

2 a)

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

find $\frac{dA}{dt}$ when $r = 40 \text{ cm}$ and $\frac{dr}{dt} = 3 \text{ cm/s}$

$$\begin{aligned} \frac{dA}{dt} &= 2\pi(40) \times 3 \\ &= 240\pi \text{ cm}^2/\text{s} \end{aligned}$$

$$\frac{dA}{dt} = 2x \frac{dx}{dt} \quad \frac{dl}{dt} = \sqrt{2} \frac{dx}{dt}$$

a) $\frac{dA}{dt}$ when $x = 70 \text{ cm}$, $\frac{dx}{dt} = 6 \text{ cm/s}$

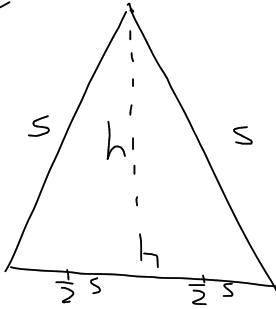
$$\frac{dA}{dt} = 2(70) \times 6$$

$$= 840 \text{ cm}^2/\text{s}$$

b) $A = 1 \text{ m}^2$
 $= 10000 \text{ cm}^2$
 $x = 100 \text{ cm}$

$$\frac{dA}{dt} = 2(100) \times 6$$
$$= \underline{\underline{1200 \text{ cm}^2/\text{s}}}$$

b)



$$A = \frac{1}{4} s^2 \sqrt{3}$$

$$h = \frac{1}{2} s \sqrt{3}$$

$$a) \frac{dA}{dt} = \frac{dA}{ds} \times \frac{ds}{dt}$$

$$= \frac{1}{2} s \sqrt{3} \times \frac{ds}{dt}$$

$$\frac{dh}{dt} = \frac{dh}{ds} \times \frac{ds}{dt}$$

$$= \frac{\sqrt{3}}{2} \times \frac{ds}{dt}$$

$$b) s = 12 \text{ cm}, \quad \frac{ds}{dt} = 3 \text{ mm/s}$$

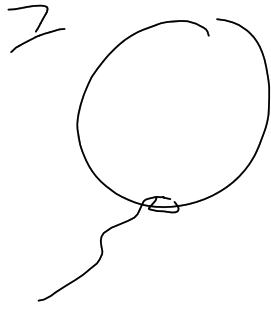
$$= \frac{3}{10} \text{ cm/s}$$

$$\frac{dA}{dt} = \frac{1}{2} (12) \sqrt{3} \times \frac{3}{10}$$

$$= \frac{9\sqrt{3}}{5} \text{ cm}^2/\text{s}$$

$$\frac{dh}{dt} = \frac{\sqrt{3}}{2} \times \frac{3}{10}$$

$$= \frac{3\sqrt{3}}{20} \text{ cm/s}$$



$$\frac{ds}{dt} = 1 \text{ cm}^2/\text{s}$$

a) $r = 3$

$$\frac{dr}{dt} = \frac{ds}{dt} \times \frac{dr}{ds}$$

$$= 1 \times \frac{1}{8\pi r}$$

when $r = 3$, $\frac{dr}{dt} = \frac{1}{24\pi} \text{ cm/s}$

$$S = 4\pi r^2$$

$$\frac{ds}{dr} = 8\pi r$$

$$\begin{aligned} \text{(ii)} \quad \frac{dV}{dt} &= \frac{dr}{dt} \times \frac{dV}{dr} \\ &= \frac{1}{24\pi} \times 4\pi(3)^2 \\ &= \underline{\underline{\frac{3}{2} \text{ cm}^3/\text{s}}} \end{aligned}$$

$$\begin{aligned} &\frac{ds}{dt}, \frac{dr}{dt} \\ V &= \frac{4}{3}\pi r^3 \\ \frac{dV}{dr} &= 4\pi r^2 = S \end{aligned}$$

$$b) \frac{dV}{dt} = 10 \text{ cm}^3/\text{s}$$

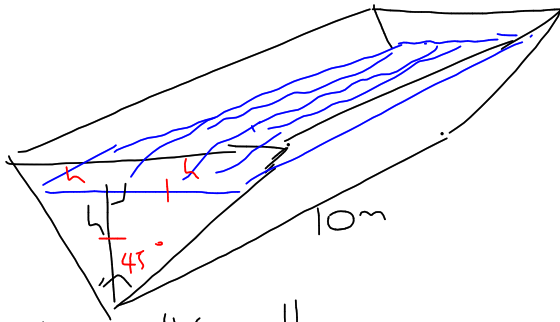
$$\frac{1}{8\pi r} \times 4\pi r^2 = 10$$

$$\frac{r}{2} = 10$$

$$r = 20$$

$$V = \frac{4}{3} \pi (20)^3$$
$$= \frac{32000\pi}{3} \text{ cm}^3$$

18



$$V = 1000h^2$$

$$A = 2000h$$

$$\begin{aligned}\frac{dh}{dt} &= \frac{dV}{dt} \times \frac{dh}{dV} \\ &= (5000) \left(\frac{1}{2000h} \right) \\ &= \frac{5}{2h}\end{aligned}$$

$$\begin{aligned}\frac{dA}{dt} &= \frac{dV}{dt} \times \frac{dh}{dV} \times \frac{dA}{dh} \\ &= \frac{5}{2h} \times 2000 \\ &= \frac{5000}{h}\end{aligned}$$

$\frac{dV}{dt} = 5000 \text{ cm}^3/\text{min}$

when $h = 60$,

$$\frac{dh}{dt} = \frac{5}{120} \text{ cm/min}$$

$$\frac{dA}{dt} = \frac{500}{6} \text{ cm}^2/\text{min}$$

b) $h = 40$, $\frac{dh}{dt} = 0.1$, $\frac{dV}{dt} = ?$, $\frac{dA}{dt} = ?$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$= 2000h \times 0.1$$

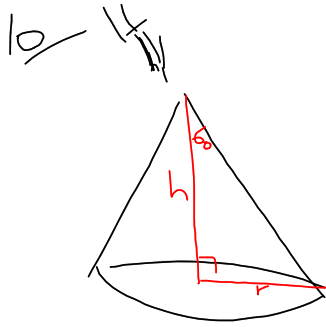
$$= 200h$$

when $h = 40$, $\frac{dV}{dt} = \underline{8000 \text{ cm}^3/\text{min}}$

$$\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$$

$$= 2000 \times 0.1$$

$$= \underline{200 \text{ cm}^2/\text{min}}$$



$$\frac{r}{h} = \tan 60^\circ$$
$$r = h \tan 60^\circ$$
$$= \sqrt{3}h$$

$$V = \frac{1}{3} \pi r^2 h$$
$$= \frac{1}{3} \pi (\sqrt{3}h)^2 h$$
$$= \underline{\underline{\pi h^3}}$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$
$$= \underline{\underline{3\pi h^2 \times \frac{dh}{dt}}}$$

$$a) \frac{dV}{dt} = 0.3 \text{ m}^3/\text{min}$$

$$\begin{aligned} A &= \pi r^2 \\ &= \pi (\sqrt{3}h)^2 \\ &= 3\pi h^2 \end{aligned}$$

$$\begin{aligned} (i) \frac{dh}{dt} &= \frac{dV}{dt} \times \frac{dh}{dV} \\ &= \frac{3}{10} \times \frac{1}{3\pi h^2} \\ &= \frac{1}{10\pi h^2} \end{aligned}$$

when $h=4$,

$$\begin{aligned} \frac{dh}{dt} &= \frac{1}{10\pi(4)^2} \\ &= \frac{1}{160\pi} \text{ m/min} \end{aligned}$$

(ii) when $r = 4$ m

$$4 = \sqrt{3} h$$

$$h = \frac{4}{\sqrt{3}}$$

$$\frac{dh}{dt} = \frac{1}{10\pi h^2}$$

$$= \frac{1}{10\pi \left(\frac{4}{\sqrt{3}}\right)^2}$$

$$= \frac{3}{160\pi} \text{ m/min}$$

$$b) \quad A = 3\pi h^2$$

$$\frac{dA}{dh} = 6\pi h$$

$$\begin{aligned} \frac{dA}{dt} &= \frac{dA}{dh} \times \frac{dh}{dt} \\ &= 6\pi h \times \frac{dh}{dt} \end{aligned}$$

(i) when $h=4$

$$\begin{aligned} \frac{dA}{dt} &= 6\pi(4) \times \frac{1}{160\pi} \\ &= \frac{3}{20} \text{ m}^2/\text{min} \end{aligned}$$

(ii) when $r=4$

$$\begin{aligned} \frac{dA}{dt} &= 6\pi\left(\frac{4}{\sqrt{3}}\right) \times \frac{3}{160\pi} \\ &= \frac{9}{20\sqrt{3}} \\ &= \frac{3\sqrt{3}}{20} \text{ m}^2/\text{min} \end{aligned}$$

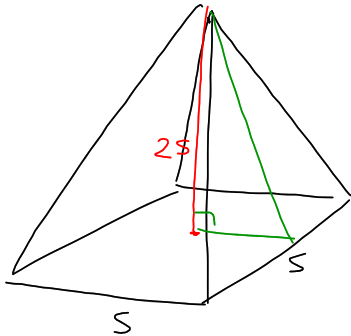
$$\begin{aligned} c) \quad \frac{dh}{dt} &= 8 \text{ cm/min} & h &= 4 \text{ m} \\ &= \frac{2}{25} \text{ m/min} \end{aligned}$$

$$\frac{dV}{dt} = 3\pi h^2 \times \frac{dh}{dt}$$

$$= 3\pi(4)^2 \times \frac{2}{25}$$

$$= \frac{96\pi}{25} \text{ m}^3/\text{min}$$

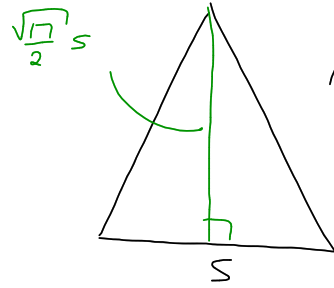
12



$$V = \frac{1}{3}(s)(s)(2s)$$

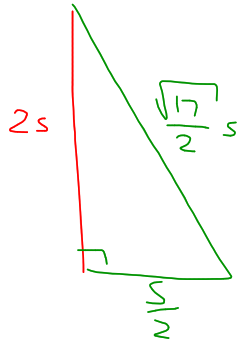
$$V = \frac{2}{3}s^3$$

$$\frac{dV}{ds} = 2s^2$$



$$A_{\Delta} = \frac{1}{2}(s)\left(\frac{\sqrt{17}}{2}s\right)$$

$$= \frac{\sqrt{17}}{4}s^2$$



$$A = 4\left(\frac{\sqrt{17}}{4}s^2\right) + s^2$$

$$= (\sqrt{17} + 1)s^2$$

$$\frac{dA}{ds} = 2(\sqrt{17} + 1)s$$

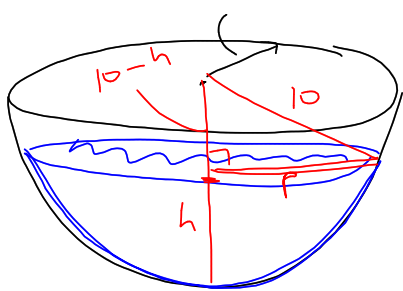
$$b) \quad s = 4 \text{ m}$$

$$\frac{ds}{dt} = -\frac{3}{1000} \text{ m/s}$$

$$\begin{aligned} \frac{dV}{dt} &= \frac{dV}{ds} \times \frac{ds}{dt} \\ &= 2s^2 \times -\frac{3}{1000} \\ &= 2(4)^2 \times -\frac{3}{1000} \\ &= -\frac{12}{125} \text{ m}^3/\text{s} \end{aligned}$$

$$\begin{aligned} \frac{dA}{dt} &= \frac{dA}{ds} \times \frac{ds}{dt} \\ &= 2(\sqrt{17}+1)(4) \left(-\frac{3}{1000}\right) \\ &= -\frac{3(\sqrt{17}+1)}{125} \text{ m}^2/\text{s} \end{aligned}$$

$$\frac{dV}{dt} = 6 \text{ cm}^3/\text{s}$$



$$V = \frac{\pi}{3} h^2 (30 - h)$$

$$= 10\pi h^2 - \frac{\pi}{3} h^3$$

$$\frac{dV}{dh} = 20\pi h - \pi h^2$$

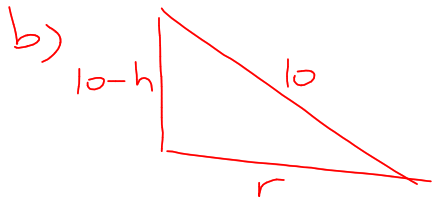
$$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$$

$$= 6 \times \frac{1}{20\pi h - \pi h^2}$$

when $h = 2$,

$$\frac{dh}{dt} = \frac{6}{40\pi - 4\pi}$$

$$= \frac{1}{6\pi} \text{ cm/s}$$



$$\begin{aligned}
 r^2 &= 10^2 - (10-h)^2 \\
 &= 100 - 100 + 20h - h^2 \\
 &= 20h - h^2 \\
 r &= (20h - h^2)^{\frac{1}{2}}
 \end{aligned}$$

$$A = \pi r^2$$

$$A = \pi (20h - h^2)$$

$$\frac{dA}{dh} = \pi (20 - 2h)$$

$$\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$$

$$= \pi (20 - 2h) \times \frac{6}{20\pi h - \pi h^2}$$

$$= \frac{6\pi (20 - 2h)}{20\pi h - \pi h^2}$$

$$h=2, \frac{dA}{dt} = \frac{96\pi}{36\pi}$$

$$= \underline{\underline{\frac{8}{3} \text{ cm}^2/\text{s}}}$$

