3d) $x^2 - 2(k-3)x + (k-1)$

$\Delta = 4(k-3)^2 - 4(k-1)$

$= 4(k^2 - 7k + 10)$

positive definite when $a > 0$, $\Delta \leqslant 0$

$a = 1 > 0$ ✓

$4(k^2 - 7k + 10) < 0$

$(k-5)(k-2) < 0$

$\underline{2 < k < 5}$

5b) $2\lambda x^2 + 2\lambda x + 1$

$a^2 + 2ab + b^2$

perfect squae if $\Delta = 0$

$4\lambda^2 - 8\lambda = 0$

$4\lambda(\lambda - 2) = 0$

$\lambda = 0$ or $\lambda = 2$

not a quadratic

$\therefore \underline{\lambda = 2}$

5d)

$(4\lambda+1)x^2 - 6\lambda x + 4$

$(a-b)^2 = a^2 - 2ab + b^2$

$b^2 = 4$ $\quad$ $2ab = 6\lambda$ $\quad$ $a^2 = 4\lambda + 1$

$b = \pm 2$

If $b = 2$ $\quad$ $\left(\frac{3}{2}\lambda\right)^2 = 4\lambda + 1$

$4a = 6\lambda$ $\quad$ $\frac{9\lambda^2}{4} = 4\lambda + 1$

$a = \frac{3}{2}\lambda$ $\quad$ $9\lambda^2 - 16\lambda - 4 = 0$

$(9\lambda+2)(\lambda-2) = 0$

$\lambda = -\frac{2}{9}$ or $\lambda = 2$

<u>OR</u>

$$\Delta = 0$$

$$36\lambda^2 - 16(4\lambda + 1) = 0$$

$$36\lambda^2 - 64\lambda - 16 = 0$$

$$9\lambda^2 - 16\lambda - 4 = 0$$

6/ $(3k-5)x^2 + 2(4-k)x + 4 = 0$

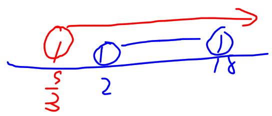
$$\Delta = 4(k^2 - 20k + 36)$$

a)

$a > 0$ $\qquad$ $\Delta < 0$

$3k - 5 > 0$ $\qquad$ $k^2 - 20k + 36 < 0$

$k > \frac{5}{3}$ $\qquad$ $(k-2)(k-18) < 0$



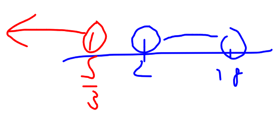
$2 < k < 18$

b) $\qquad$ $2 < k < 18$

$a < 0$

$k < \frac{5}{3}$ $\qquad$ $\Delta < 0$

$2 < k < 18$



no solutions

12/ $x^2 - xy - 2y^2 + x + 7y - 5$

$= x^2 + (1-y)x + (-2y^2 + 7y - 5)$

positive definite if $a>0$, $\Delta<0$

$a = 1 > 0$ ✓

$\Delta = (1-y)^2 - 4(-2y^2 + 7y - 5)$

$= 1 - 2y + y^2 + 8y^2 - 28y + 20$

$= 9y^2 - 30y + 21$

$= 3(3y^2 - 10y + 7)$

$3y^2 - 10y + 7 < 0$

$(3y-7)(y-1) < 0$

$\underline{1 < y < \frac{7}{3}}$

14

$$3x^2 + 2xy - 8y^2 - 8x + 14y - 3$$

$$= 3x^2 + (2y-8)x - (8y^2 - 14y + 3)$$

$$\Delta = (2y-8)^2 + 12(8y^2 - 14y + 3)$$

$$= 4y^2 - 32y + 64 + 96y^2 - 168y + 36$$

$$= 100y^2 - 200y + 100$$

$$= 100(y-1)^2 > 0$$

15/

$$4a^2 - 10ab + 10b^2 + \lambda(3a^2 - 10ab + 3b^2)$$

$$(4 + 3\lambda)a^2 - 10b.(1+\lambda)a + (10 + 3\lambda)b^2$$

perfect square if $\Delta = 0$

$$100b^2(1+\lambda)^2 - 4(4+3\lambda)(10+3\lambda)b^2 = 0$$

$$100b^2 + 200b^2\lambda + 100b^2\lambda^2 - 4b^2(40 + 42\lambda + 9\lambda^2) = 0$$

$$25 + 50\lambda + 25\lambda^2 - 40 - 42\lambda - 9\lambda^2 = 0$$

$$16\lambda^2 + 8\lambda - 15 = 0$$

$$(4\lambda - 3)(4\lambda + 5) = 0$$

$$\lambda = \frac{3}{4} \text{ or } \lambda = -\frac{5}{4}$$

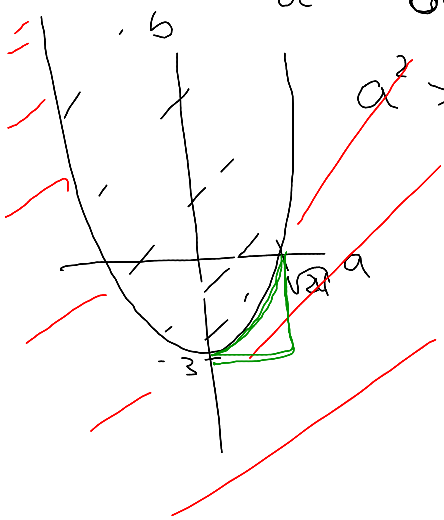
16

$2x^2 + ax + (b+3) = 0$

$8b+24=a^2$ has real roots

$b=\frac{1}{8}a^2 - 3$ $\therefore \Delta \geq 0$

$a^2 - 8(b+3) \geq 0$



$a^2 \geq 8(b+3)$

minimum

a^2+b^2

$\geq 8(b+3)+b^2$

$= b^2+8b+24$

$\min = \frac{-\Delta}{4a}$

$= \frac{-(64-4\times 24)}{4}$

17a) $f(x) = (x-5)^2 + (x+2)^2$

$= x^2 - 10x + 25 + x^2 + 4x + 4$

$= 2x^2 - 6x + 29$

$a = 2 > 0$ $\Delta = 36 - 4(2)(29) < 0$

$\therefore$ positive definite

b) $2x^2 + 4x + 10$

$= x^2 - 2x + 1 + x^2 + 6x + 9$

$= (x - 1)^2 + (x + 3)^2$

17c)

$$f(x) = 2x^2 + 2bx + c$$

$$\Delta = 4b^2 - 8c$$

$$= 4\underline{(b^2 - 2c)}$$

$$f(x) = (x-r)^2 + (x-s)^2$$

$$= 2x^2 - 2(r+s)x + (r^2+s^2)$$

$$-b = r+s \qquad c = r^2+s^2$$

If $\Delta < 0$

$$b^2 - 2c < 0$$

$$(r+s)^2 - 2(r^2+s^2) < 0$$

$$-r^2 + 2rs - s^2 < 0$$

$$-(r-s)^2 < 0$$

true for all r, s.