = \frac{(4+b)^2}{4+b}$

$16 = 4+b$

$b = 12$

$\therefore a = \frac{1}{16}$

13

$$y = \sqrt{169 - x^2}$$

$$= (169 - x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(169 - x^2)^{-\frac{1}{2}}(-2x)$$

$$= -x(169 - x^2)^{-\frac{1}{2}}$$

$$= \frac{-x}{\sqrt{169 - x^2}}$$

when $x = 12$,

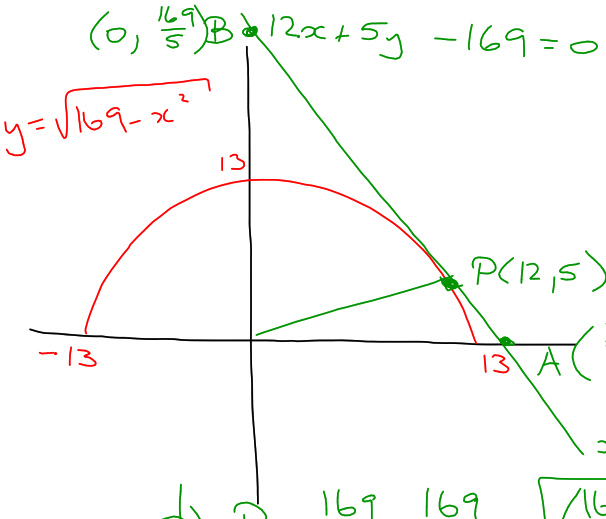
$$\frac{dy}{dx} = \frac{-12}{\sqrt{169 - 144}}$$

$$= -\frac{12}{5}$$

$$y - 5 = -\frac{12}{5}(x - 12)$$

$$5y - 25 = -12x + 144$$

$$12x + 5y - 169 = 0$$



b) $d = \dfrac{|12(0)+5(0)-169|}{\sqrt{12^2+5^2}}$

$= \dfrac{169}{13}$

$= 13$ units

c) $A = \frac{1}{2}bh$

$= \frac{1}{2} \times \frac{169}{12} \times \frac{169}{5}$

$= \dfrac{169^2}{120}$

d) $P = \frac{169}{12} + \frac{169}{5} + \sqrt{\left(\frac{169}{12}\right)^2 + \left(\frac{169}{5}\right)^2}$

$\therefore 84.5$