NORMANHURST BHS KNOW THYSELF

2014
HSC Trial
EXAMINATION

Student Number

Class (Please circle)

M1 M2

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved Calculators may be used
- A table of standard integrals is provided
- In Questions 11 – 16 show relevant mathematical reasoning and/or calculations

Total Marks – 100

Section I Questions 1 – 10 **10 marks**
Allow about 15 minutes for this section

Section II Questions 11 – 16 **90 marks**
Allow about 2 hour and 45 minutes for this section

SECTION I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

---

1 If $z = 1 + 2i$ and $w = 3 - i$,

$z - \overline{w}$ is?

(A) $3i - 2$

(B) $4 + 3i$

(C) $i - 2$

(D) $4 + i$

2 The directrices of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ are:

(A) $x = \pm\frac{16\sqrt{7}}{7}$

(B) $x = \pm\frac{\sqrt{7}}{16}$

(C) $x = \pm\frac{16}{5}$

(D) $x = \pm\frac{5}{16}$

3 The factorisation of $x^4 + 7x^2 - 18$ over the complex number field is:

(A) $\left(x - i\sqrt{2}\right)\left(x + i\sqrt{2}\right)(x-3)(x+3)$

(B) $\left(x - i\sqrt{2}\right)\left(x - i\sqrt{2}\right)(x+3)(x+3)$

(C) $(x-3i)(x-3i)\left(x+\sqrt{2}\right)\left(x+\sqrt{2}\right)$

(D) $(x-3i)(x+3i)\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)$

4 If $f(x) = \dfrac{x(x-1)}{x-2}$, which of the following lines will be an asymptote $y = f(x)$?

(A) $y = x+1$

(B) $y = x-2$

(C) $y = x-1$

(D) $y = 0$

5 If α, β, γ are the roots of $x^3 + x - 1 = 0$, then an equation with roots $\dfrac{\alpha}{2}, \dfrac{\beta}{2}, \dfrac{\gamma}{2}$ is:

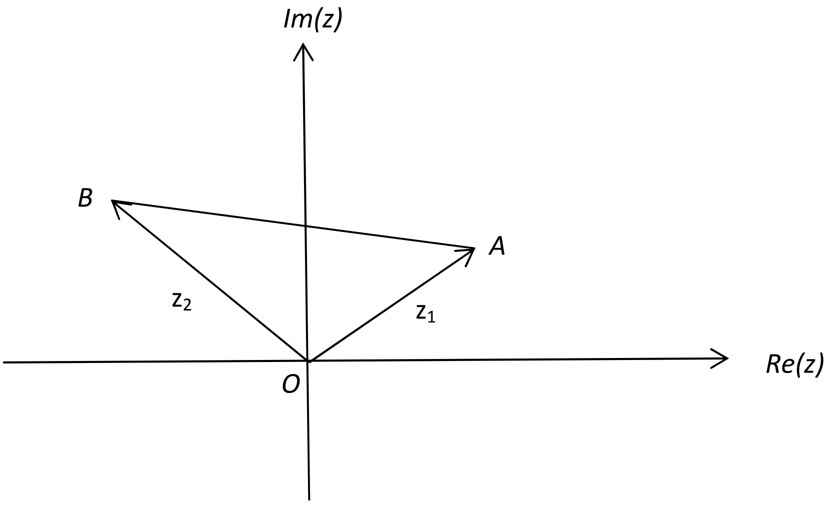
(A) $\dfrac{x^3}{8} + \dfrac{x}{2} - 1 = 0$

(B) $\dfrac{8}{x^3} + \dfrac{2}{x} - 1 = 0$

(C) $8x^3 + 2x - 1 = 0$

(D) $-8x^3 - 2x - 1 = 0$

6 In the Argand diagram below, vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent the complex numbers z_1 and z_2 respectively where $|z_2| = 2|z_1|$ and $\angle AOB = \frac{2\pi}{3}$.



Vector $\overrightarrow{AB}$ represents?

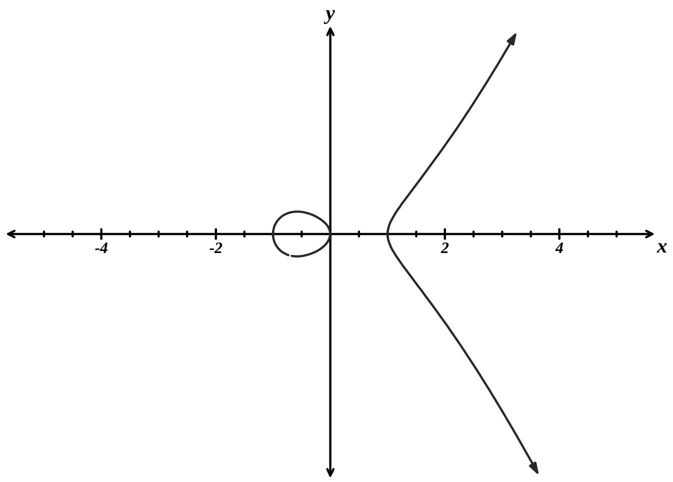
(A) $\left(-2 + i\sqrt{3}\right)z_1$

(B) $\left(2 - i\sqrt{3}\right)z_1$

(C) $\left(-\sqrt{3} + 2i\right)z_1$

(D) $\left(\sqrt{3} - 2i\right)z_1$

7 The diagram shows the graph of $y^2 = f(x)$.



Which expression best represents the function $f(x)$

(A) $x^2(x-1)$

(B) $x^2(1-x)$

(C) $x(x^2-1)$

(D) $x(1-x^2)$

8 Four female and four male athletes are arranged in a row for the presentation of prizes. In how many ways can this be done if the males and females must alternate?

(A) $4! \times 4!$

(B) $2 \times 4! \times 4!$

(C) $4! \times 5!$

(D) $2 \times 4! \times 5!$

9 What is the natural domain of the function

$$f(x) = \frac{1}{2}\left(x\sqrt{x^2 - 1} - \log_e\left(x + \sqrt{x^2 - 1}\right)\right) \text{ ?}$$

(A) $x \leq -1$ or $x \geq 1$

(B) $-1 \leq x \leq 1$

(C) $x \geq 1$

(D) $x \leq -1$

10 Four digit numbers are formed from the digits 1, 2, 3 and 4. Each digit is used once only.The sum of all the numbers that can be formed is?

(A) 266,640

(B) 66,660

(C) 44,440

(D) 6,666

SECTION II

90 marks

Attempt Questions 11-16

Allow about 2 hour and 45 minutes for this section

Answer each question in a SEPARATE Writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Use a SEPARATE writing booklet.

(a) (i) Write $\sqrt{3}+i$ in modulus/argument form. **2**

(ii) Hence, express $z=\dfrac{-1+i}{\sqrt{3}+i}$ in modulus/argument form and **3**

find the exact value of $\cos\dfrac{7\pi}{12}$ in simplest form.

(b) (i) Find constants A, B, C such that $\dfrac{x^2+3x-4}{x^2-x-2}\equiv A+\dfrac{Bx+C}{x^2-x-2}$ **2**

(ii) Hence find $\displaystyle\int \frac{x^2+3x-4}{x^2-x-2}\,dx$ **2**

(c) Sketch the region on the Argand diagram where the inequalities **2**

$|z|\le 4$ and $0\le \arg(z+2)\le \dfrac{\pi}{4}$ both hold.

(d) Using the substitution $x=2\sin\theta$, evaluate $\displaystyle\int_0^{\sqrt{3}} x^2\left(4-x^2\right)^{-\frac{5}{2}}\,dx$ **4**

Question 12 (15 Marks) Use a SEPARATE writing booklet.

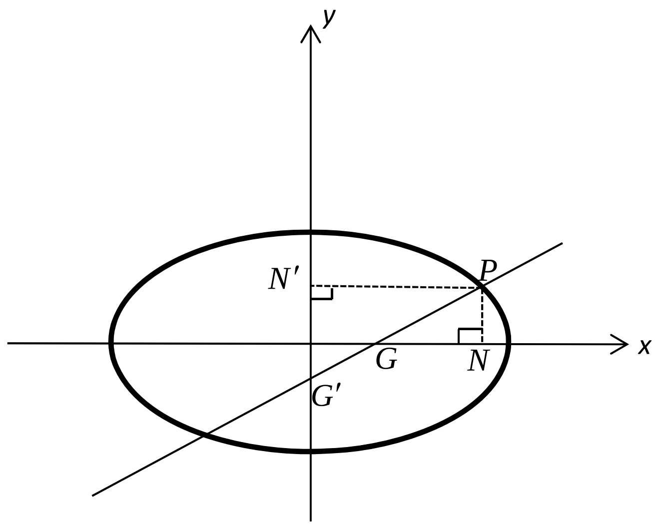
(a) (i) Show that $2+i$ is a root of $x^3 - 11x + 20 = 0$. **1**

(ii) Hence or otherwise solve $x^3 - 11x + 20 = 0$. **2**

(b) The diagram below shows the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

The point $P\left(a\cos\theta, b\sin\theta\right)$ lies on the ellipse.

The normal to the ellipse at P meets the major and minor axes of the ellipse in G and G' respectively. N and N' are the feet of the perpendiculars from P to the major and minor axes respectively.



(i) Show that the equation of the normal at P is $ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2$. **2**

(ii) Show that the ratio $OG : ON = e^2 : 1$ **2**

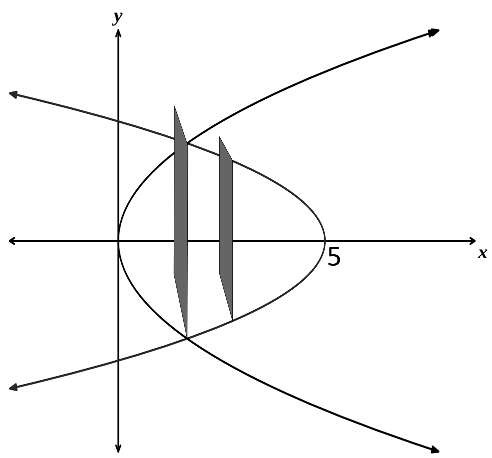
(iii) Find the ratio of the area of $\Delta PNG : \Delta PN'G'$. **1**

Question 12 continues on page 9

Question 12 (continued)

(c) Given the polynomial $P(x) = 2x^4 + 9x^3 + 6x^2 - 20x - 24 = 0$ has a root of multiplicity 3, solve $P(x) = 0$. **3**

(d) The base of a solid is the region bounded by the parabolas $x = y^2$ and $x = 3 - 2y^2$. Vertical cross-sections are squares perpendicular to the $x - axis$ as shown in the diagram. **4**

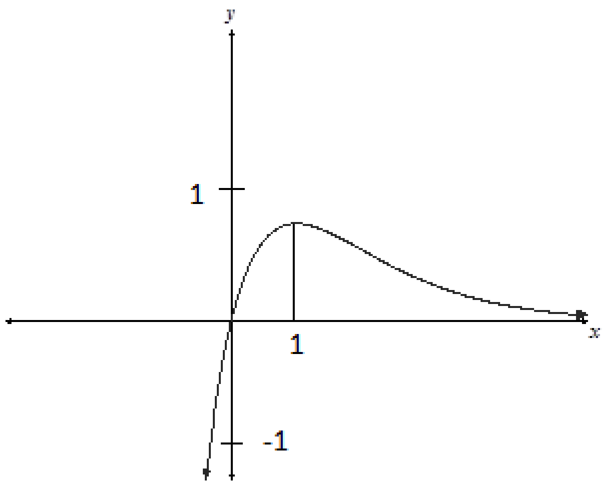


Find the volume of the solid.

End of Question 12

Question 13 (15 Marks) Use a SEPARATE writing booklet.

(a) The diagram below shows the graph of $y = f(x)$.



NOT TO SCALE

As $x \to \infty, f(x) \to 0$. Draw large (one-third page), separate sketches of the following curves. Show any important features.

(i) $y = e^{f(x)}$ **2**

(ii) $y = \sin^{-1} f(x)$ **3**

(b) From a group of eleven teachers, four will be chosen to form a committee. One teacher will be chairperson of the committee and the others will serve as members. **2**

In how many ways can the committee be formed?

Question 13 continues on page 11

Question 13 (continued)

(c) $\int \frac{1}{\sqrt{3-(x^2-2x)}}\,dx$ 3

(d) $\int \tan^{-1} x \, dx$ 2

(e) Prove $\cos x - 1 + \frac{x^2}{2} > 0$, if $x \neq 0$. 3

End of Question 13

Question 14 (15 Marks) Use a SEPARATE writing booklet.

(a) Given $f(x) = \frac{3}{14} x^{2/3} \left(x^4 - \frac{7}{2} x^2 + 7 \right)$

(i) Show that $y = f(x)$ is an even function. **1**

(ii) The graph of $y = f(x)$ has two stationary points. **2**

One stationary point is at $\left(1, \frac{27}{28} \right)$. Determine its nature.

(iii) Write down the coordinates of any point(s) where $f'(x)$ is undefined. **1**

(iv) Sketch a graph of $y = f(x)$ showing all important features. **2**

(b) (i) Using the substitution $t = \tan \frac{\theta}{2}$ show that: **3**

$$I = \int_0^{\pi} \frac{d\theta}{a + b\cos\theta} = \lim_{h \to \infty} \int_0^{h} \frac{2dt}{(a-b)(t^2 + c^2)}$$

where $c^2 = \frac{a+b}{a-b}$ and $a > b > 0$.

(ii) Hence show that $I = \frac{\pi}{\sqrt{(a^2 - b^2)}}$ **2**

(c) Using the method of cylindrical shells, find the volume of the solid of revolution formed when the area bounded by $y = \sin x$, the x-axis, between $x = 0$ and $x = \pi$, is rotated about the y-axis.

Question 15 (15 Marks) Use a SEPARATE writing booklet.

(a) (i) Show that $\left(3+\sqrt{5}\right)^n+\left(3-\sqrt{5}\right)^n$ is divisible by 2^n **1**

for $n=1$ and $n=2$.

(ii) Use mathematical induction to prove that $\left(3+\sqrt{5}\right)^n+\left(3-\sqrt{5}\right)^n$ **3**

is divisible by 2^n, for integer $n \geq 1$.

(b) A non-real number w is such that $|w| = 1$. **3**

If $z = \dfrac{1+w}{1-w}$ find the locus of z as w moves on the complex number plane.

(c) A rubber ball of mass 7 kg, falls from rest, from the top of a building.

While falling the ball experiences a resistive force $\dfrac{7v^2}{10}$, where v is the velocity

of the ball. Take g, acceleration due to gravity, as $g = 10\,\text{ms}^{-2}$.

(i) Show that $\ddot{x} = 10 - \dfrac{v^2}{10}$, where x is the distance the ball has fallen. **1**

(ii) Find the terminal velocity of the ball as it falls. **1**

(iii) Show that $v^2 = 100\left(1-e^{-\frac{x}{5}}\right)$ **3**

(iv) After hitting the ground the ball rises vertically such that **3**

$\ddot{X} = -10 - \dfrac{V^2}{10}$, where V is the velocity of the ball as it rises and

X is the distance the ball rises.

Find the time that it takes for the ball to rise to its maximum height

if initially $V = \dfrac{10}{\sqrt{3}}\,m/s$.

Question 16 (15 Marks) Use a SEPARATE writing booklet.

(a) (i) Use de Moivre's Theorem to show that: **3**

$$\tan 7\theta = \frac{7t - 35t^3 + 21t^5 - t^7}{1 - 21t^2 + 35t^4 - 7t^6} \quad \text{where } t = \tan\theta$$

(ii) Hence show that the roots of the equation $x^3 - 21x^2 + 35x - 7 = 0$ **3**

are $\tan^2\left(\pi/7\right), \tan^2\left(2\pi/7\right)$ and $\tan^2\left(3\pi/7\right)$

(iii) Deduce that: **2**

$$\sec^4\left(\pi/7\right) + \sec^4\left(2\pi/7\right) + \sec^4\left(3\pi/7\right) = 416$$

(b) (i) Show that: **2**

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots\ldots + x^{n-1} + \frac{x^n}{1-x}$$

(ii) By letting $x = -t$, show that: **3**

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots + (-1)^{n-1}\frac{x^n}{n} + T_n(x)$$

Where $T_n(x) = (-1)^n \int_0^x \frac{t^n}{1+t}\,dt$.

(iii) Show that $T_n(x) \to 0$ for $0 \le x \le 1$. **1**

(iv) Hence express $\log_e 2$ as a series writing the first 5 terms. **1**

End of paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Section 1 – 10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this part

Use the multiple-choice answer sheet overpage.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A 0 B C 0 D 0

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A 0 B C 0 D 0

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word ***correct*** and drawing an arrow as follows:

correct

A B C 0 D 0

HSC Trial
Examination
2014

Student Number

Class (Please circle)

12M1 12M2 12M3 12M4

12M5 12M6 11M1

Mathematics Extension 2

Multiple Choice Answer Sheet

Instructions: Colour in the circle next to the letter that represents the correct answer.

1. A O B O C O D O
2. A O B O C O D O
3. A O B O C O D O
4. A O B O C O D O
5. A O B O C O D O
6. A O B O C O D O
7. A O B O C O D O
8. A O B O C O D O
9. A O B O C O D O
10. A O B O C O D O

NBHS SOLUTIONS 2014 Extension 2 Trial HSC

Section 1

10 marks

Questions 1 – 10 (1 mark each)

Question 1 (1 mark)

Outcomes Assessed: E3

Targeted Performance Bands: E2

Solution	Answer	Mark
$z-\overline{w}=(1+2i)-(3+i)$ $=-2+i$	C	1

Question 2 (1 mark)

Outcomes Assessed: E3

Targeted Performance Bands: E2

Solution	Answer	Mark
$a=4, b=3$ $b^2=a^2\left(1-e^2\right)$ $9=16\left(1-e^2\right)$ $e^2=\frac{7}{16}$ $e=\frac{\sqrt{7}}{4}$ $x=\pm\frac{16\sqrt{7}}{7}$	**A**	**1**

Question 3 (1 mark)

Outcomes Assessed: E4

Targeted Performance Bands: E3

Solution	Answer	Mark
x^4+7x^2-18 $=\left(x^2+9\right)\left(x^2-2\right)$ $=\left(x^2-(3i)^2\right)\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)$ $=(x-3i)(x+3i)\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)$	**D**	**1**

Question 4 (1 mark)
Outcomes Assessed: E6
Targeted Performance Bands: E3

Solution	**Answer**	**Mark**
$\frac{x(x-1)}{x-2}=\frac{x^2-x}{x-2}$ $=\frac{x^2-x-2+2}{x-2}$ $=\frac{(x-2)(x+1)+2}{x-2}$ $=x+1+\frac{2}{x-2}$ $\therefore$ asymptote is $y=x+1$	**A**	**1**

Question 5 (1 mark)
Outcomes Assessed: E4
Targeted Performance Bands: E3

Solution	**Answer**	**Mark**
Since α,β,γ satisfy $x^3+x-1=0$ $\frac{\alpha}{2},\frac{\beta}{2},\frac{\gamma}{2}$ satisfy $(2x)^3+(2x)-1=0$ $\therefore$ the required equation is $8x^3+2x-1=0$	**C**	**1**

Question 6 (1 mark)
Outcomes Assessed: E3
Targeted Performance Bands: E3

Solution	**Answer**	**Mark**
Since $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ $=2cis\left(\frac{2\pi}{3}\right)z_1-z_1$ $=\left[2\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)-1\right]z_1$ $=\left(-2+i\sqrt{3}\right)z_1$	**A**	**1**

Question 7 (1 mark)
Outcomes Assessed: E6
Targeted Performance Bands: E2

Solution	Answer	Mark
Zeros at x = -1,1 and 0 as $x \to \infty$, $y \to \infty$	**C**	**1**

Question 8 (1 mark)
Outcomes Assessed: E5
Targeted Performance Bands: E2

Solution	Answer	Mark
With female first, e.g.: $F_1M_1F_2M_2F_3M_3F_4M_4$ 4! arrangements for females, 4! arrangements for males Similarly with male first $\therefore$ number of arrangements $= 2 \times 4! \times 4!$	**B**	**1**

Question 9 (1 mark)
Outcomes Assessed: E2
Targeted Performance Bands: E2

Solution	Answer	Mark
$x^2 - 1 \geq 0$ and $x + \sqrt{x^2 - 1} > 0$ i.e $x \leq -1, x \geq 1$ $\therefore$ domain is $x \geq 1$	**C**	**1**

Question 10 (1 mark)
Outcomes Assessed: E9
Targeted Performance Bands: E4

Solution	Answer	Mark
If we look at the number 1 it can have a value of (1000+100+10+1)=1111 with the other numbers arranged 3! ways. The value of all numbers is $3! \times (1111) \times (1+2+3+4) = 66,660$	**B**	**1**

Summary

1 C	**6 A**
2 A	**7 C**
3 D	**8 B**
4 A	**9 C**
5 C	**10 B**

Question 11 (15 marks)

(a) (i) (1 mark)

Outcomes assessed: E3

Targeted Performance Bands: E2

Criteria	**Marks**
• Both modulus and argument correct and written in correct form	**2**
• Modulus or argument correct	**1**

$$\left|\sqrt{3}+i\right| = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2$$

$$\arg\left(\sqrt{3}+i\right) = \frac{\pi}{6} \qquad \therefore \sqrt{3}+i = 2\text{cis}\frac{\pi}{6}$$

(ii) (2 marks)

Outcomes assessed: E3

Targeted Performance Bands: E2 – E3

Criteria	**Marks**
• Correct value in simplest form	**3**
• Some further progress towards solution	**2**
• Obtains $\frac{\sqrt{2}}{2}cis\left(\frac{7\pi}{12}\right)$ using De-Moivre's Theorem	**1**

Sample answer:

$$z = \frac{\sqrt{2}cis\left(\frac{3\pi}{4}\right)}{2cis\left(\frac{\pi}{6}\right)}$$

$$= \frac{\sqrt{2}}{2}cis\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$$

$$= \frac{\sqrt{2}}{2}cis\left(\frac{7\pi}{12}\right)$$

Now $\frac{-1+i}{\sqrt{3}+i} \times \frac{\sqrt{3}-i}{\sqrt{3}-i} = \frac{1-\sqrt{3}}{4} + \frac{\left(1+\sqrt{3}\right)i}{4}$

Equating real parts $\frac{\sqrt{2}}{2}\cos\left(\frac{7\pi}{12}\right) = \frac{1-\sqrt{3}}{4}$

$\therefore \cos\left(\frac{7\pi}{12}\right) = \frac{1-\sqrt{3}}{4} \times \frac{2}{\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$

11. (b) (i) (2 marks)

Outcomes assessed: E8

Targeted Performance Bands: E3

Criteria	Marks
• Finds correct values of *A*, *B* and *C*	**2**
• Makes some progress in partial fraction decomposition	**1**

Sample answer:

$$\frac{x^2+3x-4}{x^2-x-2} \equiv A+\frac{Bx+c}{x^2-x-2}$$

$$\frac{x^2-x-2+4x-2}{x^2-x-2} \equiv A+\frac{Bx+c}{x^2-x-2}$$

$$\frac{x^2-x-2}{x^2-x-2}+\frac{4x-2}{x^2-x-2} \equiv A+\frac{Bx+c}{x^2-x-2}$$

$$1+\frac{4x-2}{x^2-x-2} \equiv A+\frac{Bx+c}{x^2-x-2}$$

Equating $A=1,\ B=4$ and $C=-2$

11. (b) (ii) (2 marks)

Outcomes assessed: E8

Targeted Performance Bands: E2

Criteria	Marks
• Correct integration of partial fractions from (i). No penalty for no absolute value or no +*c*.	**2**
• Some progress towards answer.	**1**

Sample answer:

$$\int \frac{x^2+3x-4}{x^2-x-2}\,dx=\int\left(1+\frac{2(2x-1)}{x^2-x-2}\right)dx$$

$$=x+2\ln\left|x^2-x-2\right|+c$$

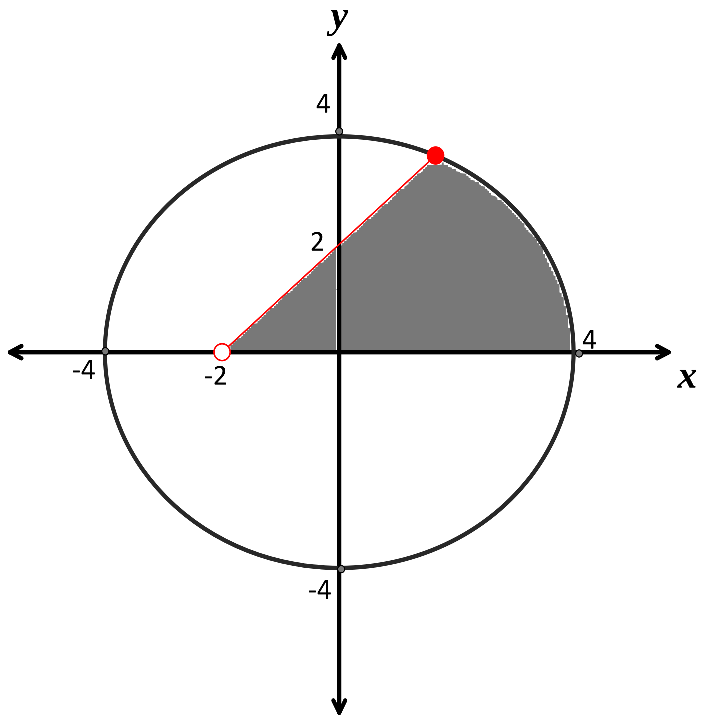
11. (c) (2 marks)Type equation here.

Outcomes assessed: E3

Targeted Performance Bands: E2

Criteria	**Marks**
• Correct region shown	**2**
• Correctly gives one region	**1**

Sample answer:Type equation here.



11. (d) (4 marks)

Outcomes assessed: E8

Targeted Performance Bands: E3 – E4

Criteria	Marks
• Correct answer	**4**
• Integrates $\tan^2\theta\sec^2\theta$	**3**
• Some progress towards evaluation of integral	**2**
• Correct substitution $x = 2\sin\theta$	**1**

Sample answer:

$$Let\ x = 2\sin\theta$$

$$\int_0^{\sqrt{3}} x^2\left(4-x^2\right)^{-\frac{5}{2}}dx = \int_0^{\frac{\pi}{3}} \frac{4\sin^2\theta\times 2\cos\theta}{\left[4\left(1-\sin^2\theta\right)\right]^{\frac{5}{2}}}d\theta$$

$$= \int_0^{\frac{\pi}{3}} \frac{8\sin^2\theta\cos\theta}{32\cos^5\theta}d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{3}} \frac{\sin^2\theta}{32\cos^4\theta}d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{3}} \tan^2\theta\sec^2\theta\, d\theta$$

$$= \frac{1}{12}\left[\tan^3\theta\right]_0^{\frac{\pi}{3}}$$

$$= \frac{\sqrt{3}}{4}$$

Question 12 (15 marks)

(a) (i) (1 mark)
Outcomes assessed: E4
Targeted Performance Bands: E2

Criteria	Marks
• Correctly shows that (2+i) is a root.	1

Sample answer:

$$(2+i)^3 - 11(2+i) + 20$$
$$= 8 + 12i + 6i^2 + i^3 - 22 - 11i + 20$$
$$= 8 + 12i - 6 - i - 22 - 11i + 20$$
$$= 0$$
$$\therefore (2+i) \text{ is a root of } x^3 - 11x + 20 = 0$$

(a) (ii) (2 marks)
Outcomes assessed: E4
Targeted Performance Bands: E2 – E3

Criteria	Marks
• Correctly solves the equation	2
• Correctly identifies that $2-i$ is also a root.	1

Sample answer:

$$(2-i) \text{ is a root, conjugate pairs.}$$
$$\text{Now } (2+i) + (2-i) + \alpha = 0$$
$$\text{So } \alpha = -4$$
$$\therefore \text{Solutions are } 2+i, 2-i \text{ and } -4$$

12. (b) (i) (2 marks)
Outcomes assessed: E3
Targeted Performance Bands: E2

Criteria	Marks
• Correctly derives the equation of the normal	**2**
• Finds gradient of normal	**1**

Sample answer:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

differentiate w.r.t.x

$$\frac{2x}{a^2}+\frac{2y}{b^2}=0$$

$$\frac{dy}{dx}=-\frac{b^2x}{a^2y}$$

At $P\left(a\cos\theta, b\sin\theta\right)$

$$\frac{dy}{dx}=-\frac{b\cos\theta}{a\sin\theta}$$

$$M_N=\frac{a\sin\theta}{b\cos\theta}$$

equation of normal

$$y-b\sin\theta=\frac{a\sin\theta}{b\cos\theta}\left(x-a\cos\theta\right)$$

$$\frac{by}{\sin\theta}-b^2=\frac{ax}{\cos\theta}-a^2$$

$$by\cos ec\theta-b^2=ax\sec\theta-a^2$$

$$\therefore\ ax\sec\theta-by\cos ec\theta=a^2-b^2$$

12. (b) (ii) (2 marks)
Outcomes assessed: E3
Targeted Performance Bands: E3

Criteria	Marks
• Correctly establishes the result.	**2**
• Some progress towards establishing the result (e.g. finds OG and attempts to use $b^2 = a^2\left(1-e^2\right)$.	**1**

Sample answer:

Put $y = 0$ into equation of normal

$ax\sec\theta = a^2 - b^2$

$x = \dfrac{a^2 - b^2}{a\sec\theta} \rightarrow OG = \dfrac{a^2 - b^2}{a\sec\theta}$

Using $b^2 = a^2\left(1-e^2\right)$

$OG = \dfrac{a^2 - a^2\left(1-e^2\right)}{a\sec\theta} = \dfrac{ae^2}{\sec\theta} = ae^2\cos\theta$

Now

$ON = a\cos\theta$

$\therefore OG : ON = ae^2\cos\theta : a\cos\theta = e^2 : 1$

12. (b) (iii) (1 mark)
Outcomes assessed: E3
Targeted Performance Bands: E4

Criteria	Marks
• Correctly establishes the ratio	**1**

Sample answer:

Using similar triangles

$OG : GN = e^2 : 1 - e^2$

$\therefore \Delta PNG : PN'G' = \left(1-e^2\right)^2 : 1$

12. (c) (3 marks)

Outcomes assessed: E4

Targeted Performance Bands: E4

Criteria	Marks
• Correctly solves the equation	**3**
• Correctly factorises $P(x)$	**2**
• Shows that $x=-2$ is a root of multiplicity 3.	**1**

Sample answer:

$P(x)=2x^4+9x^3+6x^2-20x-24=0$

$P'(x)=8x^3+27x^2+12x-20$

$P''(x)=24x^2+54x+12$

if $x=\alpha$ is a root of multiplicity 3 of $P(x)=0$

then it is a root of $P''(x)=0$.

$6(4x^2+9x+2)=0$

$6(4x+1)(x+2)=0$

$\therefore x=-2\, or -\frac{1}{4}$

now $P(-2)=32-72+24+40-24=0$

then $x=-2$ is a root with multiplicity 3.

so $P(x)=(x+2)^3(ax+b)$

from inspection $a=2$

substitute $x=0$

$-24=2^3\times b\rightarrow b=-3$

$\therefore$ solution of $P(x)=0$ is $x=-2$ or $\frac{3}{2}$

12. (d) (4 marks)

Outcomes assessed: E7

Targeted Performance Bands: E3

Criteria	Marks
• Correct answer	**4**
• Some progress towards evaluation of volume	**3**
• Some progress towards evaluation of cross-section	**2**
• Correctly finds point of intersection	**1**

Sample answer:

Solve simultaneously $x = y^2$ and $x = 3 - 2y^2$

$y^2 = 3 - 2y^2$

$3y^2 = 3$

$\because y = \pm 1$

$x = 1$

from $x = 0$ to $x = 1$, the cross section has area

$A_1 = (2y)^2 = 4y^2 = 4x$

the volume from $x = 0$ to $x = 1$ is:

$$V_1 = \int_0^1 4x\,dx = 2\left[x^2\right]_0^1 = 2$$

from $x = 1$ to $x = 3$, the cross section has area

$A_2 = (2y)^2 = 4y^2 = 2(3 - x)$

the volume from $x = 1$ to $x = 3$ is:

$$V_2 = 2\int_1^3 (3 - x)\,dx = 2\left[3x - \frac{x^2}{2}\right]_1^3$$

$$= 2\left[(9 - 4.5) - (3 - 0.5)\right] = 4$$

$\because$ the volume of the solid is $2 + 4 = 6u^3$

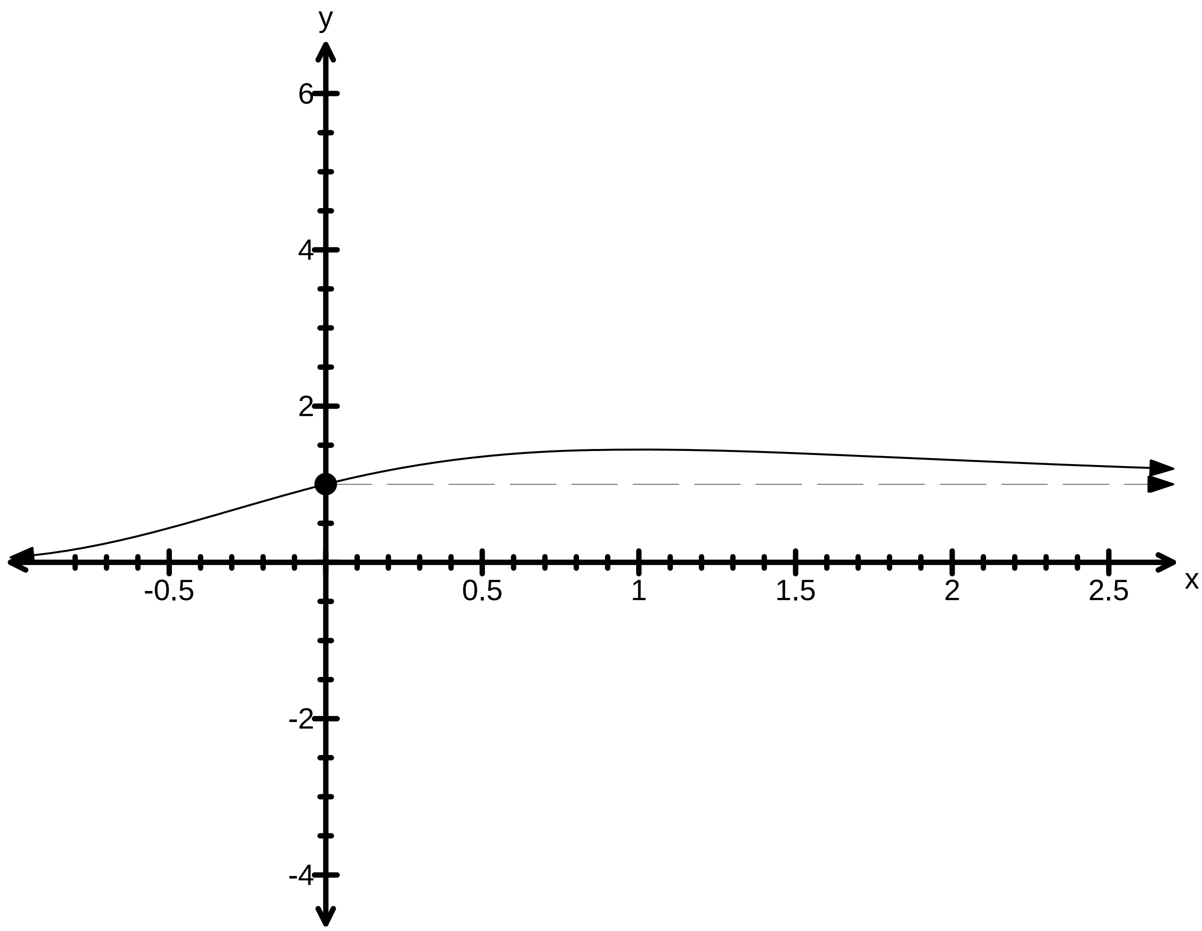
Question 13 (15 marks)
13. (a) (i) (2 marks)

Outcomes assessed: E6

Targeted Performance Bands: E2 - E3

Criteria	Marks
• Sketches correct curve	**2**
• Sketches curve for $x \geq 0$ or for $x < 0$ showing asymptote	**1**

Sample answer:



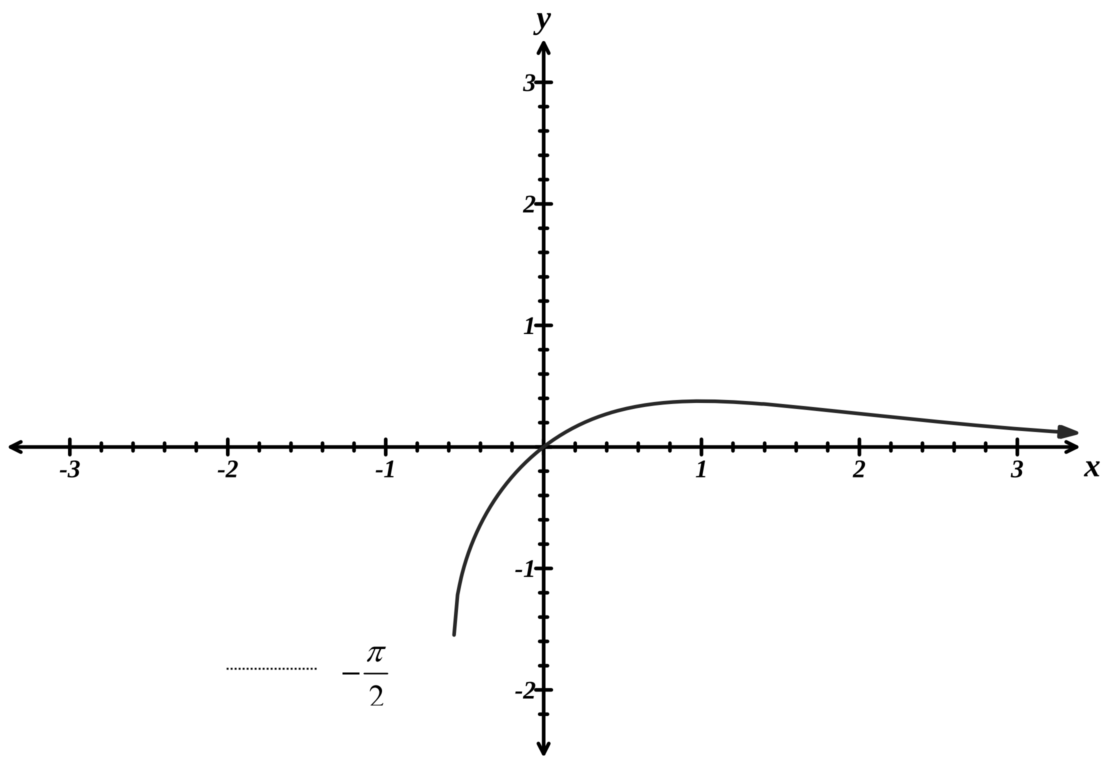
13. (a) (ii) (3 marks)

Outcomes assessed: E6

Targeted Performance Bands: E3 – E4

Criteria	Marks
• Sketches correct curve indicating $y = -\frac{\pi}{2}$.	**3**
• Sketches curve for $x < 0$	**2**
• Sketches curve for $x \geq 0$ showing asymptote	**1**

Sample answer:



13. (b) (2 marks)

Targeted Performance Bands: E3

Criteria	Marks
• Correct answer	**2**
• Some progress towards answer.	**1**

Sample answer:

The chairperson can be chosen in ${}^{11}C_1$ ways

The other 3 members can be chosen in ${}^{10}C_3$ ways

$\therefore$ number of committees $= {}^{11}C_1 \times {}^{10}C_3 = 1320$

13. (c) (3 marks)

Targeted Performance Bands: E3

Criteria	Marks
• Correctly establishes result.	**3**
• Correctly applies completing of the square and recognises to use $\sin^{-1}$.	**2**
• Attempts to use completing of the square	**1**

Sample answer:

$$\int \frac{1}{\sqrt{3-(x^2-2x)}}\,dx = \int \frac{1}{\sqrt{4-(x-1)^2}}\,dx$$

$$= \sin^{-1}\left(\frac{x-1}{2}\right) + c$$

13. (d) (2 marks)

Targeted Performance Bands: E3

Criteria	Marks
• Correctly determines the integral.	**2**
• Attempts to use integration by parts	**1**

Sample answer:

$$\int \tan^{-1} x\,dx = \int 1.\tan^{-1} x\,dx$$

$$= x\tan^{-1} x - \int \frac{x}{1+x^2}\,dx$$

$$= x\tan^{-1} x - \frac{1}{2}\log_e(1+x^2) + c$$

13. (e) (3 marks)
Outcomes assessed:
Targeted Performance Bands: E3 – E4

Criteria	**Marks**
• Correct solution	**3**
• Finds the stationary point and attempts to test itst nature	**2**
• Some progress towards answer.	**1**

Sample answer:

Let $f(x) = \cos x - 1 + \frac{x^2}{2} > 0$, if $x \neq 0$.

$f'(x) = -\sin x + x$

$f''(x) = -\cos x + 1 \geq 0$ for all x, $x \neq 0$

Stationary point occurs when $f'(x) = 0$

ie when $\sin x = x$

$x = 0$

When $x = 0, y = 0$

∴ Stationary point of inflection at (0,0)

$f''(0) = 0$, ∴ possible point of inflection at (0,0)

Test $f'(x)$ about x = 0

x	x < 0	x = 0	x > 0
$f'(x)$	-	0	+

Therefore minimum turning point

∴ there is an absolute minimum at (0,0)

∴ $f(0) < f(x)$ for all $x \neq 0$

$$\therefore 0 < \cos x - 1 + \frac{x^2}{2}$$

Question 14 (15 marks)
(a) (i) (1 mark)
Outcomes assessed: E9
Targeted Performance Bands: E2

Criteria	Marks
• Correctly shows that $y = f(x)$ is an even function.	**1**

Sample answer:

$$f(x) = \frac{3}{14}x^{2/3}\left(x^4 - \frac{7}{2}x^2 + 7\right)$$

$$f(-x) = \frac{3}{14}(-x)^{2/3}\left((-x)^4 - \frac{7}{2}(-x)^2 + 7\right)$$

$$= \frac{3}{14}x^{2/3}\left(x^4 - \frac{7}{2}x^2 + 7\right)$$

$$= f(x) \quad \therefore y = f(x) \textit{ is an even function.}$$

14. (a) (ii) (2 marks)
Outcomes assessed: E6
Targeted Performance Bands: E3

Criteria	Marks
• Correctly determines the nature of the stationary points.	**2**
• Uses appropriate method to determine the nature of the stationary point.	**1**

Sample answer:

$$f(x) = \frac{3}{14}x^{2/3}\left(x^4 - \frac{7}{2}x^2 + 7\right)$$

$$= \frac{3}{14}x^{14/3} - \frac{3}{4}x^{8/3} + \frac{3}{2}x^{2/3}$$

$$f'(x) = x^{11/3} - 2x^{5/3} + x^{-1/3}$$

$$f''(x) = \frac{11}{3}x^{8/3} - \frac{10}{3}x^{2/3} - \frac{1}{3}x^{-4/3}$$

since Stationary points are $\left(1, \frac{27}{28}\right)$ and $\left(-1, \frac{27}{28}\right)$

$$f''(1) = 0$$

x	0.9	1	1.1
$f''(x)$	-	0	+

$\therefore$ Both points are Horizontal points of inflexion since $y = f(x)$ is an even function.

14. (a) (iii) (1 mark)
Outcomes assessed: E6
Targeted Performance Bands: E3

Criteria	Marks
• Correctly answer.	**1**

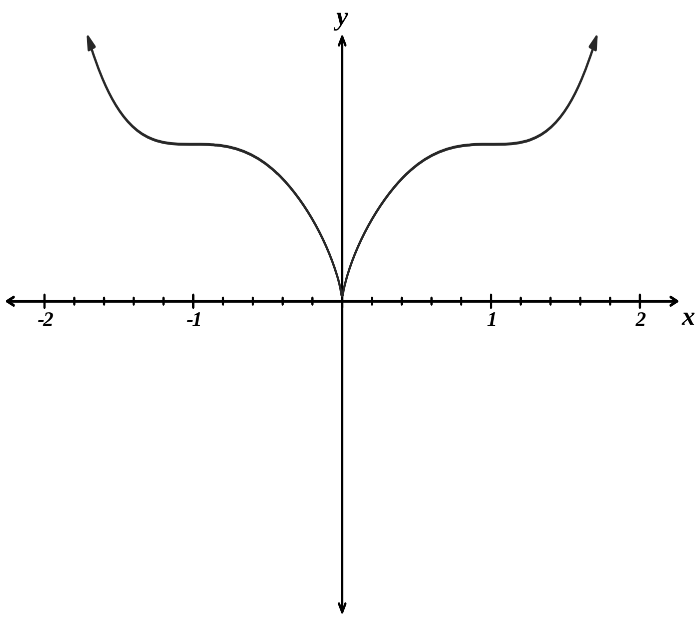
Sample answer:

$f'(x)$ is undefined at the origin.

(a) (iv) (2 marks)
Outcomes assessed: E6
Targeted Performance Bands: E3

Criteria	Marks
• Correctly draws $y = f(x)$ showing important features.	**2**
• Draws part of $y = f(x)$ correctly.	**1**

Sample answer:



14. (b) (i) (3 marks)

Outcomes assessed: E8

Targeted Performance Bands: E4

Criteria	Marks
• Correctly establishes result.	**3**
• Also correctly establishes limits	**2**
• Correct substitution of $t=\tan\frac{\theta}{2}$ and use of $\frac{d\theta}{dt}=\frac{2}{1+t^2}$ in integral	**1**

Sample answer:

If $\tan\frac{\theta}{2}=t$, then as $\theta\to\pi, t\to\infty$

Hence

$$I=\lim_{h\to\infty}\int_0^h \frac{1+t^2}{a\left(1+t^2\right)+b\left(1-t^2\right)}\times\frac{2}{1+t^2}\,dt$$

$$=\lim_{h\to\infty}\int_0^h \frac{2dt}{a+b+t^2\left(a-b\right)}$$

$$=\lim_{h\to\infty}\int_0^h \frac{2dt}{\left(a-b\right)\left[t^2+\frac{a+b}{a-b}\right]}$$

$$=\lim_{h\to\infty}\int_0^h \frac{2dt}{\left(a-b\right)\left(t^2+c^2\right)}\quad \text{where } c^2=\frac{a+b}{a-b}$$

14. (b) (ii) (2 marks)

Outcomes assessed: E8

Targeted Performance Bands: E4

Criteria	Marks
• Correctly establishes the result.	**2**
• correctly integrates $\int_0^{\infty} \frac{dt}{\left(t^2+c^2\right)}$	**1**

Sample answer:

$$2\int_0^{\infty} \frac{dt}{(a-b)\left(t^2+c^2\right)} = \frac{2}{a-b} \lim_{h\to\infty} \int_0^{h} \frac{dt}{\left(t^2+c^2\right)}$$

$$= \frac{2}{c(a-b)} \lim_{h\to\infty} \left[\tan^{-1}\frac{t}{c}\right]_0^h \quad \text{, now as } h\to\infty, \tan^{-1}\frac{t}{c} \to \frac{\pi}{2}$$

$$= \frac{2}{c(a-b)}\left[\frac{\pi}{2}-0\right]$$

$$= \frac{\pi}{c(a-b)}$$

$$= \frac{\pi}{\left[\sqrt{\frac{a+b}{a-b}}\right](a-b)} \quad , c^2 = \frac{a+b}{a-b}$$

$$= \frac{\pi}{\sqrt{\left(a^2-b^2\right)}}$$

14. (c) (3 marks)

Outcomes assessed: E9

Targeted Performance Bands: E3

Criteria	Marks
• Correct answer.	**4**
• Uses integration by parts	**3**
• Correct use of the shell method	**2**
• Attempt to use the shell method	**1**

Sample answer:

By shell Method

$$V = 2\pi \int_0^{\pi} x\sin x\,dx$$

$let\ u = x \qquad\qquad v' = \sin x$

$u' = 1$ $\qquad\qquad\qquad\qquad v = -cos\, x$

$I = uv - \int vu'$

$= 2\pi[-xcos\, x]_0^\pi - \int_0^\pi -\cos x\, dx$

$= 2\pi\, [\pi + (sin\, x)_0^\pi]$

$= 2\pi^2$

Question 15 (15 marks)

(a) (i) (1 mark)

Outcomes assessed: E9

Targeted Performance Bands: E3

Criteria	Marks
• Correctly establishes the result.	**1**

Sample answer:

$$\left(3+\sqrt{5}\right)+\left(3-\sqrt{5}\right)=6=2^1\times 3$$

$\therefore$ true $n=1$

$$\left(3+\sqrt{5}\right)^2+\left(3-\sqrt{5}\right)^2=9+6\sqrt{5}+5+9-6\sqrt{5}+5=28=2^2\times 7$$

$\therefore$ true $n=2$

15. (a) (ii) (3 marks)

Outcomes assessed: E2, E9

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correctly proof by mathematical induction.	**3**
• Makes two correct assumptions and attempts to prove for $n = k + 1$ or correctly uses one assumption	**2**
• Attempts to use one assumption	**1**

Sample answer:

Step 1: From (i) true for $n = 1$ and $n = 2$

Step 2: Let $n = k$ and $n = k - 1$ be values for which the statement is true

i.e $\left(3+\sqrt{5}\right)^k + \left(3-\sqrt{5}\right)^k = 2^k \times M$

and $\left(3+\sqrt{5}\right)^{k-1} + \left(3-\sqrt{5}\right)^{k-1} = 2^{k-1} \times N$

M and N are integers.

Step 3: Prove true for $n = k + 1$

$$\begin{aligned}
Now\ \left(3+\sqrt{5}\right)^{k+1} + \left(3-\sqrt{5}\right)^{k+1} &= p^{k+1} + q^{k+1}\ ,\ p = 3+\sqrt{5} \text{ and } q = 3-\sqrt{5} \\
&= p^k p + q^k q \\
&= \left[2^k \times M - q^k\right]p + \left[2^k \times M - p^k\right]q \text{ from step 2} \\
&= 2^k \times M\left(p+q\right) - pq\left(q^{k-1} + p^{k-1}\right) \\
&= 2^k \times 6M - 4\left[2^{k-1} \times N\right] \text{ from step 2} \\
&= 2^{k+1} \times 3M - 2^{k+1} \times N \\
&= 2^{k+1}\left[3M - N\right]
\end{aligned}$$

$\therefore$ True by mathematical induction

(b) (3 marks)

Outcomes assessed: E3

Targeted Performance Bands: E4

Criteria	Marks
• Finds correct locus with restriction.	**3**
• Substitutes $a^2 + b^2 = 1$	**2**
• Finds expression of z in terms of a and b	**1**

Sample answer:

$$\text{let } z = x + iy \text{ and } w = a + ib$$

$$\therefore x + iy = \frac{1 + a + ib}{1 - a - ib} \times \frac{1 - a + ib}{1 - a + ib}$$

$$= \frac{1 - a + ib + a - a^2 + iab + ib - iab - b^2}{(1-a)^2 + b^2}$$

$$= \frac{1 - a^2 - b^2 + 2ib}{(1-a)^2 + b^2}$$

$$= \frac{2ib}{(1-a)^2 + b^2}, \text{ since } |w| = 1 \rightarrow a^2 + b^2 = 1$$

$$\therefore x = 0$$

z is purely imaginary

$\therefore$ locus of z is the y – axis, excluding the origin.

15. (c) (i) (1 mark)

Outcomes assessed: E5

Targeted Performance Bands: E3

Criteria	Marks
• Correctly establishes the result.	**1**

Sample answer:

$$\downarrow mg \quad \uparrow R = \frac{7v^2}{10}$$

$$\begin{aligned} F &= m\ddot{x} \\ m\ddot{x} &= mg - R \\ &= mg - \frac{7v^2}{10} \\ \ddot{x} &= g - \frac{7v^2}{10m}, \ m = 7 \ and \ g = 10 \\ \therefore \ddot{x} &= 10 - \frac{v^2}{10} \end{aligned}$$

15. (c) (ii) (1 mark)

Outcomes assessed: E2

Targeted Performance Bands: E3

Criteria	Marks
• Correctly establishes the result.	**1**

Sample answer:

$$\ddot{x} = 10 - \frac{v^2}{10}$$

$$\ddot{x} \to 0$$

$$v^2 \to 100$$

$$\therefore \text{Terminal velocity is } v = 10m/s$$

15. (c) (iii) (3 marks)

Outcomes assessed: E5

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct answer	**3**
• Correct integration.	**2**
• Correctly uses $\ddot{x} = v \, {}^{dv}\!/_{dx}$ plus some further progress.	**1**

Sample answer:

$$\ddot{x} = 10 - \frac{v^2}{10}$$

$$v\frac{dv}{dx} = 10 - \frac{v^2}{10}$$

$$\frac{dv}{dx} = \frac{10}{v} - \frac{v}{10} = \frac{100 - v^2}{10v}$$

$$\frac{dx}{dv} = \frac{10v}{100 - v^2}$$

$$x = \int \frac{10v}{100 - v^2}\, dv$$

$$\therefore x = -5\ln\left(100 - v^2\right) + c$$

$$x = 0, v = 0$$
$$c = 5\ln(100)$$

$$\therefore x = 5\ln\left(\frac{100}{100 - v^2}\right)$$

$$e^{-x/5} = \frac{100 - v^2}{100}$$

$$\therefore v^2 = 100\left(1 - e^{-x/5}\right)$$

15. (c) (iv) (3 marks)

Outcomes assessed: E5

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct answer.	**3**
• Integrates with correct limits.	**2**
• Correctly recognises $\ddot{x} = {dV}/{dt}$ plus some further progress.	**1**

Sample answer:

$$\ddot{X} = -10 - \frac{V^2}{10}$$

$$\frac{dV}{dt} = -10 - \frac{V^2}{10}$$

$$\frac{dt}{dV} = \frac{-10}{100+V^2}, \text{ at maximum height } V = 0$$

$$\therefore t = -10 \int_{10/\sqrt{3}}^{0} \frac{dV}{100+V^2} = 10 \int_{0}^{10/\sqrt{3}} \frac{1}{100+V^2}\, dV$$

$$= \left[\tan^{-1} \frac{V}{10} \right]_0^{10/\sqrt{3}}$$

$$= \left[\left(\frac{\pi}{6} \right) - 0 \right]$$

$$\therefore t = \frac{\pi}{6} \text{ seconds}$$

Question 16 (15 marks)

(a) (i) (3 marks)

Outcomes assessed: E3

Targeted Performance Bands: E3-E4

Criteria	Marks
• Establish Correct result	**3**
• Expanding using Binomial Theorem	**2**
• Correct use of De Moivre's Theorem	**1**

Sample answer:

By De Moivre's Theorem

$$\cos 7\theta + i\sin 7\theta = \left(\cos\theta + i\sin\theta\right)^7$$

$$= \cos^7\theta\left(1+it\right)^7, \text{where } t = \tan\theta$$

Epanding by the Binomial Theorem

$$= \cos^7\theta\left(1 + {}^7C_1\left(it\right) - {}^7C_2t^2 - {}^7C_3(it^3) + {}^7C_4t^4 + {}^7C_5\left(it^5\right) - {}^7C_6t^6 - it^7\right)$$

equating real and imaginary parts

$$\cos 7\theta = \cos^7\theta\left(1 - {}^7C_2t^2 + {}^7C_4t^4 - {}^7C_6t^6\right)..........(1)$$

$$\sin 7\theta = \cos^7\theta\left({}^7C_1t - {}^7C_3t^3 + {}^7C_5t^5 - t^7\right)...........(2)$$

$(2) \div (1)$

Hence

$$\tan 7\theta = \frac{7t - 35t^3 + 21t^5 - t^7}{1 - 21t^2 + 35t^4 - 7t^6}$$

16. (a) (ii) (3 marks)

Outcomes assessed: E3

Targeted Performance Bands: E4

Criteria	Marks
• Establish Correct result	**3**
• Establish equation $x^3 - 21x^2 + 35x - 7 = 0$	**2**

• Find roots of $7t-35t^3+21t^5-t^7=0$	**1**

Sample answer:

Consider the equation

$\tan 7\theta = 0 \to 7\theta = n\pi$, for integer n.

If $\tan 7\theta = 0$, then

$7t-35t^3+21t^5-t^7=0$, when

$t=0, \tan\left(\frac{\pi}{7}\right), \tan\left(\frac{2\pi}{7}\right), \tan\left(\frac{3\pi}{7}\right), \tan\left(\frac{4\pi}{7}\right), \tan\left(\frac{5\pi}{7}\right)$ and $\tan\left(\frac{6\pi}{7}\right)$

Now divide by $-t$ we get $(t \neq 0)$

$t^6-21t^4+35t^2-7=0$

Let $t = x^2$

$x^3-21x^2+35x-7=0$

$\therefore x=\tan^2\left(\frac{\pi}{7}\right), \tan^2\left(\frac{2\pi}{7}\right)$ and $\tan^2\left(\frac{3\pi}{7}\right)$ are solutions of the equation.

16. (a) (iii) (2 marks)

Outcomes assessed: E4

Targeted Performance Bands: E3

Criteria	Marks
• Correctly establishes the result.	**2**
• Some progress towards the result.	**1**

Sample answer:

Let p, q and r be the roots of the equation

Now the required sum is

$$(1+p)^2+(1+q)^2+(1+r)^2 = 3+2(p+q+r)+p^2+q^2+r^2$$
$$=3+2(p+q+r)+(p+q+r)^2-2(pq+rq+pq)$$
$$=3+42+441-70$$
$$=416$$

16. (b) (i) (2 marks)

Outcomes assessed: E9

Targeted Performance Bands: E3

Criteria	Marks
• Correctly establishes the result.	**2**
• Some progress towards the result.	**1**

Sample answer:

$$S_n = 1 + x + x^2 + ... + x^n$$

$$xS_n = x + x^2 + x^3 + ... + x^{n+1}$$

$$S_n(1-x) = 1 - x^{n+1}$$

$$\therefore S_n = \frac{1-x^{n+1}}{1-x}$$

i.e $1 + x + x^2 + ... + x^{n-1} + x^n = \dfrac{1-x^{n+1}}{1-x}$

$$1 + x + x^2 + ... + x^{n-1} = \frac{1-x^{n+1}}{1-x} - x^n$$

$$= \frac{1 - x^{n+1} - x^n + x^{n+1}}{1-x}$$

$$= \frac{1-x^n}{1-x} = \frac{1}{1-x} - \frac{x^n}{1-x}$$

thus $\dfrac{1}{1-x} = 1 + x + x^2 + ... + x^{n-1} + \dfrac{x^n}{1-x}$

16. (b) (ii) (3 marks)

Outcomes assessed: E8

Targeted Performance Bands: E4

Criteria	Marks
• Derives correct result	**3**
• Integrates with correct limits	**2**
• Attempts to integrate both sides after making substitution	**1**

Sample answer:

Let $x = -t$, then

$$\frac{1}{1+t} = 1 + (-t) + (-t)^2 + ... + (-t)^{n-1} + \frac{(-t)^n}{1+t}$$

Integrating both sides

$$\int_0^x \frac{1}{1+t} dt = \int_0^x \left(1 - t + t^2 - ...... + (-t)^{n-1} + \frac{(-t)^n}{1+t}\right) dt$$

$$\left[\log_e(1+t)\right]_0^x = \left[t - \frac{t^2}{2} + \frac{t^3}{3} - ......(-1)^{n-1}\frac{t^n}{n}\right]_0^x + (-1)^n \int_0^x \frac{t^n}{1+t} dt$$

$$\therefore \log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ......(-1)^{n-1}\frac{x^n}{n} + T_n(x)$$

where $T_n(x) = (-1)^n \int_0^x \frac{t^n}{1+t}$

16. (b) (iii) (1 marks)

Outcomes assessed: E8

Targeted Performance Bands: E4

Criteria	Marks
• Correctly establishes the result.	**1**

Sample answer:

$$\left|T_n(x)\right| = \int_0^x \frac{t^n}{1+t} dt \le \int_0^x t^n\, dt = \frac{x^{n+1}}{n+1} \to 0 \text{ as } n \to \infty \text{ and } 0 \le x \le 1$$

16. (b) (iv) (1 marks)

Outcomes assessed: E9

Targeted Performance Bands: E3

Criteria	Marks
• Correct answer	**1**

Sample answer:

Let $x = 1$ in (ii)

$$\therefore \log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}......$$