

||

$$(x-a)(x-b) = c^2$$

$$x^2 - ax - bx + ab = c^2$$

$$x^2 - (a+b)x + (ab - c^2) = 0$$

$$\Delta = (a+b)^2 - 4(ab - c^2)$$

$$= a^2 + 2ab + b^2 - 4ab + 4c^2$$

$$= a^2 - 2ab + b^2 + 4c^2$$

$$= (a-b)^2 + 4c^2 > 0$$

\therefore roots are real.

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$$y = x^2 - 5x - 3$$

$$\text{tangent is } \parallel 3x - y - 7 = 0$$

$$\therefore m = 3$$

$$\therefore \text{tangent is } 3x - y - k = 0 \quad \underline{\text{OR}} \quad \frac{dy}{dx} = 2x - 5$$

$$y = 3x - k$$

$$\frac{dy}{dx} = 3$$

$$3x - k = x^2 - 5x - 3$$

$$2x - 5 = 3$$

$$x^2 - 8x + (k - 3) = 0$$

$$x = 4, y = -7$$

$$\text{if tangent } \Delta = 0$$

$$y + 7 = 3(x - 4)$$

$$64 - 4(k - 3) = 0$$

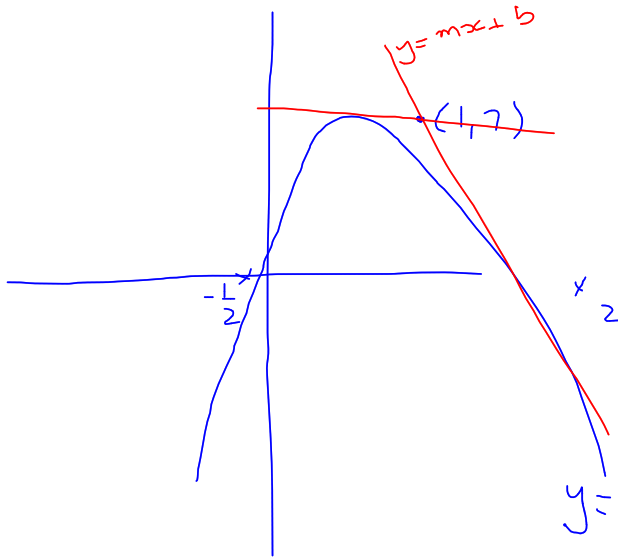
$$y + 7 = 3x - 12$$

$$4k = 76$$

$$3x - y - 19 = 0$$

$$k = 19$$
$$\text{tangent is } \underline{3x - y - 19 = 0}$$

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(1, 7)

$$7 = ma + b$$

$$b = 7 - m$$

\therefore line

$$y = mx + 7 - m$$

$$\begin{aligned} y &= (2-x)(1+3x) \\ &= -3x^2 + 5x + 2 \end{aligned}$$

$$mx + 7 - m \quad -3x^2 + 5x + 2$$

$$3x^2 + (m-5)x + (5-m) = 0$$

tangent if $\Delta = 0$

$$(m-5)^2 - 12(5-m) = 0$$

$$(m-5)(m-5+12) = 0$$

$$m = 5 \text{ or } m = -7$$

\therefore tangents are

$$\underline{y = 5x + 2 \text{ and } y = -7x + 14}$$

18c)

$$a) (x-4)^2 + y^2 = 4$$

$$b) y = mx$$

$$(x-4)^2 + m^2x^2 = 4$$

21b)

$$y = (2b-3)x^2 + (5b-1)x + (3b+2)$$

two roots when $\Delta > 0$

$$(5b-1)^2 - 4(2b-3)(3b+2) > 0$$

$$21d) (3k-2)x^2 + 2(k+6)x + (k-4) = 0$$

$$\Delta > 0$$

$$4(k+6)^2 - 4(3k-2)(k-4) > 0$$

$$k^2 + 12k + 36 - 3k^2 + 14k - 8 > 0$$

$$2k^2 - 26k - 28 < 0$$

$$k^2 - 13k - 14 < 0$$

$$(k-14)(k+1) < 0$$

$$\underline{-1 < k < 14}$$

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$$\frac{x^2 - mn}{2x - m - n} = \lambda$$

$$x^2 - mn = 2\lambda x - \lambda m - \lambda n$$

$$x^2 - 2\lambda x + (\lambda(m+n) - mn) = 0$$

$$\Delta = 4\lambda^2 - 4(\lambda(m+n) - mn)$$

$$= 4\lambda^2 - 4(m+n)\lambda + 4mn$$

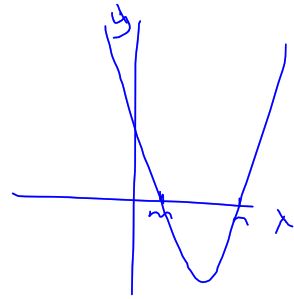
$$= 4(\lambda - m)(\lambda - n)$$

no solutions if $\Delta < 0$

$$(x-m)(x-n) < 0$$

$$m < x < n$$

\therefore has no solutions between
 m and n .



$$\frac{24}{(x-m)(x-n) + (x-n)(x-l) + (x-l)(x-m)} = 0$$

$$3x^2 - 2(l+m+n)x + (lm+ln+mn) = 0$$

$$\Delta = 4(l+m+n)^2 - 12(lm+ln+mn)$$

$$= 4(l^2+m^2+n^2) - 4(lm+ln+mn)$$

$$\Delta = 0$$

$$l^2+m^2+n^2 = lm+ln+mn$$