2014

TRIAL

HIGHER SCHOOL CERTIFICATE

EXAMINATION

GIRRAWEEN HIGH SCHOOL

MATHEMATICS EXTENSION 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board – approved calculators may be used
- A table of standard integrals is provided
- Show all necessary working in Questions 11-16

Total marks - 100

Section 1 pages 2-4

10 marks

- Attempt Questions 1-10
- Allow about 20 minutes for this section

Section 2 pages 5 - 13

- Attempt Questions 11 - 16
- Allow about 2 hours 40 minutes for this section

SECTION 1

Multiple Choice (10 marks) Circle your answer on the question paper.

1. For any complex number z, $\arg(z)+\arg(\bar{z})$ equals

(A) 0 **(B)** $n\pi$ **(C)** $-n\pi$ **(D)** $\frac{\pi}{4}$

2. The conjugate of $\frac{1+2i}{3-i}$ is

(A) $1+7i$ **(B)** $1-7i$ **(C)** $\frac{1+7i}{10}$ **(D)** $\frac{1-7i}{10}$

3. The roots of $z^3-1=0$ in modulus argument form are

(A) $cis0, cis\frac{2\pi}{3}$ **(C)** $cis0, cis\frac{\pi}{3}$

(B) $\text{cis}0, \text{cis}\frac{2\pi}{3}, cis\left(-\frac{2\pi}{3}\right)$ **(D)** $cis0, cis\frac{\pi}{3}, cis\left(-\frac{\pi}{3}\right)$

4. For the ellipse with equation $\frac{x^2}{16}+\frac{y^2}{36}=1$, what is the eccentricity?

(A) $\frac{2\sqrt{7}}{6}$ **(B)** $\frac{2\sqrt{11}}{6}$ **(C)** $\frac{2\sqrt{5}}{6}$ **(D)** $\frac{2\sqrt{3}}{6}$

5. For the hyperbola $\frac{(x+2)^2}{9}-\frac{(y-1)^2}{16}=1$, the coordinates of one of the focus is

(A) (7,1) **(B)** (-7, -1) **(C)** (3,1) **(D)** (-3, 1)

6. The value of $\int_0^1 x(1-x)^{99}\,dx$ is

(A) $\frac{1}{10010}$ **(B)** $\frac{1}{10100}$ **(C)** $\frac{11}{10100}$ **(D)** $\frac{11}{10010}$

7.

Reduce into partial fractions: $\dfrac{3x+1}{(x-2)^2(x+2)}$

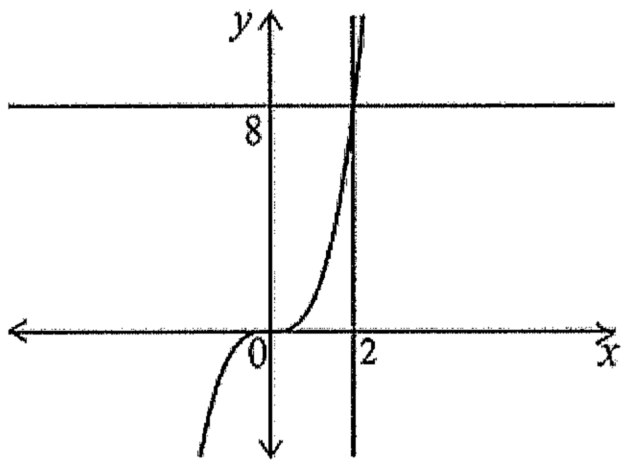
(A) $\dfrac{5}{16(x-2)}+\dfrac{7}{4(x-2)^2}-\dfrac{5}{16(x+2)}$

(B) $\dfrac{5}{16(x-2)}-\dfrac{7}{4(x-2)^2}+\dfrac{5}{16(x+2)}$

(C) $\dfrac{5}{16(x-2)}-\dfrac{7}{4(x-2)^2}-\dfrac{5}{16(x+2)}$

(D) $\dfrac{5}{16(x-2)}+\dfrac{7}{4(x-2)^2}+\dfrac{5}{16(x+2)}$

8. The volume of the solid generated when the region bounded by $y=x^3, x=0, y=8$ is revolved about the line $x=2$.



(A) $\dfrac{963\pi}{5}$ **(B)** $\dfrac{144\pi}{5}$ **(C)** $\dfrac{153\pi}{5}$ **(D)** $\dfrac{320\pi}{5}$

9. What is the angle at which a road must be banked so that a car may round a curve with a radius of 200 metres at 100 km/h without sliding. Assume that the road is smooth.

(A) 24.49° **(B)** 23.49° **(C)** 22.49° **(D)** 21.49°

10. The polynomial $P(x) = x^4 + ax^2 + bx + 28$ has a double root at $x = 2$. What are the values of a and b?

(A) $a = -11$ and $b = -12$

(C) $a = -11$ and $b = 12$

(B) $a = -5$ and $b = -12$

(D) $a = -5$ and $b = 12$

END OF SECTION 1

Question 11 (15 marks) **Marks**

(a) $\displaystyle\int_0^{\ln 2} \frac{e^x dx}{1+e^x}$ 2

(b) $\displaystyle\int \frac{dx}{3x^2+6x+10}$ 2

(c) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{dx}{5-4\cos x} dx$ 3

(d) (i) If $I_n = \displaystyle\int_{-1}^{0} x^n \sqrt{1+x}\,dx$ for $n = 0,1,2,\ldots$ show that $I_n = -\dfrac{2n}{2n+3} I_{n-1}$

for $n = 1,2,3\ldots.$ 3

(ii) Hence evaluate $\displaystyle\int_{-1}^{0} x^2 \sqrt{1+x}\,dx$ 2

(e) Use the result $\displaystyle\int_0^a f(x)dx = \int_0^a f(a-x)dx$ to evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ 3

Examination continues on next page

Question 12 (15 marks)

(a) If $z=\sqrt{3}+i$ and $w=1-i$

(i) Write $\frac{z}{w}$ in the form $a+ib$ where a and b are real numbers. 1

(ii) write $\frac{z}{w}$ in mod - arg form. 2

(iii) Hence find the exact values of $\sin\frac{5\pi}{12}$ and $\cos\frac{5\pi}{12}$ 2

(b) Sketch the following on separate Argand diagrams.

(i) $\arg(z-(-1+2i))=-\frac{2\pi}{3}$ 2

(ii) $2<|z|\leq 4$ and $-\frac{\pi}{4}<\arg(z)\leq\frac{\pi}{2}$ 2

(c) If n is a positive integer, prove that $(\sqrt{3}+i)^n+(\sqrt{3}-i)^n=2^{n+1}\cos\frac{n\pi}{6}$ 2

(d) z satisfies $|z-2i|=1$, and the point P represents z on an Argand diagram.

(i) Sketch the locus of P as z varies. 1

(ii) Find the maximum and minimum values of arg z, where $-\pi<\arg z\leq\pi$. 2

(iii) Find the value of z when arg z takes the maximum value, and express in modulus-argument form. 1

Examination continues on next page

Question 13 (15 marks)

(a) If $2-3i$ is a zero of the polynomial z^3+pz+q where p and q are real, find the values of p and q. 2

(b) If α, β and γ are the roots of the equation $x^3+6x+1=0$, find the polynomial equation whose roots are $\alpha\beta, \alpha\gamma$ and $\beta\gamma$. 3

(c) Given that the quartic polynomial $P(x)=x^4-5x^3-9x^2+81x-108$ has a zero of multiplicity3, factorise $P(x)$ completely and find all its zeros. 3

(d) **(i)** Solve $z^8+1=0$. Leave answers in mod- arg form. 2

(ii) Factorise z^8+1 into real quadratic factors. 2

(iii) Show that $\cos 4\theta = 8\left(\cos^2\theta - \cos^2\frac{\pi}{8}\right)\left(\cos^2\theta - \cos^2\frac{3\pi}{8}\right)$ 3

Examination continues on next page

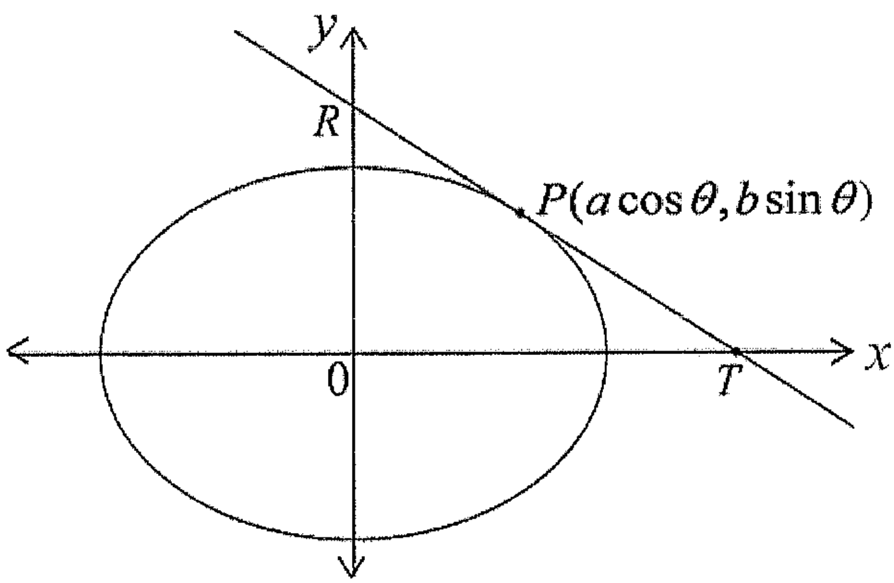
Question 14 (15 marks)

(a) The ellipse with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has a tangent at the point $P(a\cos\theta, b\sin\theta)$.

The tangent cuts the $x-$ axis at T and the $y-$axis at R.

(i) Show that the equation of the tangent at the point P is

$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$$ 3



(ii) If T is the point of intersection between the tangent at point P and one of the directrices of the ellipse, show that $\cos\theta = e$ 3

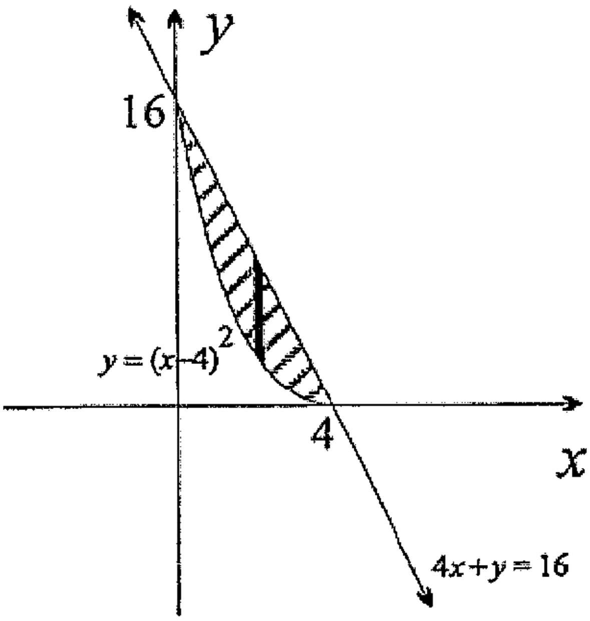
(iii) Hence find the angle that the focal chord through P makes with the

$x-$axis. 1

(iv) Using similar triangles or otherwise show that $RP = e^2 RT$. 3

Examination continues on next page

(b)



The region enclosed by the curve $y=(x-4)^2$ and the line $4x+y=16$ is shaded in the diagram above. A solid is formed with this region as its base. When the solid is sliced perpendicular to the $x-$axis, each cross-section is an equilateral triangle with its base in the $xy-$ plane.

(i) Show that the area of the cross-section x units to the right of the $y-$axis is

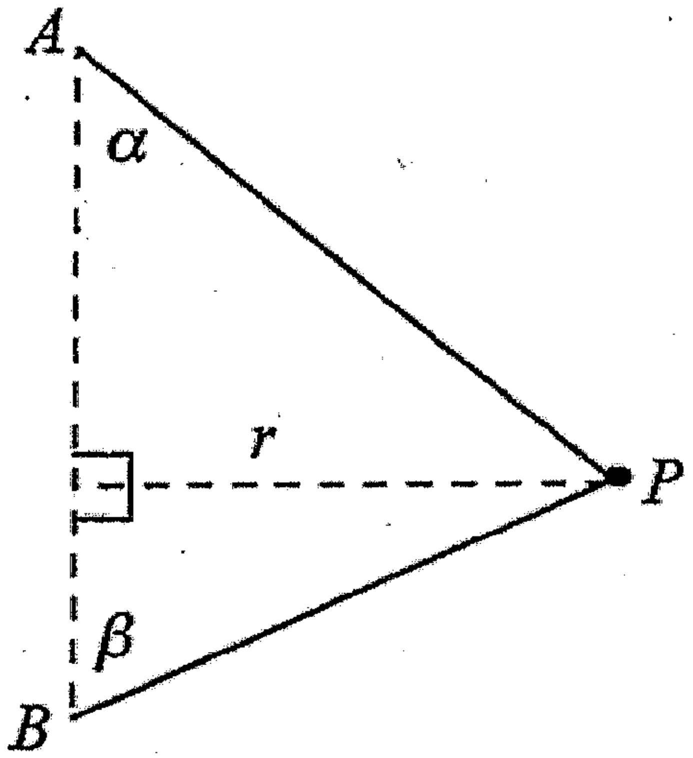
$$\frac{\sqrt{3}x^2(4-x)^2}{4}, \text{ where } 0 \le x \le 4.$$ 2

(ii) Hence find the volume of the solid. 3

Examination continues on next page

Question 15 (15 marks)

(a) A and B are two fixed points with B vertically below A. P is a particle with mass m kg. Two strings with ends fixed at A and B are fastened to P. Particle P moves in a horizontal circle of radius r metres with a constant angular velocity ω radians per second so that both strings remain taut, making angles α, β respectively with vertical. The tension in the string AP and BP are T_1 Newtons and T_2 Newtons respectively. The acceleration due to gravity is gm/s^2.



(i) Draw a diagram showing the forces acting on the particle P. 1

(ii) By resolving the forces, find an expression for the tension in each part of the string in terms of m, r, ω^2 and $\sin(\alpha + \beta)$. 5

(iii) Show that $w > \sqrt{\dfrac{g \tan \alpha}{r}}$ for both strings to be taut. 1

Examination continues on next page

(b) A body of mass m in falling from rest, experiences air resistance of magnitude kv^2 per unit mass, where k is a positive constant.

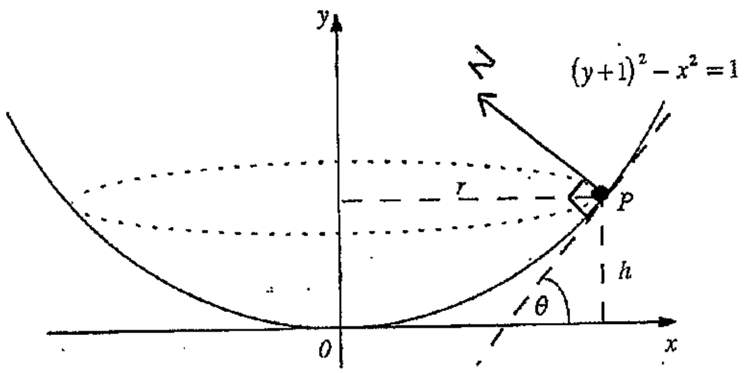
(i) Write the equation of motion of the body and find the value of the terminal velocity V of the body in terms of k and g (acceleration due to gravity) [Take $g = 9.8 m/s^2$] 2

(ii) If w is the velocity of the body when it reaches the ground, show that the distance S fallen is given by $S = -\frac{1}{2k}\ln\left(1 - \frac{w^2}{V^2}\right)$. 3

(iii) With air resistance remaining the same, prove that if the body is projected vertically upwards from the ground with velocity U, then it will attain its greatest height H where $H = \frac{1}{2k}\ln\left(1 + \frac{U^2}{V^2}\right)$, and return to the ground with velocity w given by $w^{-2} = U^{-2} + V^{-2}$. 3

Examination continues on next page

Question 16 (15 marks)



(a) A smooth bowl is formed by rotating the hyperbola $(y+1)^2-x^2=1$ around the $y-axis$. A particle P of mass m kg travels around the inside of the bowl with constant angular velocity ω radians per second in a horizontal circle of radius r metres at a height h metres above the bottom of the bowl.

(i) Show that if the tangent to the hyperbola $(y+1)^2-x^2=1$ at the point (x_1, y_1) makes an angle θ with the positive $x-$axis, then $\tan\theta=\dfrac{x_1}{1+y_1}$. 2

(ii) Copy the diagram and write the equations of motion by resolving forces on P. 2

(iii) Show that $\omega^2=\dfrac{g}{1+h}$. 2

(iv) Show that the force N Newtons exerted by the particle P on the bowl is given by $N=mg\sqrt{2-\dfrac{1}{(1+h)^2}}$. 2

(v) If the linear speed of the particle is $\sqrt{\dfrac{3g}{2}}\ m/s$, find h and the force exerted by the particle on the bowl. 2

Examination continues on next page

(b) **(i)** A truck of mass 5 tonne is travelling around a circular section of the highway which is banked at an angle $\theta = \tan^{-1}\left(\frac{1}{5}\right)$. The radius of the curve is 450 m. Find the speed of the truck, so that there is no lateral friction on the tyres. (Take $g = 10m/s^2$) 2

(ii) Find the sideways frictional force between the tyres and the road for the truck of mass 5 tonne, if the speed of the truck is $126\ km/h$. 3

END OF EXAMINATION

Extension 2 Trial HSC 2014 - solutions

Section I

1. (A)

2. $\frac{1+2i}{3-i} \times \frac{3+i}{3+i}$

$= \frac{3+i+6i+2i^2}{3^2-i^2}$

$= \frac{3+7i-2}{9+1}$

$= \frac{1+7i}{10}$ (D)

3. (B)

4. $\frac{x^2}{16}+\frac{y^2}{36}=1$

$e=\sqrt{1-\frac{a^2}{b^2}}$

$=\sqrt{1-\frac{16}{36}}$

$=\sqrt{\frac{20}{36}}$

$=\frac{2\sqrt5}{6}$ (C)

5. Foci are $(h\pm ae, k)$

$=(-2\pm 3\times\frac{5}{3}, 1)$ (C)

$=(3,1)$ and $(-7,1)$

6. $\int_0^1 x(1-x)^{99}\,dx$

$= x\times\frac{(1-x)^{100}}{-100}\Big]_0^1 - \int_0^1 1\times\frac{(1-x)^{100}}{-100}\,dx$

$=0+\frac{1}{100}\int_0^1 (1-x)^{100}\,dx$

$=\frac{1}{100}\left[\frac{(1-x)^{101}}{-101}\right]_0^1$

$=\frac{-1}{100}\times\frac{1}{101}\left[(1-x)^{101}\right]_0^1$

$=\frac{-1}{100\times101}(0-1)=\frac{1}{10100}$ (B)

7. (A)

8. $R=2 \quad r=2-x$

$R+r=4-x \; ; \; R-r=x$

Area $=\pi(R+r)(R-r)$

$=\pi(4-x)\times x$

$=\pi(4x-x^2)$

$y=x^3$

$x=y^{\frac{1}{3}}$

$x^2=y^{\frac{2}{3}}$

$\Delta V=\pi(4x-x^2)\Delta y$

$V=\pi\int_0^8\left(4y^{\frac{1}{3}}-y^{\frac{2}{3}}\right)dy$

$=\pi\left[\frac{4y^{\frac{4}{3}}}{\frac{4}{3}}-\frac{y^{\frac{5}{3}}}{\frac{5}{3}}\right]_0^8$

$$= \pi \left[4y^{\frac{4}{3}} \times \frac{3}{4} - y^{\frac{5}{3}} \times \frac{3}{5} \right]_0^8$$

$$= \pi \left[3y^{\frac{4}{3}} - \frac{3}{5} y^{\frac{5}{3}} \right]_0^8$$

$$= \pi \left\{ \left(3 \times 8^{\frac{4}{3}} - \frac{3}{5} \times 8^{\frac{5}{3}} \right) - 0 \right\}$$

$$= \pi \left(3 \times 16 - \frac{3}{5} \times 32 \right)$$

$$= \frac{144\pi}{5} \quad B$$

$$2a + b = -22$$
$$4a + b = -32$$
$$2a = -10 \quad a = -5$$

$$b = -22 - 2a$$
$$= -22 + 10 = -12$$

B

9. $\tan\theta = \frac{V^2}{rg}$

$V = 100 \text{km/h}$

$= \frac{1000}{36}$ m/s

$\tan\theta = \frac{1000 \times 1000}{36 \times 36 \times 200 \times 9.8}$

$\theta = 21.49°$ D

10. $p(2) = 0$ and $p'(2) = 0$

$p(2) = 16 + 4a + 2b + 28$

$4a + 2b = -44$

$2a + b = -22$

$p'(x) = 4x^3 + 2ax + b$

$p'(2) = 4 \times 8 + 4a + b$

$= 32 + 4a + b$

1 A	6 B
2 D	7 A
3 B	8 B
4 C	9 D
5 C	10 B

page 3

Question 11

(a) $\displaystyle\int_0^{\ln 2} \frac{e^x dx}{1+e^x}$

Let $u = 1+e^x$

$\frac{du}{dx} = e^x$

$e^x dx = du$

when $x=0$, $u = 1+e^0 = 2$

when $x = \ln 2$, $u = 1+e^{\ln 2}$
$= 3$

$I = \displaystyle\int_2^3 \frac{du}{u} = \left[\log u\right]_2^3$

$= \log 3 - \log 2$

$= \log \frac{3}{2}$

(b) $\displaystyle\int \frac{dx}{3x^2+6x+10}$

$3x^2+6x+10$

$= 3\left(x^2+2x+\frac{10}{3}\right)$

$= 3\left(x^2+2x+1-1+\frac{10}{3}\right)$

$= 3\left((x+1)^2 + \frac{10-3}{3}\right)$

$= 3\left((x+1)^2 + \frac{7}{3}\right)$

$\displaystyle\int \frac{dx}{3x^2+6x+10} = \int \frac{dx}{3\left((x+1)^2+\frac{7}{3}\right)}$

$= \frac{1}{3}\displaystyle\int \frac{dx}{(x+1)^2 + \left(\frac{\sqrt{7}}{\sqrt{3}}\right)^2}$

$= \frac{1}{3} \times \frac{1}{\frac{\sqrt{7}}{\sqrt{3}}} \tan^{-1}\left(\frac{x+1}{\frac{\sqrt{7}}{\sqrt{3}}}\right) + C$

$= \frac{1}{3} \times \frac{\sqrt{3}}{\sqrt{7}} \tan^{-1}\left(\frac{\sqrt{3}(x+1)}{\sqrt{7}}\right)$

$= \frac{1}{\sqrt{21}} \tan^{-1}\left(\frac{\sqrt{3}(x+1)}{\sqrt{7}}\right)$

(c) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{dx}{5-4\cos x}$

$t = \tan\frac{x}{2}$; $\frac{dt}{dx} = \left(\sec^2\frac{x}{2}\right)\frac{1}{2}$

$= \frac{1}{2}\sec^2\frac{x}{2}$

$dt = \frac{1}{2}\sec^2\frac{x}{2}\, dx$

$dt = \frac{1}{2}\left(1+\tan^2\frac{x}{2}\right) dx$

$dt = \frac{1}{2}(1+t^2)\, dx$

$dx = \frac{2\,dt}{1+t^2}$

when $x=0$, $t=\tan\frac{0}{2}=0$

when $x=\frac{\pi}{3}$, $t=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

$$5-4\cos x = 5-4\left(\frac{1-t^2}{1+t^2}\right)$$

$$=\frac{5(1+t^2)-4(1-t^2)}{1+t^2}$$

$$=\frac{5+5t^2-4+4t^2}{1+t^2}$$

$$=\frac{1+9t^2}{1+t^2}$$

$$I=\int_0^{\frac{1}{\sqrt{3}}}\frac{1+t^2}{1+9t^2}\times\frac{2\,dt}{1+t^2}$$

$$=2\int_0^{\frac{1}{\sqrt{3}}}\frac{dt}{9(t^2+\frac{1}{9})}$$

$$=\frac{2}{9}\int_0^{\frac{1}{\sqrt{3}}}\frac{dt}{t^2+\left(\frac{1}{3}\right)^2}$$

$$=\left[\frac{2}{9}\times\frac{1}{\frac{1}{3}}\tan^{-1}\left(\frac{t}{\frac{1}{3}}\right)\right]_0^{\frac{1}{\sqrt{3}}}$$

$$=\frac{2}{9}\times 3\left[\tan^{-1}3t\right]_0^{\frac{1}{\sqrt{3}}}$$

$$=\frac{2}{3}\left(\tan^{-1}3\times\frac{1}{\sqrt{3}}-\tan^{-1}0\right)$$

$$=\frac{2}{3}\left(\tan^{-1}\sqrt{3}-\tan^{-1}(0)\right)$$

$$=\frac{2}{3}\left(\frac{\pi}{3}-0\right)=\underline{\underline{\frac{2\pi}{9}}}$$

(d) $$I_n=\int_{-1}^{0}x^n\sqrt{1+x}\,dx$$

$$=\frac{2}{3}\left[x^n(1+x)^{\frac{3}{2}}\right]_{-1}^{0}-\frac{2}{3}\int_{-1}^{0}nx^{n-1}(1+x)\sqrt{1+x}\,dx$$

$$=0-\frac{2n}{3}\left\{\int_{-1}^{0}x^{n-1}\sqrt{1+x}\,dx+\int_{-1}^{0}x^n\sqrt{1+x}\,dx\right\}$$

$$I_n=\frac{-2n}{3}I_{n-1}-\frac{2n}{3}I_n$$

$$\left(1+\frac{2n}{3}\right)I_n=\frac{-2n}{3}I_{n-1}$$

$$(3+2n)I_n=-2n\,I_{n-1}$$

$$\underline{\underline{I_n=\frac{-2n}{2n+3}I_{n-1}}}$$

(ii) $\int_{-1}^{0} \sqrt{1+x}\, dx$

$= \frac{2}{3}\left[(1+x)^{\frac{3}{2}}\right]_{-1}^{0}$

$= \frac{2}{3}(1-0) = \frac{2}{3}$

$I_2 = \frac{-4}{7} I_1$

$= \left(\frac{-4}{7}\right)\left(\frac{-2}{5}\right) I_0$

$= \frac{-4}{7} \times \frac{-2}{5} \times \frac{2}{3}$

$= \frac{16}{105}$

(e) Let $I = \int_{0}^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$ ——— ①

Also

$I = \int_{0}^{\frac{\pi}{2}} \frac{\sqrt{\sin(\frac{\pi}{2} - x)}}{\sqrt{\sin(\frac{\pi}{2} - x)} + \sqrt{\cos(\frac{\pi}{2} - x)}}\, dx \quad \left(\because \int_{0}^{a} f(x)\, dx = \int_{0}^{a} f(a-x)\, dx\right)$

$= \int_{0}^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}$ ——— ②

$\left(\because \begin{array}{l} \sin(90-\theta) = \cos\theta \\ \cos(90-\theta) = \sin\theta \end{array}\right)$

① + ② gives

$2I = \int_{0}^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx = \int_{0}^{\frac{\pi}{2}} dx = \frac{\pi}{2}$

Question 12

(a) (i) $z = \sqrt{3}+i$, $w = 1-i$

$$\frac{z}{w} = \frac{\sqrt{3}+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(\sqrt{3}+i)(1+i)}{(1-i)(1+i)}$$

$$= \frac{\sqrt{3}+\sqrt{3}i+i+i^2}{1-i^2}$$

$$= \frac{\sqrt{3}-1+i(\sqrt{3}+1)}{2}$$

$$= \frac{\sqrt{3}-1}{2} + \frac{i(\sqrt{3}+1)}{2}$$

(ii) $z = \sqrt{3}+i$

$|z| = \sqrt{3+1} = 2$

$\tan\alpha = \frac{1}{\sqrt{3}}$ $\quad \alpha = \frac{\pi}{6}$

$\theta = \frac{\pi}{6}$

$z = 2\,\text{cis}\,\frac{\pi}{6}$

$w = 1-i$

$|w| = \sqrt{1+1} = \sqrt{2}$

$\tan\alpha = 1$ $\quad \alpha = \frac{\pi}{4}$

$\theta = -\frac{\pi}{4}$

$w = \sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)$

$$\frac{z}{w} = \frac{2\,\text{cis}\,\frac{\pi}{6}}{\sqrt{2}\,\text{cis}\left(-\frac{\pi}{4}\right)}$$

$$= \frac{2}{\sqrt{2}}\,\text{cis}\left(\frac{\pi}{6}+\frac{\pi}{4}\right)$$

$$= \sqrt{2}\,\text{cis}\,\frac{5\pi}{12}$$

(iii) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

$$= \frac{\sqrt{3}-1}{2} + \frac{i(\sqrt{3}+1)}{2}$$

Equating real and imaginary parts we get

$$\cos\frac{5\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

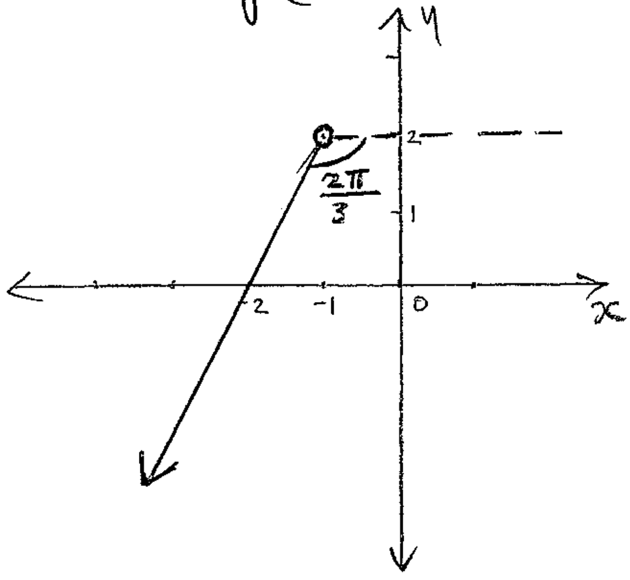
$$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

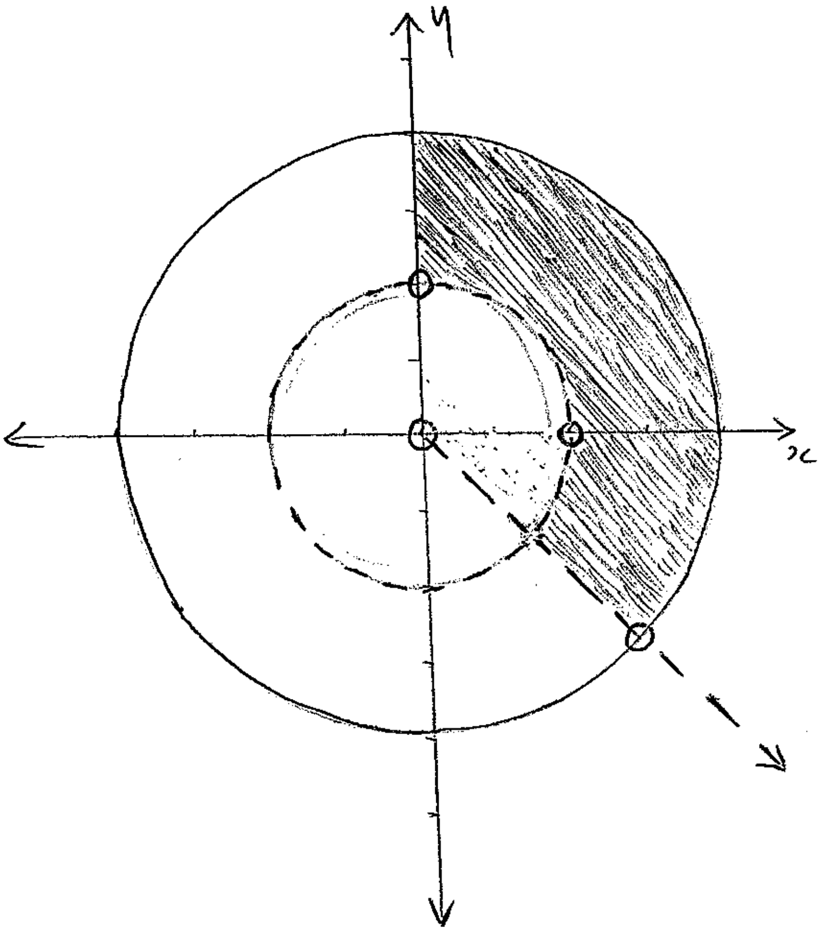
$$\sin\frac{5\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

(b) (i) $\arg\left(z-(-1+2i)\right) = -\frac{2\pi}{3}$



(ii) $2 < |z| \le 4$ and $\frac{-\pi}{4} < \arg z \le \frac{\pi}{2}$



(c) $\sqrt{3}+i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$$(\sqrt{3}+i)^n = 2^n\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^n$$

$$= 2^n\left(\cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6}\right)$$ by De Moivre's theorem

$$\sqrt{3}-i = 2\left(\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6}\right)$$

$$= 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$$

$\left(\because \sin(-\theta) = \sin\theta,\ \cos(-\theta) = \cos\theta\right)$

$$(\sqrt{3}-i)^n = 2^n\left(\cos\frac{n\pi}{6} - i\sin\frac{n\pi}{6}\right)$$

(by De Moivre's theorem)

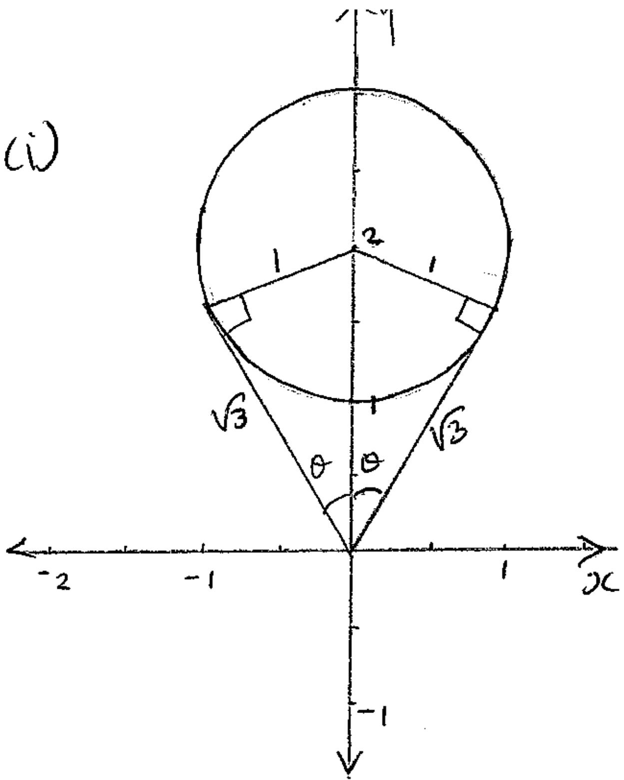
$$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n$$

$$= 2^n\left(\cos\frac{n\pi}{6} + i\sin\frac{n\pi}{6} + \cos\frac{n\pi}{6} - i\sin\frac{n\pi}{6}\right)$$

$$= 2^n \times 2\cos\frac{n\pi}{6}$$

$$= 2^{n+1}\cos\frac{n\pi}{6}$$

(d)(i)



(ii) $\tan\theta = \frac{1}{\sqrt{3}}$ $\quad \theta = \frac{\pi}{6}$

minimum $\arg(z) = \frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{\pi}{3}$

Maximum $\arg(z) = \frac{\pi}{2} + \frac{\pi}{6}$

$= \frac{2\pi}{3}$

(iii) $z = \sqrt{3}\,\text{cis}\,\frac{2\pi}{3}$

<u>Question 13</u>

(a) $z^3 + pz + q = 0$

$(2-3i)^3 + (2-3i)p + q = 0$

$(2-3i)^3 = (2-3i)^2(2-3i)$

$= (-5-12i)(2-3i)$

$= -10 + 15i - 24i + 36i^2$

$= -10 - 9i - 36$

$= -46 - 9i$

$-46 - 9i + (2-3i)p + q = 0$

$-46 - 9i + 2p - i3p + q = 0$

$-46 + 2p + q - 9i - 3pi = 0$

Equating real and imaginary parts

$-46 + 2p + q = 0$

$2p + q = 46$ — ①

$-9 - 3p = 0$ — ②

From ② $\quad 3p = -9$

$p = -3$

substitute ② in ①

$q = 46 - 2p$

$= 46 - 2 \times -3$

$= 52$

$\underline{\underline{p = -3,\ q = 52}}$

(b) $x^3 + 6x + 1 = 0$

$\alpha\beta\gamma = -1$

$\alpha\beta = \frac{-1}{\gamma}$

$\beta\gamma = \frac{-1}{\alpha}$

$\alpha\gamma = \frac{-1}{\beta}$

Let $y = \frac{-1}{x} \Rightarrow x = \frac{-1}{y}$

Required polynomial is

$$\left(\frac{-1}{y}\right)^3 + 6\left(\frac{-1}{y}\right) + 1 = 0$$

$$-1 - 6y^2 + y^3 = 0$$

$$y^3 - 6y^2 - 1 = 0$$

$$\underline{\underline{x^3 - 6x^2 - 1 = 0}}$$

(c) $p(x) = x^4 - 5x^3 - 9x^2 + 81x - 108$

$p'(x) = 4x^3 - 15x^2 - 18x + 81$

$p''(x) = 12x^2 - 30x - 18$

$p''(x) = 0 \Rightarrow 2x^2 - 5x - 3 = 0$

$x = 3, \frac{-1}{2}$

$p(3) = 3^4 - 5 \times 3^3 - 9 \times 3^2 + 81 \times 3 - 108$
$= 0$

$\therefore$ 3 is a root of multiplicity 3.

Let β be the fourth root

$27\beta = -108$

$\beta = \frac{-108}{27} = -4$

$$\underline{\underline{p(x) = (x-3)^3(x+4)}}$$

(d) (i) $z^8 = 1 = \text{cis}\,\pi$

$= \text{cis}(\pi + 2k\pi) \quad k \in J$

$z = \left[\text{cis}\,(2k+1)\pi\right]^{\frac{1}{8}}$

$= \text{cis}\frac{(2k+1)\pi}{8}, \quad k = 0, 1, 2, 3 \cdots 7$

$k=0, \; z_1 = \text{cis}\frac{\pi}{8}$

$k=1, \; z_2 = \text{cis}\frac{3\pi}{8}$

$k=2, \; z_3 = \text{cis}\frac{5\pi}{8}$

$k=3, \; z_4 = \text{cis}\frac{7\pi}{8}$

$k=4, \; z_5 = \text{cis}\frac{9\pi}{8} = \text{cis}\frac{-7\pi}{8}$

$k=5, \; z_6 = \text{cis}\frac{11\pi}{8} = \text{cis}\frac{-5\pi}{8}$

$k=6, \; z_7 = \text{cis}\frac{13\pi}{8}$

$= \text{cis}\frac{-3\pi}{8}$

$k=7, \; z_8 = \text{cis}\frac{15\pi}{8} = \text{cis}\frac{-\pi}{8}$

$$z^8+1=\left(z-\operatorname{cis}\frac{\pi}{8}\right)\left(z-\operatorname{cis}\frac{-\pi}{8}\right)\left(z-\operatorname{cis}\frac{3\pi}{8}\right)\left(z-\operatorname{cis}\frac{-3\pi}{8}\right)$$
$$\left(z-\operatorname{cis}\frac{5\pi}{8}\right)\left(z-\operatorname{cis}\frac{-5\pi}{8}\right)\left(z-\operatorname{cis}\frac{7\pi}{8}\right)\left(z-\operatorname{cis}\frac{-7\pi}{8}\right)$$

$$\left(z-\operatorname{cis}\frac{\pi}{8}\right)\left(z-\operatorname{cis}\frac{-\pi}{8}\right)=\left(z-\cos\frac{\pi}{8}-i\sin\frac{\pi}{8}\right)\left(z-\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)$$
$$=\left(z-\cos\frac{\pi}{8}\right)^2-\left(i\sin\frac{\pi}{8}\right)^2$$
$$=z^2-2\cos\frac{\pi}{8}z+\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}$$
$$=z^2-2\cos\frac{\pi}{8}z+1$$

Similarly $\left(z-\operatorname{cis}\frac{3\pi}{8}\right)\left(z-\operatorname{cis}\frac{-3\pi}{8}\right)=z^2-2\cos\frac{3\pi}{8}z+1$

$$\left(z-\operatorname{cis}\frac{5\pi}{8}\right)\left(z-\operatorname{cis}\frac{-5\pi}{8}\right)=z^2-2\cos\frac{5\pi}{8}z+1$$

$$\left(z-\operatorname{cis}\frac{7\pi}{8}\right)\left(z-\operatorname{cis}\frac{-7\pi}{8}\right)=z^2-2\cos\frac{7\pi}{8}z+1$$

$$z^8+1=\left(z^2-2\cos\frac{\pi}{8}z+1\right)\left(z^2-2\cos\frac{3\pi}{8}z+1\right)\left(z^2-2\cos\frac{5\pi}{8}z+1\right)$$
$$\left(z^2-2\cos\frac{7\pi}{8}z+1\right)$$

$\div$ by z^4

$$z^4+\frac{1}{z^4}=\left(z-2\cos\frac{\pi}{8}+\frac{1}{z}\right)\left(z-2\cos\frac{3\pi}{8}+\frac{1}{z}\right)\left(z-2\cos\frac{5\pi}{8}+\frac{1}{z}\right)$$
$$\left(z-2\cos\frac{7\pi}{8}+\frac{1}{z}\right)$$

$$2\cos4\theta=\left(2\cos\theta-2\cos\frac{\pi}{8}\right)\left(2\cos\theta-2\cos\frac{3\pi}{8}\right)$$
$$\left(2\cos\theta-2\cos\frac{5\pi}{8}\right)\left(2\cos\theta-2\cos\frac{7\pi}{8}\right)$$

$$2\cos 4\theta = 2^4\left(\cos\theta - \cos\frac{\pi}{8}\right)\left(\cos\theta - \cos\frac{3\pi}{8}\right)\left(\cos\theta - \cos\frac{5\pi}{8}\right)\left(\cos\theta - \cos\frac{7\pi}{8}\right)$$

divide by 2

$$\cos 4\theta = 8\left(\cos\theta - \cos\frac{\pi}{8}\right)\left(\cos\theta - \cos\frac{3\pi}{8}\right)\left(\cos\theta - \cos\frac{5\pi}{8}\right)\left(\cos\theta - \cos\frac{7\pi}{8}\right)$$

$$\cos\frac{5\pi}{8} = \cos\left(\pi - \frac{3\pi}{8}\right) = -\cos\frac{3\pi}{8}$$

$$\cos\frac{7\pi}{8} = \cos\left(\pi - \frac{\pi}{8}\right) = -\cos\frac{\pi}{8}$$

$$\therefore \cos 4\theta = 8\left(\cos\theta - \cos\frac{\pi}{8}\right)\left(\cos\theta - \cos\frac{3\pi}{8}\right)\left(\cos\theta + \cos\frac{3\pi}{8}\right)\left(\cos\theta + \cos\frac{\pi}{8}\right)$$

$$= 8\left(\cos^2\theta - \cos^2\frac{\pi}{8}\right)\left(\cos^2\theta - \cos^2\frac{3\pi}{8}\right)$$

Question 14

(a) (i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-b^2x}{a^2y}$$

$$\frac{dy}{dx} \text{ at } P = \frac{-b^2 a\cos\alpha}{a^2 b\sin\alpha}$$

$$= \frac{-b\cos\alpha}{a\sin\alpha}$$

Equation of tangent

$$y - b\sin\theta = \frac{-b\cos\alpha}{a\sin\alpha}(x - a\cos\theta)$$

$$a\sin\theta\, y - ab\sin^2\theta = -b\cos\theta\, x + ab\cos^2\theta$$

$$b\cos\theta\, x + a\sin\theta\, y = ab(\sin^2\theta + \cos^2\theta)$$

$\div$ by ab

$$\frac{x\cos\alpha}{a} + \frac{y\sin\alpha}{b} = 1$$

(ii) At T, $y = 0$ and T is a point on the tangent

Substitute $y = 0$ in $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

$\frac{x\cos\theta}{a} = 1 \quad \therefore x = \frac{a}{\cos\theta}$

$T\left(\frac{a}{\cos\theta}, 0\right)$

Equation of directrix is: $x = \frac{a}{e}$

$\frac{a}{\cos\theta} = \frac{a}{e} \quad \therefore \underline{\underline{\cos\theta = e}}$

(iii) Since $\cos\theta = e$, the x coordinate of P is ae.

Coordinates of focus $= (ae, 0)$

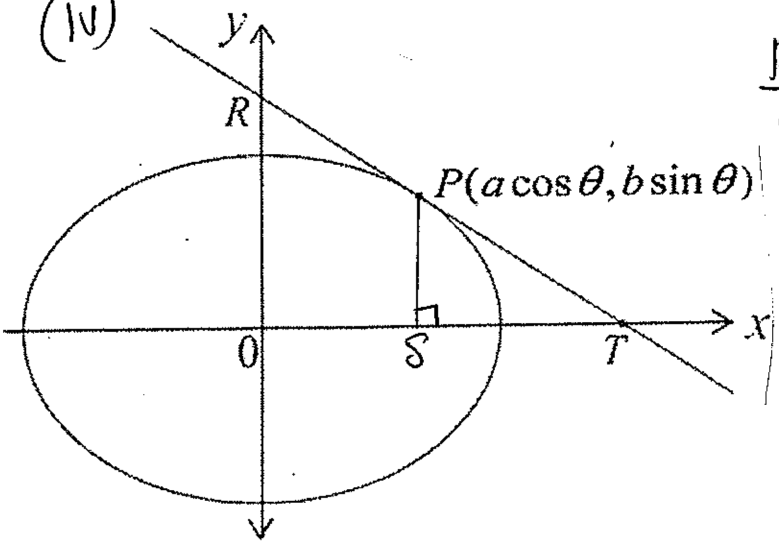
Point P and focus S are on the vertical line

$x = ae$.

$\therefore$ the focal chord makes an angle of $90°$ with the x axis

$\Delta TOR ||| \Delta TSP$ (equiangular)

(iv)



$\frac{RT}{OT} = \frac{RP}{OS}$ (ratio of intercepts)

$\frac{RT}{\frac{a}{e}} = \frac{RP}{ae}$

$\frac{RT \times e}{a} = \frac{RP}{ae}$

$RP = \frac{RT \times e \times ae}{a}$

$\underline{\underline{RP = RT \times e^2}}$

page 13

(b) (i) $y = 16 - 4x$

$y = (x-4)^2$

Side of equilateral Δ

$= 16 - 4x - (x-4)^2$

$= 16 - 4x - (x^2 - 8x + 16)$

$= 16 - 4x - x^2 + 8x - 16$

$= 4x - x^2$

Area of equilateral Δ

$= \frac{1}{2}(4x - x^2)^2 \times \sin 60$

$= \frac{1}{2} x^2 (4-x)^2 \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4} x^2 (4-x)^2$

(ii) $\Delta V = \frac{\sqrt{3}}{4} x^2 (4-x)^2 \Delta x$

$V = \lim_{\Delta x \to 0} \sum_{x=0}^{4} \frac{\sqrt{3}}{4} x^2 (4-x)^2 \Delta x$

$= \frac{\sqrt{3}}{4} \int_0^4 x^2 (4-x)^2 \, dx$

$= \frac{\sqrt{3}}{4} \int_0^4 x^2 (16 - 8x + x^2) \, dx$

$= \frac{\sqrt{3}}{4} \int_0^4 (16x^2 - 8x^3 + x^4) \, dx$

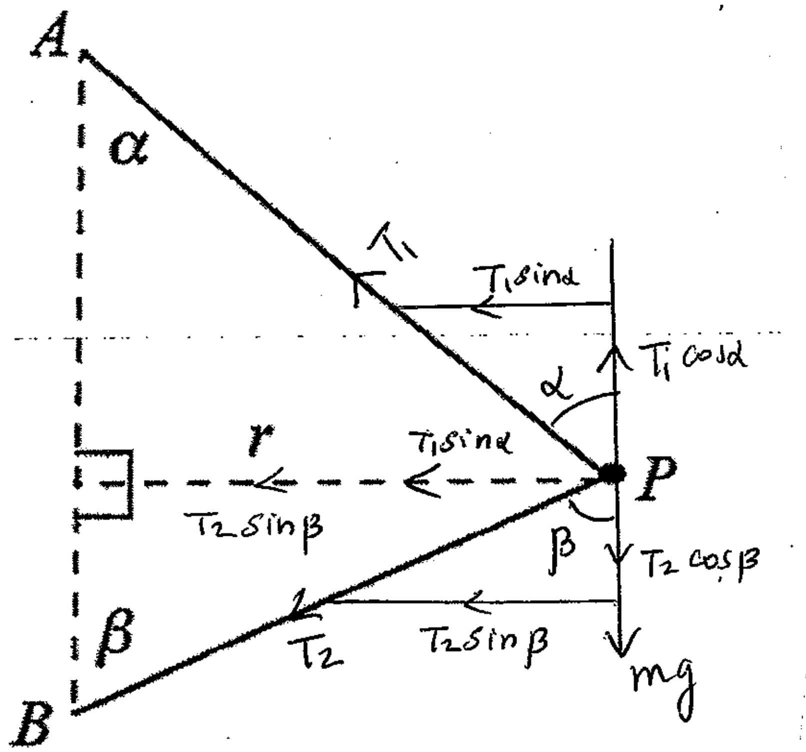
$= \frac{\sqrt{3}}{4} \left[\frac{16x^3}{3} - \frac{8x^4}{4} + \frac{x^5}{5} \right]_0^4$

$= \frac{\sqrt{3}}{4} \left\{ \frac{16 \times 64}{3} - \frac{8 \times 256}{4} + \frac{1024}{5} \right) - 0$

$= \underline{\underline{14 \cdot 78 \ U^3}}$

Question 15

(i)



(ii) $T_1 \cos\alpha - T_2 \cos\beta = mg$ ——(1) (since there is no vertical motion)

$T_1 \sin\alpha + T_2 \sin\beta = m r\omega^2$ ——(2) (centripetal force is provided by $T_1 \sin\alpha$ and $T_2 \sin\beta$)

(1) × $\sin\beta$

$T_1 \cos\alpha \sin\beta - T_2 \cos\beta \sin\beta = mg \sin\beta$ ——(3)

(2) × $\cos\beta$

$T_1 \sin\alpha \cos\beta + T_2 \sin\beta \cos\beta = m r\omega^2 \cos\beta$ ——(4)

(3)+(4) $T_1(\sin\alpha \cos\beta + \cos\alpha \sin\beta) = m(r\omega^2 \cos\beta + g\sin\beta)$

$T_1 \times \sin(\alpha+\beta) = m(r\omega^2 \cos\beta + g\sin\beta)$

$$T_1 = \frac{m(r\omega^2 \cos\beta + g\sin\beta)}{\sin(\alpha+\beta)}$$

① $\times \sin\alpha$

$$T_1 \cos\alpha \sin\alpha - T_2 \cos\beta \sin\alpha = mg\sin\alpha \quad \text{—— (5)}$$

② $\times \cos\alpha$

$$T_1 \sin\alpha \cos\alpha + T_2 \sin\beta \cos\alpha = m r w^2 \cos\alpha \quad \text{—— (6)}$$

(6) − (5)

$$T_2 (\sin\beta \cos\alpha + \sin\alpha \cos\beta) = m(rw^2\cos\alpha - g\sin\alpha)$$

$$T_2 \sin(\alpha+\beta) = m(rw^2\cos\alpha - g\sin\alpha)$$

$$T_2 = \frac{m(rw^2\cos\alpha - g\sin\alpha)}{\sin(\alpha+\beta)}$$

$T_1 > 0$ since $m, r, \omega, \cos\beta, \sin\beta, g > 0$

$$T_2 > 0 \Longrightarrow rw^2\cos\alpha - g\sin\alpha > 0$$

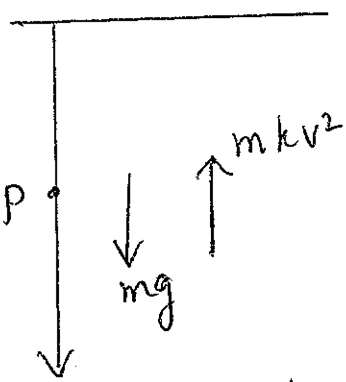
$$rw^2\cos\alpha > g\sin\alpha$$

$$\omega^2 > \frac{g\sin\alpha}{r\cos\alpha}$$

$$\omega^2 > \frac{g\tan\alpha}{r}$$

$$\omega > \sqrt{\frac{g\tan\alpha}{r}}$$

(b) (i)



Equation of motion is

$$m\ddot{x} = mg - mkv^2$$

$$\ddot{x} = g - kv^2$$

The terminal velocity occurs when the acceleration of the particle becomes zero.

$$g - kV^2 = 0$$

$$g = kV^2$$

$$V^2 = \frac{g}{k}$$

$$\underline{\underline{V = \sqrt{\frac{g}{k}}}}$$

(ii) $\ddot{x} = g - kv^2$

substitute $\ddot{x} = v\frac{dv}{dx}$

$$v\frac{dv}{dx} = g - kv^2$$

$$v\,dv = (g - kv^2)\,dx$$

$$\frac{v\,dv}{g - kv^2} = dx$$

$$\int_0^S dx = \int_0^w \frac{v\,dv}{g - kv^2}$$

$$[x]_0^S = \int_0^w \frac{-2kv\,dv}{-2k(g - kv^2)}$$

$$S = \frac{-1}{2k}\int_0^w \frac{-2kv\,dv}{g - kv^2}$$

$$= \frac{-1}{2k}\left[\log(g - kv^2)\right]_0^w$$

$$= \frac{-1}{2k}\left(\log(g - kw^2) - \log g\right)$$

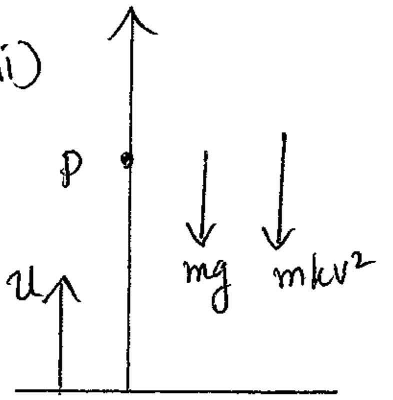
$$= \frac{-1}{2k}\log\left(\frac{g - kw^2}{g}\right)$$

$$= \frac{-1}{2k}\log\left(1 - \frac{kw^2}{g}\right)$$

$$\underline{\underline{= \frac{-1}{2k}\log\left(1 - \frac{w^2}{V^2}\right)}}$$

since $\frac{k}{g} = \frac{1}{V^2}$

(iii)



$$m\ddot{x} = -mg - mkv^2$$

$$\ddot{x} = -g - kv^2$$

$$v\frac{dv}{dx} = -g - kv^2$$

$$v\,dv = -(g+kv^2)\,dx$$

$$\frac{v\,dv}{-(g+kv^2)} = dx$$

$$\int_0^H dx = \int_u^0 \frac{v\,dv}{-(g+kv^2)}$$

$$[x]_0^H = -\int_u^0 \frac{v\,dv}{g+kv^2}$$

$$= \int_0^u \frac{v\,dv}{g+kv^2}$$

$$= \int_0^u \frac{2kv\,dv}{2k(g+kv^2)}$$

$$H = \frac{1}{2k}\int_0^u \frac{2kv\,dv}{g+kv^2}$$

$$= \frac{1}{2k}\Big[\log(g+kv^2)\Big]_0^u$$

$$= \frac{1}{2k}\left(\log(g+ku^2) - \log g\right)$$

$$= \frac{1}{2k}\log\left(\frac{g+ku^2}{g}\right)$$

$$= \frac{1}{2k}\log\left(1+\frac{ku^2}{g}\right)$$

$$= \frac{1}{2k}\log\left(1+\frac{u^2}{V^2}\right) \qquad \because \frac{k}{g} = \frac{1}{V^2}$$

(iii) The distances travelled by the particle in going up and coming down are the same. $\therefore S = H$

$$-\frac{1}{2k}\log\left(1-\frac{w^2}{V^2}\right) = \frac{1}{2k}\ln\left(1+\frac{u^2}{V^2}\right)$$

$$\frac{1}{2k}\log\left(1-\frac{w^2}{v^2}\right)^{-1} = \frac{1}{2k}\log\left(1+\frac{u^2}{v^2}\right)$$

$$\left(1-\frac{w^2}{v^2}\right)^{-1} = 1+\frac{u^2}{v^2}$$

$$\left(\frac{v^2-w^2}{v^2}\right)^{-1} = \frac{v^2+u^2}{v^2}$$

$$\frac{1}{\left(\frac{v^2-w^2}{v^2}\right)} = \frac{v^2+u^2}{v^2}$$

$$\frac{v^2}{v^2-w^2} = \frac{v^2+u^2}{v^2}$$

$$v^4 = (v^2-w^2)(v^2+u^2)$$

$$v^4 = v^4+v^2u^2-w^2v^2-w^2u^2$$

$$0 = v^2u^2-w^2v^2-w^2u^2$$

$\div$ by $u^2v^2w^2$

$$0 = \frac{v^2u^2}{u^2v^2w^2}-\frac{w^2v^2}{u^2v^2w^2}-\frac{w^2u^2}{u^2v^2w^2}$$

$$0 = \frac{1}{w^2}-\frac{1}{u^2}-\frac{1}{v^2}$$

$$0 = w^{-2}-u^{-2}-v^{-2}$$

$$\underline{\underline{w^{-2} = u^{-2}+v^{-2}}}$$

Question 16

(a) (i) $(y+1)^2 - x^2 = 1$

$$2(y+1)\frac{dy}{dx} - 2x = 0$$

$$\frac{dy}{dx} = \frac{2x}{2(y+1)} = \frac{x}{1+y}$$

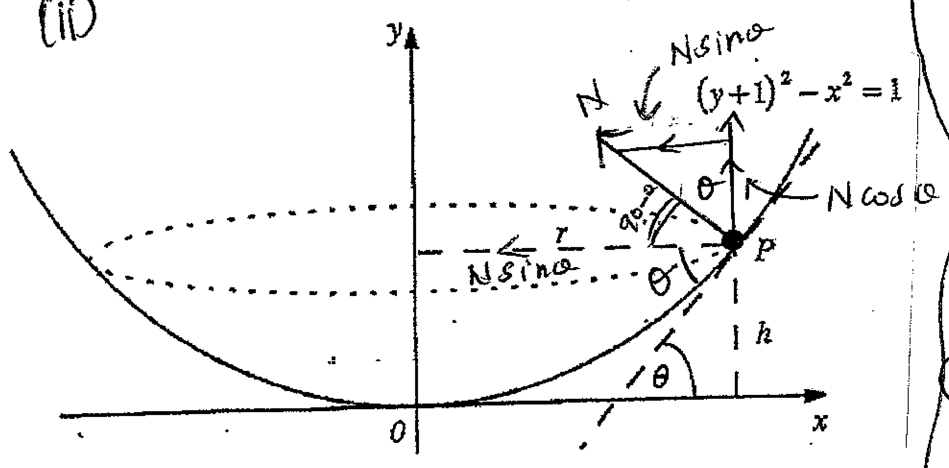
$\frac{dy}{dx}$ at (x_1, y_1)

$= \frac{x_1}{1+y_1}$ = gradient of the tangent at (x_1, y_1)

Also gradient $= \tan\theta$

$$\therefore \tan\theta = \frac{x_1}{1+y_1}$$

(ii)



Vertical component $N\cos\theta$ balances mg and horizontal component $N\sin\theta$ provides

Centripetal force

$$N\cos\theta = mg \quad \text{—— (1)}$$

$$N\sin\theta = mr\omega^2 \quad \text{—— (2)}$$

(2) ÷ (1)

$$\frac{N\sin\theta}{N\cos\theta} = \frac{mr\omega^2}{mg}$$

$$\tan\theta = \frac{r\omega^2}{g}$$

$$\frac{x_1}{1+y_1} = \frac{r\omega^2}{g}$$

$P(r, h)$

$x_1 = r \quad y_1 = h$

$$\frac{r}{1+h} = \frac{r\omega^2}{g}$$

$$\omega^2 = \frac{rg}{r(1+h)} = \frac{g}{1+h}$$

(iv) From (1)

$$N = \frac{mg}{\cos\theta} = mg\sec\theta$$

$$N = mg\sqrt{1+\tan^2\theta}$$

$$N = mg\sqrt{1 + \frac{r^2}{(1+h)^2}}$$

But (r, h) is on the parabola

$$(y+1)^2 - x^2 = 1$$

$$(h+1)^2 - r^2 = 1$$

$$r^2 = (h+1)^2 - 1$$

$$\frac{r^2}{(1+h)^2} = 1 - \frac{1}{(1+h)^2}$$

$$N = mg\sqrt{1 + 1 - \frac{1}{(1+h)^2}}$$

$$= mg\sqrt{2 - \frac{1}{(1+h)^2}}$$

(v) linear speed $V = r\omega$

$$V^2 = r^2\omega^2$$

$$\frac{3g}{2} = r^2 \times \frac{g}{1+h}$$

$$\frac{3}{2} = \frac{r^2}{1+h}$$

$$\frac{3}{2} = \frac{(h+1)^2 - 1}{1+h}$$

$$3(1+h) = 2(1+h)^2 - 2$$

$$2(1+h)^2 - 3(1+h) - 2 = 0$$

This is a quadratic in $1+h$

$-4, 1$ $\quad pq = -4$, $p+q = -3$

$$2(1+h)^2 - 4(1+h) + (1+h) - 2 = 0$$

$$2(1+h)\left[(1+h) - 2\right] + (1+h) - 2 = 0$$

$$\left[(1+h) - 2\right]\left[2(1+h) + 1\right] = 0$$

$$(h-1)(2+2h+1) = 0$$

$$(h-1)(2h+3) = 0$$

$h = 1$ ($\because$ h is positive)

$$N = mg\sqrt{2 - \frac{1}{(1+h)^2}}$$

$$= mg\sqrt{2 - \frac{1}{4}} = mg\sqrt{\frac{8-1}{4}}$$

$$= mg\sqrt{\frac{7}{4}} = \frac{1}{2}mg\sqrt{7}$$

(b) $g = 10 m/s^2$, $r = 450 m$

$\theta = \tan^{-1}\left(\frac{1}{5}\right)$

$\tan\theta = \frac{1}{5}$

design speed

$V = \sqrt{rg\tan\theta}$

$= \sqrt{450 \times 10 \times \frac{1}{5}}$

$= 30 m/s$

$= 108 km/h$

(i) when the vehicle travels with the design speed of the track, frictional force is zero.

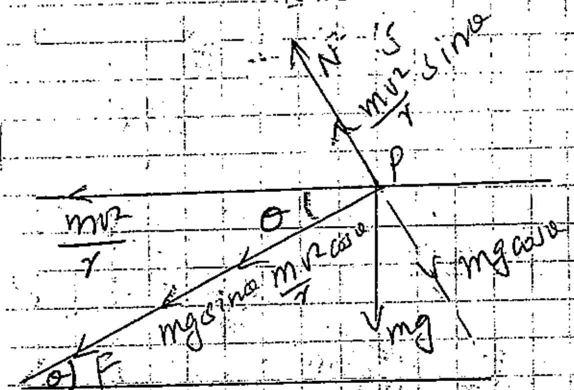
(ii)

126 km/h.

The vehicle is travelling at a speed higher than the design speed.

$126 km/h = \frac{126000}{3600} m/s$

$= 35 m/s$



$\frac{mv^2}{r}\cos\theta = F + mg\sin\theta$

$F = mg\cos\theta\left(\frac{v^2}{rg} - \tan\theta\right)$

$= \frac{5000 \times 10 \times 5}{\sqrt{26}}\left(\frac{35 \times 35}{450 \times 10} - \frac{1}{5}\right)$

$= \frac{250000}{\sqrt{26}}\left(\frac{1225}{4500} - \frac{1}{5}\right)$

$= 3540.98$

$= \underline{\underline{3541 N}}$ down track