(h) $(x^2-4x) + 8 = \dfrac{48}{x^2-4x}$

let $m = x^2-4x$

$m + 8 = \dfrac{48}{m}$

$m^2 + 8m = 48$

$m^2 + 8m - 48 = 0$

$(m+12)(m-4) = 0$

$m = -12$ or $m = 4$

$x^2 - 4x = -12$ or $x^2 - 4x = 4$

$x^2 - 4x + 12 = 0$

no real solutions

$x^2 - 4x - 4 = 0$

$$x = \frac{4 \pm \sqrt{32}}{2}$$

$$x = 2 \pm 2\sqrt{2}$$

2d) $\cot^2 x = \operatorname{cosec} x + 1$

$\operatorname{cosec}^2 x - 1 = \operatorname{cosec} x + 1$

$\operatorname{cosec}^2 x - \operatorname{cosec} x - 2 = 0$

$(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 1) = 0$

$\operatorname{cosec} x = 2$ or $\operatorname{cosec} x = -1$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -1$

$x = 30°, 150°$ $\quad$ $x = 270°$

4b)

$$x(x+1)(x+2)(x+3) = 35$$

$$(x^2+3x)(x^2+3x+2) = 35$$

let $m = x^2+3x$

$$m(m+2) = 35$$

$$m^2+2m-35 = 0$$

$$(m+7)(m-5) = 0$$

$$m = -7 \text{ or } m = 5$$

$x^2 + 3x = -7$ or $x^2 + 3x = 5$

$x^2 + 3x + 7 = 0$

no real solutions

$x^2 + 3x - 5 = 0$

$$x = \frac{-3 \pm \sqrt{29}}{2}$$

5 c)

$$\sqrt{5+x} + \sqrt{x} = 5$$

$$5+x + 2\sqrt{5x+x^2} + x = 25$$

$$2\sqrt{5x+x^2} = 20 - 2x$$

$$\sqrt{5x+x^2} = 10 - x$$

$$5x + x^2 = 100 - 20x + x^2$$

$$25x = 100$$

$$x = 4$$

6a)

$$\frac{x+5}{x-5} - \frac{x-6}{x+6} = \frac{x+4}{x-4} - \frac{x-7}{x+7}$$

$$\frac{x+5}{x-5} - \frac{x+4}{x-4} = \frac{x-6}{x+6} - \frac{x-7}{x+7}$$

$$\frac{(x+5)(x-4)-(x+4)(x-5)}{(x-5)(x-4)} = \frac{(x-6)(x+7)-(x-7)(x+6)}{(x+6)(x+7)}$$

$$\frac{2x}{(x-5)(x-4)} = \frac{2x}{(x+6)(x+7)}$$

$$x(x^2-9x+20) = x(x^2+13x+42)$$

$$22x = -22$$

$$x = -1 \quad \text{or} \quad x = 0$$

9a) $x^4 - 5x^3 + 8x^2 - 5x + 1 = 0$

$x^2\left(x^2 - 5x + 8 - \frac{5}{x} + \frac{1}{x^2}\right) = 0$

$x^2\left(x^2 + \frac{1}{x^2} - 5\left(x + \frac{1}{x}\right) + 8\right) = 0$

$x^2 = 0$ not a solution

$\therefore x^2 + \frac{1}{x^2} - 5\left(x + \frac{1}{x}\right) + 8 = 0$

$\left(x^2 + 2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) + 6 = 0$

$m = x + \frac{1}{x}$

$m^2 = x^2 + 2 + \frac{1}{x^2}$

$m^2 - 5m + 6 = 0$

$(m-3)(m-2) = 0$

$x + \frac{1}{x} = 3$ or $x + \frac{1}{x} = 2$

$x^2 - 3x + 1 = 0$ $x^2 - 2x + 1 = 0$

$x = \frac{3 \pm \sqrt{5}}{2}$ $(x-1)^2 = 0$

$x = 1$

$\therefore x = 1, \; x = \frac{3 \pm \sqrt{5}}{2}$