6a) $f(x) = \dfrac{x^2}{x+1}$

$$f'(x) = \frac{(x+1)(2x) - (x^2)(1)}{(x+1)^2}$$

$$= \frac{x^2 + 2x}{(x+1)^2}$$

$$f'(c) = -3$$

$$\frac{c^2 + 2c}{(c+1)^2} = -3$$

$$c^2 + 2c = -3c^2 - 6c - 3$$

$$4c^2 + 8c + 3 = 0$$

$$(2c+1)(2c+3) = 0$$

$$c = -\frac{1}{2} \text{ or } c = -\frac{3}{2}$$

8/ a) $x = \frac{t}{t+1}$ $y = \frac{t}{t-1}$

when $t=2$, $T\left(\frac{2}{3}, 2\right)$

$$\frac{dx}{dt} = \frac{(t+1)(1) - (t)(1)}{(t+1)^2} \qquad \frac{dy}{dt} = \frac{(t-1)(1) - (t)(1)}{(t-1)^2}$$

$$= \frac{1}{(t+1)^2} \qquad = \frac{-1}{(t-1)^2}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{-1}{(t-1)^2} \times (t+1)^2$$

$$= \frac{-(t+1)^2}{(t-1)^2}$$

When $t = 2$, $\frac{dy}{dx} = \frac{-9}{1}$

$\therefore$ slope of normal is $\frac{1}{9}$

$$y - 2 = \frac{1}{9}\left(x - \frac{2}{3}\right)$$

$$y - 2 = \frac{1}{9}x - \frac{2}{27}$$

$$\underline{y = \frac{1}{9}x + \frac{52}{27}}$$

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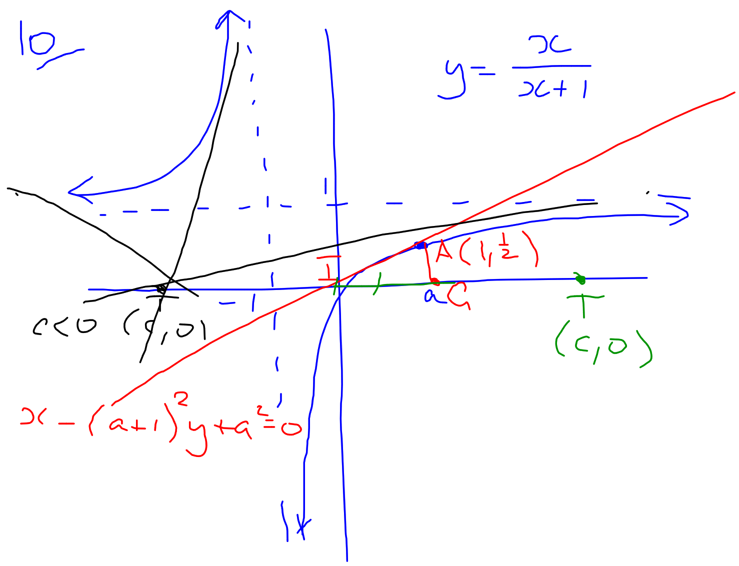


(i)

$x - (a+1)^2 y + a^2 = 0$

$\underline{y = 0}$

$x = -a^2$

$c = -a^2$

not possible as $c > 0$.

(iii)

$x = -a^2$

$c = -a^2$

but $c < 0$

$a = \pm\sqrt{-c}$

$\therefore$ two tangents possible

$-1 < c < 0$ : same branch

$c < -1$ : different branches.