Name and Number ....

Trial HSC Mathematics

Extension 1 - 2014

Time Allowed - 2 hours + 5minutes reading

Instructions: Calculators may be used in any parts of the task. For 1 Mark Questions, the correct answer is sufficient to receive full marks. For Questions worth more than 1 Mark, necessary working MUST be shown to receive full marks.

Multiple Choice	/10
Question 11	/15
Question 12	/15
Question 13	/15
Question 14	/15
Total	/70

Student name / number ______________________

Marks

Section I

10 Marks
Attempt Questions 1-10.
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1-10.

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1 What is the acute angle to the nearest degree between the lines $y = 1 - 3x$ and $4x - 6y - 5 = 0$? 1

(A) 15°
(B) 38°
(C) 52°
(D) 75°

2 Which of the following is a solution of the equation $2^x = 5$? 1

(A) $x = \sqrt{5}$
(B) $x = \log_e 5$
(C) $x = \dfrac{\log_e 5}{\log_e 2}$
(D) $x = \dfrac{\log_e 2}{\log_e 5}$

3 What is the equation of the normal to $x = 2at$, $y = at^2$ at the point $t = p$? 1

(A) $x - py = 2ap + ap^2$
(B) $x - py = 2ap + ap^3$
(C) $x + py = 2ap + ap^2$
(D) $x + py = 2ap + ap^3$

4 Which of the following is equivalent to the expression $\sqrt{\dfrac{4 + 4\cos 2x}{1 - \cos 2x}}$? 1

(A) $2\cot^2 x$
(B) $2\cot x$
(C) $2\tan x$
(D) $2\tan^2 x$

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Marks

5 If $f(x)=1-\cos\frac{x}{2}$ what is the inverse function $f^{-1}(x)$? 1

(A) $f^{-1}(x)=2\cos^{-1}(1-x)$

(B) $f^{-1}(x)=\frac{1}{2}\cos^{-1}(1-x)$

(C) $f^{-1}(x)=\frac{1}{2}\cos^{-1}(1+x)$

(D) $f^{-1}(x)=2\cos^{-1}(1+x)$

6 Let α, β and γ be the roots of $x^3-4x+1=0$. 1

What is the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$?

(A) −4

(B) −1

(C) 1

(D) 4

7 It is known that two of the roots of the equation $3x^3+x^2-kx+6=0$ are reciprocals of each other. What is the value of k? 1

(A) −2

(B) 6

(C) 7

(D) 17

8 What is the value of $f'(x)$ if $f(x)=\tan^{-1}x+x\tan^{-1}x$? 1

(A) $\frac{1}{1+x^2}$

(B) $\frac{x+1}{1+x^2}$

(C) $\tan^{-1}x+\frac{1}{1+x^2}$

(D) $\tan^{-1}x+\frac{x+1}{1+x^2}$

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Marks

9 What is the coefficient of x^{-5} in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$? 1

(A) $-{}^{20}C_{16} \times 2^4$

(B) $-{}^{20}C_{16} \times 2^5$

(C) $-{}^{20}C_{15} \times 2^4$

(D) $-{}^{20}C_{15} \times 2^5$

10 What is the domain and range of the function $y = 6\cos^{-1}(3x)$? 1

(A) Domain $-\frac{1}{3} \le x \le \frac{1}{3}$; Range $0 \le y \le 6\pi$.

(B) Domain $-\frac{1}{3} \le x \le \frac{1}{3}$; Range $0 \le y \le 3\pi$.

(C) Domain $0 \le x \le 6\pi$; Range $-\frac{1}{3} \le y \le \frac{1}{3}$.

(D) Domain $0 \le x \le 3\pi$; Range $-\frac{1}{3} \le y \le \frac{1}{3}$.

End of Section I

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Marks

Section II

60 Marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section.

All necessary working should be shown in every question.

Question 11 (15 marks) **Use a separate writing booklet.**

a) (i) Show that $x+1$ is a factor of $P(x) = -x^3 + 3x + 2$ 1

(ii) Fully factorise $P(x)$ 3

(iii) Hence sketch the graph of $y = P(x)$ 2

(b) Solve the inequality $\dfrac{x^2+x-6}{x} \geq 0$. 3

(c)

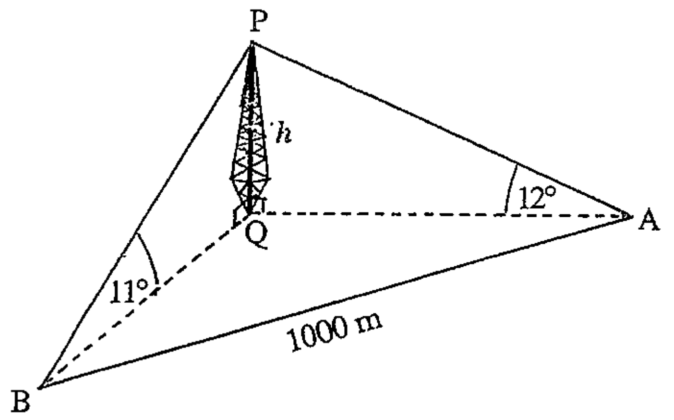

Figure not to scale

The angle of elevation of a tower PQ of height h metres at a point A due east of it is $12°$. From another point B, the bearing of the tower is $051°T$ and the angle of elevation is $11°$. The points A and B are 1000 metres apart and on the same level as the base Q of the tower.

(i) Show that $\angle AQB = 141°$. 1

(ii) Consider the triangle APQ and show that $AQ = h \tan 78°$. 1

(iii) Find a similar expression for BQ. 1

(iv) Use the cosine rule in the triangle AQB to calculate h to the nearest metre. 3

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Marks

Question 12 (15 marks) **Use a separate writing booklet.**

(a) Use the method of Mathematical Induction to prove **4**

$$\sum_{r=1}^{n} \frac{r^2}{(2r-1)(2r+1)} = \frac{n(n+1)}{2(2n+1)} \text{ for all positive integers } n.$$

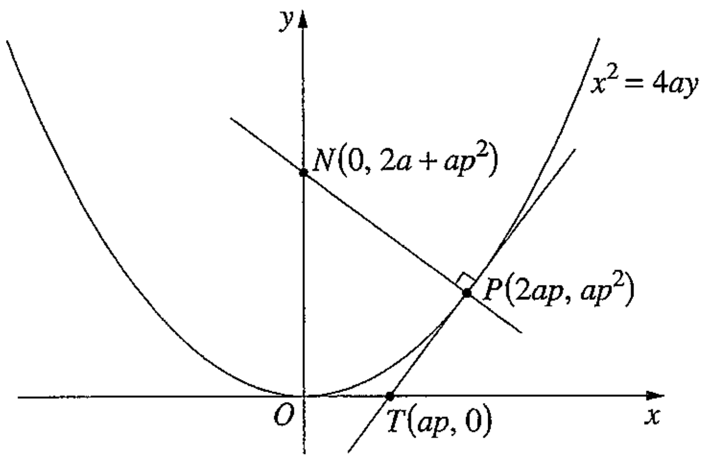
(b) (i) Solve the equation **2**

$$\sqrt{3}\cos x - \sin x = 1 \text{ for } 0 \le x \le 2\pi.$$

(ii) What is the general solution of the equation? **2**

(c) Find the volume generated when $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{3}$ is rotated around the x axis. **3**

(d) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$. The tangent to the parabola at P meets the x-axis at $T(ap, 0)$. The normal to the tangent at P meets the y-axis at $N(0,\ 2a + ap^2)$.



The point G divides NT externally in the ratio $2:1$.

(i) Show that the coordinates of G are $(2ap, -2a - ap^2)$. **2**

(ii) Show that G lies on a parabola with the same directrix and focal length as the original parabola. **2**

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Marks

Question 13 (15 marks) **Use a separate writing booklet.**

(a) Greg designs a sky-diving simulator for a video game. He simulates the rate of change of the velocity of the skydivers as they fall by:

$\frac{dV}{dt} = -k(V-P)$, where k and P are constants.

The constant P represents the terminal velocity of the skydiver in the prone position which is 55 m/s.

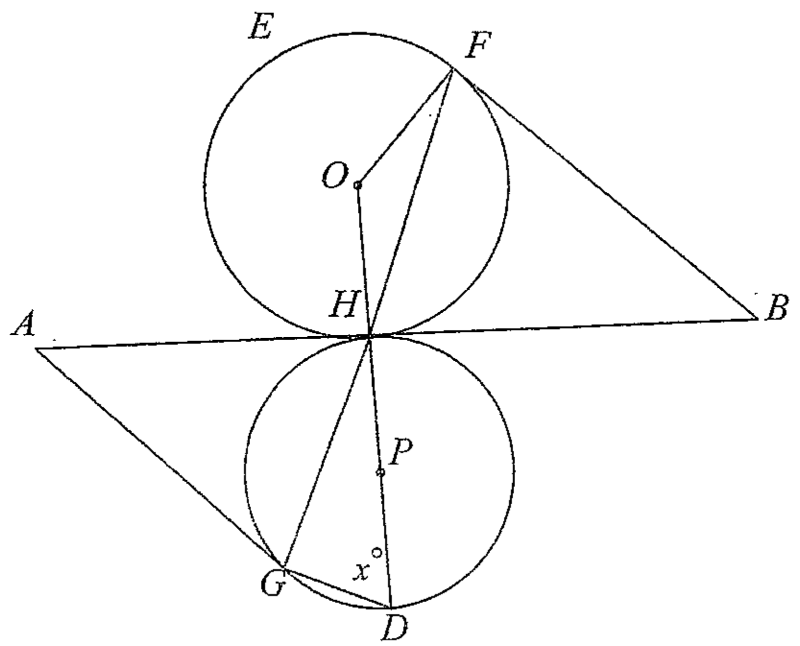
(i) Show that $V = P + Ae^{-kt}$ is solution of this differential equation. 1

(ii) Initially the velocity of the skydiver is 0 m/s and the velocity after 10 seconds is 27 m/s. Find values for A and k. 2

(iii) Find the velocity of the skydiver after 17 seconds. 1

(iv) How long does it take the skydiver to reach a velocity of 50 m/s? 2

(b) The diagram shows two circles with centres O and P respectively, which touch at the point H. OP is produced to meet the smaller circle at D. AB is a common tangent drawn through H. A secant is drawn through H meeting the respective circles at F and G. FB and AG are tangents to the respective circles. GD and OF are joined. $\angle GDH = x°$



i) Show that $\angle HFB = \angle GDH$. 2

ii) Show that $\angle HAG = 2 \times \angle OFH$ 3

(c) (i) Show that $x^n(1+x)^n\left(1+\frac{1}{x}\right)^n = (1+x)^{2n}$. **1**

(ii) Hence prove that

$$1 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{n}^2 = \binom{2n}{n}.$$ 3

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Marks

Question 14 (15 marks) **Use a separate writing booklet.**

(a) A particle moves on a line so that its distance from the origin at time t is x.

(i) Prove that $\frac{d^2x}{dt^2} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ where v denotes velocity. **2**

(ii) If $\frac{d^2x}{dt^2} = -2x\left(x^2 - 20\right)$ and $v = 0$ at $x = 2$ find v^2 in terms of x. **2**

(b) Using $x = 2$ as an initial approximation to the root of $f(x) = lnx - sinx$ use one application of Newton's Method to find a better approximation. **3**

(c) (i) Use the substitution $y = \sqrt{x}$ to find **3**

$$\int \frac{dx}{\sqrt{x(1-x)}}.$$

(ii) Use the substitution $z = x - \frac{1}{2}$ to find another expression for **3**

$$\int \frac{dx}{\sqrt{x(1-x)}}.$$

(iii) Use the results of parts (i) and (ii) to express $\sin^{-1}(2x-1)$ in terms of $\sin^{-1}\left(\sqrt{x}\right)$ for $0 < x < 1$. **2**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Ext 1.

13c	13a + 14a	Total.
/4	/10	

Solutions to 2014 Ext 1 Trial HSC

1. $m_1 = -3$, $m_2 = \frac{2}{3}$

$\tan\theta = \left| \dfrac{-3 - \frac{2}{3}}{1 + -3 \times \frac{2}{3}} \right| = 3\frac{2}{3}$

$\theta \doteqdot 75$ (D)

2. $x = \log_2 5$

$x = \dfrac{\log_e 5}{\log_e 2}$ (C)

3. (D)

4. $\sqrt{\dfrac{4(1+\cos 2x)}{1-\cos 2x}} = \sqrt{\dfrac{4(1+2\cos^2 x - 1)}{1-(1-2\sin^2 x)}} = \sqrt{\dfrac{4 \times 2\cos^2 x}{2\sin^2 x}}$

$= 2\cot x$ (B)

5. $y = 1 - \cos\frac{x}{2}$

$\therefore x = 1 - \cos\frac{y}{2}$

$\cos\frac{y}{2} = 1 - x$

$\frac{y}{2} = \cos^{-1}(1-x)$

$y = 2\cos^{-1}(1-x)$

$f^{-1}(x) = 2\cos^{-1}(1-x)$ (A)

6. $\dfrac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \dfrac{-4}{-1} = 4$ (D)

7. Roots are $\alpha, \frac{1}{\alpha}, \beta$

$\therefore$ product

$\alpha . \frac{1}{\alpha} . \beta = -\frac{6}{3} = -2$

$\beta = -2$ is a root

$\therefore 3(-8) + 4 + 2k + 6 = 0$

$2k = 14$

$k = 7$ (C)

8. $f(x) = \tan^{-1} x + x\tan^{-1} x$

$f'(x) = \dfrac{1}{1+x^2} + x . \dfrac{1}{1+x^2} + \tan^{-1} x . 1$

$= \dfrac{x+1}{1+x^2} + \tan^{-1} x$ (D)

9. ${}^{20}C_{20-r} \, (2x^2)^r . \left(-\frac{1}{x}\right)^{20-r}$

$x^{2r} . (x^{-1})^{20-r} = x^{-5}$

equate indices $2r - 20 + r = -5$

$3r = 15$

$r = 5$

$\therefore -{}^{20}C_{15}\, 2^5$ (D)

10. D for $\cos^{-1} x$ $\quad -1 \le x \le 1$

$\therefore$ D $\quad -\frac{1}{3} \le x \le \frac{1}{3}$

Range for $\cos^{-1} x$ $\quad 0 \le y \le \pi$

$\therefore 6\cos^{-1} x$ $\quad 0 \le y \le 6\pi$ (A)

Q11

a) i)

$$
\begin{array}{r}
-x^2 + x + 2 \\
x+1 \overline{) -x^3 \qquad + 3x + 2} \\
-x^3 - x^2 \qquad\qquad \\
\hline
x^2 + 3x + 2 \\
x^2 + x \qquad \\
\hline
2x + 2 \\
2x + 2 \\
\hline
0 + 0
\end{array}
$$

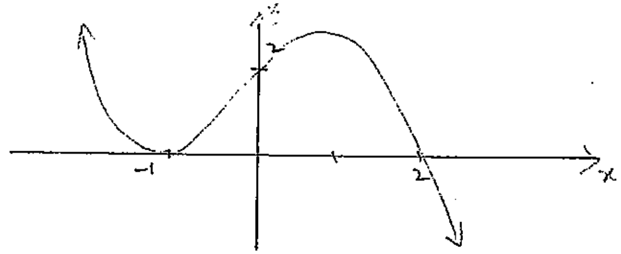
Zero remainder $\therefore$ $(x+1)$ is a factor

ii) $\therefore$ $P(x) = -(x+1)(x^2 - x - 2)$

$= -(x+1)(x-2)(x+1)$

$= -(x+1)^2 (x-2)$

iii)

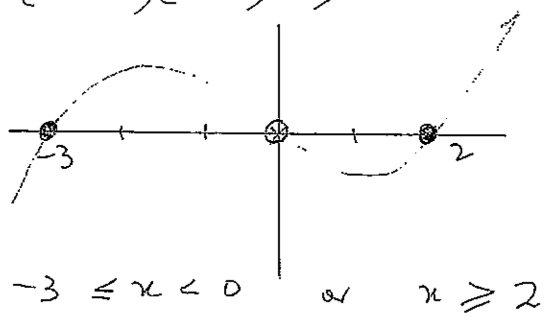


b) $\dfrac{x^2 + x - 6}{x} \geq 0$ Note $x \neq 0$

Mult b/s by x^2

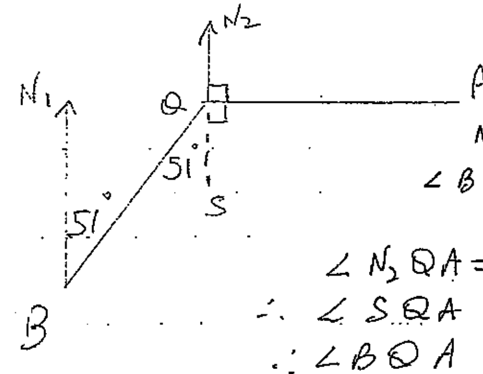
$x(x^2 + x - 6) \geq 0$

$x(x+3)(x-2) \geq 0$



$\therefore$ $-3 \leq x < 0$ or $x \geq 2$

11 c)



$N_1 BQ = 51°$

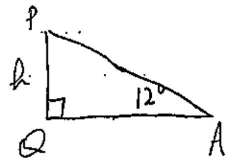
$\angle BQS = 51°$ (alternate $\angle$'s $N_1 B \parallel N_2 S$)

$\angle N_2 QA = 90°$ (due east)

$\therefore \angle SQA = 90°$

$\therefore \angle BQA = 90 + 51 = 141°$

ii)



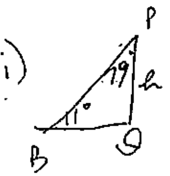
$\angle QPA = 90 - 12 = 78°$

(angle sum of $\triangle AQP$)

$\therefore \dfrac{AQ}{h} = \tan 78°$

$AQ = h \tan 78°$

iii)



$\dfrac{BQ}{h} = \tan 79°$

$BQ = h \tan 79°$

iv) $AB^2 = BQ^2 + QA^2 - 2BQ.QA.\cos 141°$

$1000^2 = h^2 \tan^2 79° + h^2 \tan^2 78° - 2h^2 \tan 79° \tan 78° \cos 141°$

$1000^2 = h^2 \{\tan^2 79 + \tan^2 78 - 2\tan 79 \tan 78 \cos 141\}$

$h^2 = \dfrac{1000^2}{\tan^2 79 + \tan^2 78 - 2\tan 79 \tan 78 \cos 141}$

$= 11598.39$

$h = 107.6958$

$h \doteqdot 108$ metres to nearest m

Q12

a) $\frac{1^2}{1\times3} + \frac{2^2}{3\times5} + \frac{3^2}{5\times7} + \cdots \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)}$

Prove true for $n=1$

$LHS = \frac{1^2}{1\times3} = \frac{1}{3}$ $\quad RHS = \frac{1\times2}{2\times3} = \frac{1}{3}$

$\therefore$ True for $n=1$

Assume true for $n=k$. ie assume

$\frac{1^2}{1\times3} + \cdots \frac{k^2}{(2k-1)(2k+1)} = \frac{k(k+1)}{2(2k+1)}$ ——— $*$

Using $*$, prove true for $n=k+1$

ie prove $\frac{1^2}{1\times3} + \frac{2^2}{3\times5} \cdots \frac{(k+1)^2}{(2k+1)(2k+3)} = \frac{(k+1)(k+2)}{2(2k+3)}$

Add $(k+1)^{th}$ term to b/s of $*$

$RHS = \frac{k(k+1)}{2(2k+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)}$

$= \frac{k(k+1)(2k+3) + 2(k+1)^2}{2(2k+1)(2k+3)}$

$= \frac{(k+1)\{k(2k+3) + 2(k+1)\}}{2(2k+1)(2k+3)}$

$= \frac{(k+1)\{2k^2+3k+2k+2\}}{2(2k+1)(2k+3)}$

$= \frac{(k+1)(2k^2+5k+2)}{2(2k+1)(2k+3)}$ $\quad$ $2k \; +1$ / $k \; +2$

$= \frac{(k+1)(2k+1)(k+2)}{2(2k+1)(2k+3)} = \frac{(k+1)(k+2)}{2(2k+3)}$

Since statement is true for $n=1$, and if true for $n=k$, is also true for $n=k+1$, then it is true for all $n \geq 1$ by mathematical induction

b) i) Put $\sqrt{3}\cos x - \sin x \equiv R\cos(x+\alpha)$ where $R>0$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

$\therefore R\cos\alpha = \sqrt{3}$, $R\sin\alpha = 1$ $\Rightarrow \tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$R^2 = 3+1 \Rightarrow R=2$

$\therefore 2\cos(x+\frac{\pi}{6}) = 1$

$\cos(x+\frac{\pi}{6}) = \frac{1}{2}$

$(x+\frac{\pi}{6})$ lies in 1st or 4th quad

$x+\frac{\pi}{6} = \frac{\pi}{3}$ or $2\pi - \frac{\pi}{3}$

$x = \frac{\pi}{6}$ or $\frac{3\pi}{2}$ for $0 \leq x \leq 2\pi$

ii) For general solution

$\cos(x+\frac{\pi}{6}) = \frac{1}{2}$

$x+\frac{\pi}{6} = 2n\pi \pm \cos^{-1}\frac{1}{2}$

$x+\frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$

$x = 2n\pi + \frac{\pi}{3} - \frac{\pi}{6}$ or $2n\pi - \frac{\pi}{3} - \frac{\pi}{6}$

$x = 2n\pi + \frac{\pi}{6}$ or $2n\pi - \frac{\pi}{2}$

or $x = (12n+1)\frac{\pi}{6}$ or $(4n-1)\frac{\pi}{2}$

12 c) $V = \pi\int y^2\,dx$

$V = \pi\int_0^{\pi/3} \sin^2 x\,dx$

$= \frac{\pi}{2}\int^{\pi/3} 1 - \cos 2x\,dx$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2}\{1-\cos 2x\}$

$$= \frac{\pi}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/3}$$

$$= \frac{\pi}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) - 0$$

$$= \frac{\pi^2}{6} - \frac{\sqrt{3}\pi}{8}$$

d) i) $x = \frac{kx_2 + lx_1}{k+l}$ $\quad k:l = 2:-1$

$x = \frac{2 \times ap - 1 \times 0}{2-1}$

$x = 2ap$ ——— (1)

$y = \frac{ky_2 + ly_1}{k+l}$

$= \frac{2\times 0 - 1\times(2a+ap^2)}{2-1}$

$y = -2a - ap^2$ ——— (2)

$\therefore$ G is $(2ap, -2a-ap^2)$

ii) From (1) $p = \frac{x}{2a}$

Sub into (2) $y = -2a - a\left(\frac{x^2}{4a^2}\right)$

$y = -2a - \frac{x^2}{4a}$

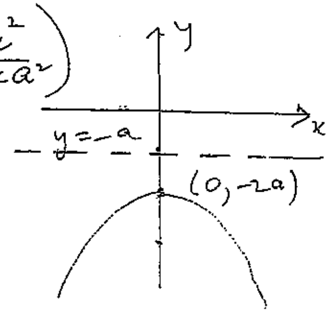
$4ay = -8a^2 - x^2$

$x^2 = -4ay - 8a^2$

$x^2 = -4a(y+2a)$

which is a parabola, vertex $(0,-2a)$

focal length a, focus $(0, -3a)$



13 a) $\frac{dV}{dt} = -k(V-P)$

i) $V = P + Ae^{-kt}$ ——— (1)

Differentiating w r t t:

$\frac{dV}{dt} = 0 + Ae^{-kt} \cdot -k$

From (1) $V - P = Ae^{-kt}$

$\therefore \frac{dV}{dt} = (V-P) \cdot -k$

$= -k(V-P)$ as required.

ii) $t=0 \quad V=0 \qquad t=10 \quad V=27$

Into (1) $0 = 55 + A$

$A = -55$

$27 = 55 - 55e^{-10k}$

$55e^{-10k} = 28$

$e^{-10k} = \frac{28}{55}$

or $e^{10k} = \frac{55}{28}$

$10k = \ln(55/28)$

$k = \frac{\ln\left(\frac{55}{28}\right)}{10} \doteq 0.0675$

iii) $V = 55 - 55e^{-0.0675t}$

$t = 17 \quad V = 55 - 55e^{-0.0675\times 17}$

$V = 37.545$ m/s

iv) $V = 55 - 55e^{-0.0675t}$

$50 = 55 - 55e^{-0.0675t}$

$55e^{-0.0675t} = 5$

$e^{-0.0675t} = \frac{5}{55} = \frac{1}{11}$

$e^{0.0675t} = 11$

$0.0675t = \ln 11$

$t = \frac{\ln 11}{0.0675}$

13 b) i)

$\angle HGD = 90^\circ$ (angle in a semicircle $= 90^\circ$)

$\therefore \angle GHD = 90 - x$ (Complementary $\angle$'s, $\angle$ Sum of $\triangle GHD$)

$\therefore \angle OHF = 90 - x$ (Vertically opposite to $\angle GHD$)

$\therefore \angle OFH = \angle OHF = 90 - x$ (Base $\angle$'s of isosceles $\triangle OHF$ as $OH = OF$ equal radii)

But $\angle OFB = 90^\circ$ (Angle between radius & tangent $= 90^\circ$)

$\therefore \angle HFB = 90 - (90 - x)$
$= x$ (complementary $\angle$'s)
$= \angle GDH$

ii) $\angle AHG = \angle HGD = x^\circ$ (Angle between tangent & chord drawn to point of contact equals any angle in the alternate segment)

$AH = AG$ (Tangents from external point A are equal)

$\therefore \angle AGH = x$ (Base $\angle$'s of isosceles $\triangle AGH$)

$\therefore \angle HAG = 180 - 2x$ (Angle sum of $\triangle HAG$)
$= 2(90 - x)$
$= 2 \angle OFH$ from (i)

13 c) i)

$x^n(1+x)^n\left(1+\frac{1}{x}\right)^n$

$= x^n(1+x)^n\left(\frac{x+1}{x}\right)^n$

$= x^n(1+x)^n \frac{(x+1)^n}{x^n}$

$= (1+x)^{2n}$ $\quad (x \neq 0)$

or

$\rightarrow x^n\left(1+\frac{1}{x}\right)^n = \left[x\left(1+\frac{1}{x}\right)\right]^n$
$= [(x+1)]^n$

ii) $x^n(1+x)^n\left(1+\frac{1}{x}\right)^n$

$= x^n\left[\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\dots+\binom{n}{n}x^n\right]\left[\binom{n}{0}+\binom{n}{1}\frac{1}{x}+\binom{n}{2}\frac{1}{x^2}+\dots+\binom{n}{n}\frac{1}{x^n}\right]$

Term in x^n is term independent of x in [ ][ ] and has coefficient:

$\binom{n}{0}\binom{n}{0}+\binom{n}{1}\binom{n}{1}+\binom{n}{2}\binom{n}{2}+\dots\binom{n}{n}\binom{n}{n}$

$= 1 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2$

Now term in x^n from $(1+x)^{2n}$ is $\binom{2n}{n}x^n$

$\therefore 1 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}$

Q14 a) $RHS = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{1}{2} \cdot 2v \cdot \frac{dv}{dx}$

$= v\frac{dv}{dx}$

$= \frac{dx}{dt} \times \frac{dV}{dx}$

$= \frac{dv}{dt}$ (using chain rule)

$= \ddot{x}$ or $\frac{d^2x}{dt^2}$

ii) $\ddot{x} = -2x(x^2-20)$

$= -2x^3 + 40x$

$\frac{1}{2}v^2 = \int 40x - 2x^3 \, dx$

$= 20x^2 - \frac{x^4}{2} + c$

or $V^2 = 40x^2 - x^4 + c_2$

$v=0,\ x=2 \quad 0 = 160 - 16 + c_2 \Rightarrow c_2 = -144$

$\therefore V^2 = 40x^2 - x^4 - 144$

b) $f(x) = \ln x - \sin x$

$f'(x) = \frac{1}{x} - \cos x$

$f(2) = \ln 2 - \sin 2 \doteqdot -0.216\ldots$

$f'(2) = \frac{1}{2} - \cos 2 \doteqdot 0.916\ldots$

For root of $f(x) = 0$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x=2$ is first approximation

$x_2 = 2 - \frac{-0.216}{0.916}$

$x_2 = 2.2359$

or $x \doteqdot 2.24$ is a better approximation.

14 c) $\int \frac{dx}{\sqrt{x(1-x)}}$

Put $y = \sqrt{x}$

or $y^2 = x$

$\therefore 2y\,dy = dx$

$= \int \frac{2y\,dy}{\sqrt{y^2(1-y^2)}}$

$= \int \frac{2dy}{\sqrt{1-y^2}} = 2\sin^{-1} y + c.$

$= 2\sin^{-1}\sqrt{x} + c.$

ii) $\int \frac{dx}{\sqrt{x(1-x)}}$

Put $z = x - \frac{1}{2}$

$dz = dx$

$= \int \frac{dz}{\sqrt{(z+\frac{1}{2})(1-(z+\frac{1}{2}))}} = \int \frac{dz}{\sqrt{(z+\frac{1}{2})(\frac{1}{2}-z)}}$

$= \int \frac{dz}{\sqrt{\frac{1}{4}-z^2}} = \int \frac{2dz}{\sqrt{1-4z^2}}$

$= \sin^{-1} 2z$

$= \sin^{-1}(2x-1) + c_2$

iii) $\therefore 2\sin^{-1}\sqrt{x} = \sin^{-1}(2x-1) + C$

Put $x = 0$

$0 = \sin^{-1}(-1) + C$

$\frac{\pi}{2} = C$

$\therefore 2\sin^{-1}\sqrt{x} = \sin^{-1}(2x-1) + \frac{\pi}{2}$

or $\sin^{-1}(2x-1) = 2\sin^{-1}\sqrt{x} - \frac{\pi}{2}$